Collatz Conjecture solution

The Conjecture

n – natural number

- If n is even, divide it by 2
- If n is odd, multiply it by 3 and add 1

Example of n=96: 96 -> 48 -> 24 -> 12 -> 6 -> 3 -> 10 -> 5 -> 16 -> 8 -> 4 -> 2 -> 1

Conjecture states that any natural number after repeating this process indefinitely, will reach 1.

Terms that I'll use in my solution:

Path - The process of going from a natural number to 1 by following the steps of the Collatz conjecture.(or the steps of a different conjecture in general)

Connects – leads to another number by following the flipped (inverted as some call it) conjecture steps.

Line – all numbers of a group of numbers ordered from smallest to largest.(For odd numbers that are connected to a number we don't count those odd numbers that the first odd numbers connect to (for simplicity's sake). For example if 5 connects to 3 and 1 connects to 5, that means that 1 connects to just 5 and not to 5 and 3)

Solution:

1. **Structure.**

   1.1. **Flipping the conjecture.**

   You can flip (invert as some call it) the conjecture to go from 1 to any natural number (What I'll prove).

   Start with 1 as n.

   - If n is even do one of the following steps
     - multiply it by 2, this gives you an even number.
     - In some cases, you can **subtract 1 and divide by 3** to get an **odd number**. This works only if the result is a whole number. This is for the numbers 4; 10; 16; 22 and so on. The result is always an odd number, because to divide a number by 3 it has to be 3s, where s is a whole number. To get 3s by removing 1 from an even number, that even number has to be 3s+1. 3s+1 is only even if s is an odd number. If it would be even, then 3s+1 would be odd, not even, because an odd times an even results in an even number, then adding 1 to the even number makes it an odd number. s can be odd, because an odd multiplied by an odd results in an odd number and adding 1 to an odd results in an even number. After removing 1 from 3s+1 and then dividing by 3 you get s, an odd number((3s+1-1)/3=3s/3=s.

       I'll call this step -1)/3
   - If n is odd, multiply it by 2, this gives you an even number.

   Example (starting from 1 to reach 168): 1 -> 2 -> 4 -> 8 -> 16 -> 32 -> 64 -> 21 -> 42 -> 84 -> 168

If the flipped conjecture allows us to go from 1 to any natural number, then the Collatz conjecture is solved, since we can just flip it back to normal. We can do this, because all that flipping the conjecture is, is that the paths will go the opposite direction as the not flipped conjecture.

1.2. **Explaining why even numbers are connected with odd numbers.**

Any even number can be reached by doubling an odd number once or multiple times depending on the even number we want to reach.

For example: To get 6, we need to double 3 once, but to get 4, we need to double 1 twice.

1.3. **Dividing the odd numbers into 3 groups.**

All odd numbers fall into 3 groups.

- 6m-3 (nowhere group)
- 6m-1 (backwards group)
- 6m+1 and 1 (forward group)

m - natural numbers

I'll be using this grouping method for my solution further on.

1.4. **Explaining how odd numbers connect to other odd numbers and why.**

This is how odd numbers are connected to more odd numbers:

1 -> 1; 5; 21; 85; 341; 1365; 5461…

Explanation: 1 -> 2 -> 4 -> 8 -> 16 -> 32 -> 64 -> 128 -> 256 -> 512 -> 1024 -> 2048 -> 4096 -> 8192 -> 16384

(4-1)/3=1; (16-1)/3=5; (64-1)/3=21; (256-1)/3=85; (1024-1)/3=341; (4096-1)/3=1365; (16384-1)/3=5461

5 -> 3; 13; 53; 213; 853; 3413; 13653…

3 -> Nothing

13 -> 17; 69; 277; 1109; 4437; 17749; 70997…

17 -> 11; 45; 181; 725; 2901; 11605; 46421…

…

Nowhere group doesn't make new connections, even after multiplying by 2 however many times, subtracting 1 makes them of the form 3s + 2, which is **not divisible by 3**.

