Test # 4 Review Pg. 317 - Question 1 d.

1.

d. $\sum_{n=0}^{\infty} \frac{n(n+1)(x-2)^n}{16^n} (x-2)^n$

$= \sum_{n=0}^{\infty} \frac{n(n+1)(x-2)^n}{16^n} = \sum_{n=0}^{\infty} a_n$

$a_n = \frac{n(n+1)(x-2)^n}{16^n}$

$a_{n+1} = \frac{(n+1)(n+2)(x-2)^{n+1}}{16^{n+1}}$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(n+1)(n+2)(x-2)^{n+1}}{16^{n+1}} \cdot \frac{16^n}{n(n+1)(x-2)^n}\right|$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(n+1)(n+2)(x-2)^{n+1-n}}{16^{n+1-n}} \cdot \frac{1}{n(n+1)}\right|$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{n(1+\frac{1}{n})n(1+\frac{2}{n})(x-2)^1}{16^1} \cdot \frac{1}{n^2(1+\frac{1}{n})}\right|$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(1+\frac{1}{n})(1+\frac{2}{n})(x-2)}{16} \cdot \frac{1}{(1+\frac{1}{n})}\right|$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(1+\frac{1}{\infty})(1+\frac{2}{\infty})(x-2)}{16} \cdot \frac{1}{(1+\frac{1}{\infty})}\right|$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(1+0)(1+0)(x-2)}{16} \cdot \frac{1}{(1+0)}\right|$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x-2}{16}\right|$

$= \left|\frac{x-2}{16}\right| < 1$

$-1 < \frac{x-2}{16} < 1$

$-16 + 2 < x 16 < +2$

$-14 < x < 18$

Open interval of convergence:

$\boxed{(-14, 18)}$

Radius of convergence:

$R = \frac{18-(-14)}{2}$

$R = \frac{32}{2}$

$\boxed{R = 16}$