Polynomial Modeling Writing Task: Using Division and Factoring in Real-World Applications

In this assignment, you will apply polynomial concepts to model and solve a real-world scenario involving the design of a container. You'll use polynomial division, factoring techniques, and your understanding of polynomial behaviors to find optimal dimensions.

The Assignment

A manufacturing company needs to design a rectangular box with an open top. The box will be constructed by cutting equal-sized squares from each corner of a rectangular sheet of material and folding up the sides.

Part 1: Developing the Polynomial Model

The company starts with a rectangular sheet that is 4 feet longer than it is wide. After cutting 2-foot squares from each corner, the resulting box must have a volume of 160 cubic feet.

1. Let x represent the width of the original sheet in feet. Express the length of the original sheet in terms of x.
2. After cutting out the 2-foot squares from each corner, what are the dimensions of the resulting box? Express these in terms of x.
3. Write a polynomial equation for the volume of the box in terms of x.
4. Use the given volume requirement to write a polynomial equation that can be solved to find the possible values of x.

Part 2: Using Polynomial Division and Factoring

5. Use polynomial division or synthetic division to factor your polynomial equation from Part 1.
6. Find all possible values of x that satisfy your equation. Show all your work.
7. For each possible value of x, calculate the corresponding dimensions of the original sheet and the final box.
8. Determine which solution is physically meaningful in this context and explain why.

Part 3: Polynomial Behavior Analysis

9. Graph your polynomial function from Part 1. Clearly mark all x-intercepts, y-intercept, and any turning points.
10. Analyze the end behavior of your polynomial function and explain how it relates to the physical constraints of this problem.
11. How does the multiplicity of any zeros in your polynomial relate to the behavior of the graph at those points?
12. If the company wanted to maximize the surface area of the box while maintaining the 160 cubic feet volume, how would this change your approach to the problem?

Part 4: Application and Reflection

13. In manufacturing, tolerances are important. If the width of the original sheet can vary by ±0.25 feet from your solution, what would be the range of possible volumes for the resulting box?
14. Write a brief reflection (1-2 paragraphs) about how polynomial division and factoring were essential to solving this problem. Could you have solved it using other methods?
15. Describe another real-world scenario where polynomial modeling, division, and factoring would be useful tools for finding a solution.

Your submission should include:

- Clear mathematical work showing how you developed and solved the polynomial equation
- Step-by-step factoring work using polynomial or synthetic division
- A well-labeled graph of your polynomial function
- Thorough explanations that connect the mathematics to the physical constraints of the problem
- Proper mathematical notation and units throughout

Remember to:

- Show all your work for calculations
- Clearly explain what each variable and term represents in the context of the problem
- Justify why some solutions may be rejected based on physical constraints
- Use appropriate units (feet, square feet, cubic feet) in your answers

This assignment is worth 20 points. Your work will be assessed on mathematical accuracy, proper application of polynomial division and factoring techniques, clear explanations of your process, and the correctness of your final answers.

Rubric

Criteria	Proficient	Developing	Not Evident	Points
Polynomial Modeling	Correctly develops the polynomial equation representing the volume constraint. Variables and terms are accurately defined in context. All work is clearly shown with proper notation.	Develops the polynomial equation with minor errors. Most variables and terms are defined in context. Some work shown but may lack clarity.	Significant errors in developing the polynomial equation. Variables and terms poorly defined or missing context. Little or no work shown.	___/5
Division and Factoring	Polynomial division is performed correctly with all steps shown. All solutions are found and verified. Clear distinction between meaningful and extraneous solutions with proper justification.	Division contains minor errors or incomplete steps. Most solutions are found with some verification. Some attempt to distinguish between meaningful and extraneous solutions.	Major errors in division or factoring. Missing solutions or incorrect verification. No meaningful distinction between valid and invalid solutions.	___/6
Polynomial Behavior Analysis	Graph is accurate with all key features properly labeled. Thorough analysis of end behavior and zeros as they relate to the context. Insightful connections between mathematical properties and physical constraints.	Graph contains most key features with minor labeling issues. Basic analysis of end behavior and zeros with some connection to context. Some connections between mathematical properties and physical constraints.	Graph is missing or severely inaccurate. Limited or incorrect analysis of end behavior and zeros. Few or no connections between mathematical properties and physical constraints.	___/5

Application and Reflection	Thoroughly addresses tolerance analysis with accurate calculations. Reflection demonstrates deep understanding of polynomial methods and their applications. Creative and realistic alternative scenario provided.	Tolerance analysis contains minor computational errors. Reflection shows basic understanding of polynomial methods. Alternative scenario is reasonable but may lack detail.	Tolerance analysis missing or contains major errors. Reflection is superficial or missing. Alternative scenario is unrealistic or missing.	___/4
Total				___/20