The backwards group makes connections with an odd number that's smaller than the backwards groups' number by 2m and to numbers that are 4× larger +1 than the previous number.

Backwards group makes connections with an odd number that's smaller than the backwards groups' number by 2m, because:

The backwards group consists of only 6m-1. 6m after getting multiplied by 2 however many times will always be able to divide by 3, because $(2^n \times 6m)/3 = 2^n \times 2m$. But the -1 after getting multiplied by 2 any amount of times has to be reduced by 1 and that leaves us with only doubling the -1 an odd amount of times, because to divide something with 3 and get a natural number, the number you're dividing has to be divisible by 3, in other words those numbers have to equal 3s, where s is a whole number. Adding a -1 to the -1 multiplied by 2 any amount of times is the same as $-2^n - 1 = -(2^n + 1)$. $2^n + 1$, which equals 3s when $2^n = 3s + 2$, to find out which n values give such a result, we need to know how the result changes depending on n:

Starting with n=1

1. $2^n = 3s + 2$
2. $(3s + 2) \times 2 = 3s + 1$

   3s after multiplied by 2 is still a 3s number, because s is a whole number and after multiplying a whole number it turns into another whole number. The 2 after multiplied by 2 becomes a 4, which is 3s+1

3. $(3s + 1) \times 2 = 3s + 2$

After doubling $3s + 1$ we get $3s + 2$

After doubling $3s + 2$ we get $3s + 1$

It's a loop, where you get 3s+2 when n is an odd natural number and you get 3s+1 when n is an even natural number.

Therefore, only the even numbers that were gotten through an odd amount of times of doubling a number from the backwards group are connected to different odd numbers.

To get the first number that the backwards group connects to, we need to double it once.

$\frac{2(6m-1)-1}{3} = \frac{12m-3}{3} = 4m - 1 = 6m - 1 - 2m$

Side note: To get the next number that can -1)/3 in the line of even numbers connected to the number from the backwards group, we need to double and then double again (quadruple) the known number that can -1)/3, because the difference in the line of odd numbers (1; 3; 5; 7; 9) is 2.

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backwards group makes connections with odd numbers that are 4× larger +1 than the previous odd number connected to in the line of odd numbers that the backwards group connects to, because, let's say that the previous number is w, to get back in the line of even numbers that the backwards group connects to, we need to reverse the step taken to get w (multiply w by 3 and add 1). Then to get the next even number that can be -1)/3, we need to multiply 3w+1 by 4, as explained in the side note. Then to get the odd number that the even number connects to, we need to -1)/3 the 4(3w+1)=12w+4. We start from w and end up with (12w+4-1)/3=(12w+3)/3=4w+1, which is 4× larger +1 than w.

The forward group makes connections with an odd number that's larger than the forward groups' number by 2m and to numbers that are 4× larger +1 than the previous number.

Forward group makes connections with an odd number that's larger than the forward groups' number by 2m, because:

The forward group consists of only 6m-5. 6m after getting multiplied by 2 however many times will always be able to divide by 3, because $(2^n \times 6m)/3 = 2^n \times 2m$. But the +1 after getting multiplied by 2 any amount of times has to be reduced by 1, that leaves us with only doubling the +1 an even natural amount of times, because to divide something with 3 and get a natural number, we need to get 3s, adding a -1 to the +1 multiplied by 2 any amount of times is the same as $2^n - 1$, it needs to equal 3s —> $2^n - 1 = 3s$, we can rewrite it as $2^n = 3s + 1$

To find out which n values give such a result, we will take the previous results, which are:

After doubling $3s + 1$ we get $3s + 2$

After doubling $3s + 2$ we get $3s + 1$

It's a loop, where you get 3s+2 when n is an odd natural number and <u>you get 3s+1 when n is an even natural number</u>.

Therefore, only the even numbers that were gotten through an even amount of times of doubling a number from the backwards group are connected to different odd numbers.

To get the first number that the forward group connects to, we need to double it twice.

$\frac{4(6m+1)-1}{3} = \frac{24m+3}{3} = 8m + 1 = 6m + 1 + 2m$

Side note: To get the next number that can be -1)/3 in the line of even numbers connected to the number from the forward group, we need to double and then double again (quadruple) the known number that can get -1)/3, because the difference in the line of even numbers (2; 4; 6; 8; 10…) is 2.

Forward group makes connections with numbers that are 4× larger +1 than the previous number in the line of numbers that the forward group connects to, because, let's say that the previous number is w, to get back in the line of even numbers that the forward group connects to, we need to reverse the step taken to get w (multiply w by 3 then add 1). Then to get the next even number that can be -1)/3, we need to multiply 3w+1 by 4, as explained in the side note. Then to get the odd number that the even number connects to, we need to -1)/3 the 4(3w+1)=12w+4. We start from w and end up with (12w+4-1)/3=(12w+3)/3=4w+1, which is 4× larger +1 than w.

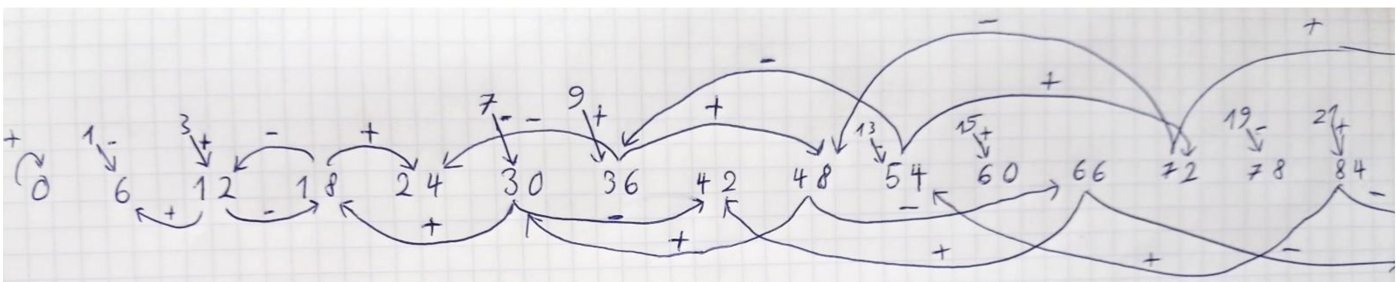
The numbers from the backwards and forward groups connect to all groups of numbers (nowhere, forward, backwards in that order starting from either one), because:

$(6m_q - 1) \times 4 + 1 = 24m_q - 3 = 6m_{q\times4} - 3 = 6m - 3$ and
$(6m_q + 1) \times 4 + 1 = 24m_q + 5 = 6m_{q\times4+1} - 1 = 6m - 1,$ and
$(6m_q - 3) \times 4 + 1 = 24m_q - 11 = 12m_q + 1 = 6m_{q\times2} + 1.$

q represents how big m is.

1.5. How backwards and forward group numbers connect and why.

These numbers are connected to each other in this way:



Explanation: The – on the arrows indicates that the number that is pointed towards is lower by 1 and the + on the arrows indicates that the number that is pointed towards is larger by 1.

The rest in this step does not rely on these observations. These observations are here to make it easier for you to understand how backwards and forward group numbers connect and not to understand why.

The numbers that 6m-1 connects to alter in a +; -; +; -; +; -… way, because 6m-1-2m=4m–1, which is a:

- Forward groups' number if m = 2; 5; 8; 11…, because 4m - 1 can equal a 6m+1 of a different m value, we can rewrite it as $4m_1 = 6m_2 + 2$
- Backwards groups' number if m = 3; 6; 9; 12…, because 4m - 1 can equal a 6m - 1 of a different value, we can rewrite it as $4m_1 = 6m_2$
- Nowhere groups' number if m=1; 4; 7; 10; 13…, because 4m - 1 can equal a 6m + 3 of a different value, we can rewrite it as $4m_1 = 6m_2 + 4$.

To find out which m_1 values give such results, we need to know how the result changes depending on the m_1 value:

Starting from m_1=1, I'll show the change of $4m_1$ by adding 4 to the next, since 4m increases by 4 for every 1 that the m is increased by.

1. $6m_2 + 4 = 6m_2 + 4$
2. $6m_2 + 4 + 4 = 6m_2 + 2$

Because, if you add 6 to $6m_2$, we can increase m_2 by 1 instead.

3. $6m_2 + 2 + 4 = 6m_2$
4. $6m_2 + 4 = 6m_2 + 4$

After adding 4 to $6m_2$ you get $6m_2 + 4$

After adding 4 to $6m_2 + 4$ you get $6m_2 + 2$

After adding 4 to $6m_2 + 2$ you get $6m_2$

It's a loop, where: the result is $6m_2 + 4$ when m=1; 4; 7; 10… and the result is $6m_2 + 2$ when m=2; 5; 8; 11…, and the result is $6m_2$ when m=3; 6; 9; 12….

There are **no gaps** made by the 6m numbers that connect to the nowhere group, because the 2m number increases by 2 for every 1 added to m, so after every third 6m number the 2m increases by 6 ($2 \times 3 = 6$) and when connecting skips 1 more number further back in the line.

The numbers that the 6m+1 connects to, connect in a -; +; 2 gaps; -; +; 2 gaps… way, because 6m+1+2m=8m+1, which is a:

- Forward groups' number when m = 0; 3; 6; 9; …, because 8m + 1 can equal a 6m + 1 of a different m value, we can rewrite it as $8m_1 = 6m_2$
- Backwards groups' number when m = 2; 5; 8; 11…, because 8m + 1 can equal a 6m -1 of a different m value, we can rewrite it as $8m_1 = 6m_2 - 2$, we can rewrite that as $8m_1 = 6m_2 + 4$
- Nowhere groups' number when m = 1; 4; 7; 10; 13…, because 8m + 1 can equal a 6m + 3 of different m value, we can rewrite it as $8m_1 = 6m_2 + 2$.

To find out which m_1 gives such results, we need to know how the result changes depending on m_1:

Starting from m_1=0, I'll show the change of m_1 by adding 8 to the next, since 8m increases by 8 for every 1 that the m is increased by.

0. $0 = 6m_2$
1. $6m_2 + 8 = 6m_2 + 2$

   because if you add 6 to $6m_2$, we can increase m_2 by 1 instead.(8-6=2)

2. $6m_2 + 2 + 8 = 6m_2 + 4$
3. $6m_2 + 4 + 8 = 6m_2$

After adding 8 to $6m_2$ you get $6m_2 + 2$

After adding 8 to $6m_2 + 2$ you get $6m_2 + 4$

After adding 8 to $6m_2 + 4$ you get $6m_2$

It's a loop, where the result is $6m_2 + 2$ when m=1; 4; 7; 10… and the result is $6m_2 + 4$ when m=2; 5; 8; 11…, and the result is $6m_2$ when m=0; 3; 6; 9…

There is a gap made by the 6m numbers that connect to the nowhere group and another gap made by the difference increase: after every third 6m number the 2m increases by 6 ($2 \times 3 = 6$) and when connecting skips 1 more number further forward in the line, because the 2m number increases by 2 for every 1 added to m.

The gaps are located on the 6m line neighboring each other. We'll call the smaller 6m of the 2 the first gap and the larger one - the second gap. The gaps are located where m=1; 2; 5; 6; 9; 10; 13; 14….

Explanation: 6+1 connects to a number from the nowhere group and 12+1 connects to 18-1, which is $6 \times 3 - 1$ and not $6 \times 2 \pm 1$or $6 \times 1 \pm 1$, so the first 2 numbers in the 6m line are gaps. Every fourth number after the first and the second will also be gaps, because the loop consists of 3 values ($6m_2 + 2$; $6m_2 + 4$; $6m_2$), 1 of the values is the connection to the nowhere group creating a gap and the difference increase applies to the loop creating the other gap.

First gap is filled with $\frac{6m-2}{4}$, because the number in the first gap already has a + input from the backwards group, it needs a – input and to get 6m-1 when m=1; 5; 9; 13… we can lower it by 1 and then divide it by 4 to find the number that connects to it.

$\frac{6m-1-1}{4} = \frac{6m-2}{4}$, this number is divisible by 4 when m=1; 5; 9; 13…, because $\frac{6\times1-2}{4} = \frac{4}{4}$, which divides. The other numbers will also divide, because the difference in the line of m numbers in the first gap is 4 and adding 4 more sixes to the side that gets divided by 4, doesn't prohibit the division, because 4 sixes equal to 24 and 24 is divisible by 4.

Second gap is filled with $\frac{6m}{4}$, because the number in the second gap already has a – input from the backwards group, it needs a + input and to get 6m+1 when m=2; 6; 10; 14… we can lower it by 1 and then divide it by 4 to find the number that connects to it.

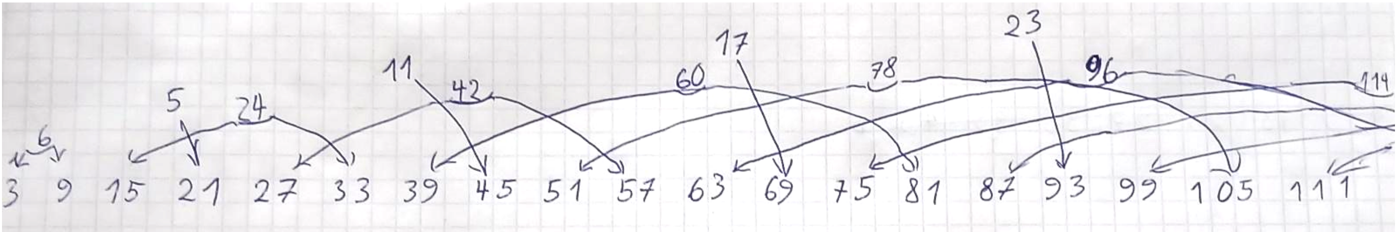
$\frac{6m+1-1}{4} = \frac{6m}{4}$, this number is divisible by 4 when m=2; 6; 10; 14…, because $\frac{6\times 2}{4} = \frac{12}{4}$, which divides. The other numbers will also divide, because the difference in the line of m numbers in the second gaps is 4 and adding 4 more sixes to the side that gets divided by 4, doesn't prohibit the division, because 4 sixes equal to 24 and 24 is divisible by 4.

The backwards and forward group are fully connected, because numbers from the backwards group connect to every 6m number in a +; -; +; -; +; - way starting from m=1 and the numbers from the forward group connect to a half of the 6m numbers in a -; +; 2 gaps; -; +; 2 gaps… way starting from m=1. The gaps are filled in a -; + way. And because of that, every 6m number is connected to by a - and a +, meaning that every number from backwards and forward group is fully connected without any skipped numbers.

This covers not just the first few numbers, but all numbers, because the results of used functions in this part of the solution depend on the m value. The m value can be any natural number, so it spans out to infinity and not to only the first few natural numbers.

1.6. **How the nowhere group is connected by the backwards and forward group numbers and why.**

The nowhere group is connected to the odd numbers in this way:



The rest in this step does not rely on these observations. These observations are here to make it easier for you to understand how the nowhere group is connected and not to understand why.

Every 6m, where m=1; 4; 7; 10; 13 … connects to 2 nowhere group numbers:
Connects to a number that is smaller than the 6m number by 2m+1, because $6m - 1 \rightarrow 6m - 1 - 2m$
Connects to a number that is larger than the 6m number by 2m+1, because $6m + 1 \rightarrow 6m + 1 + 2m$
This covers every nowhere group number except for every fourth number, because:

6m, where m=1; 4; 7; 10; 13…, connects to the first and every second number after the first in the nowhere group line when connecting to a smaller number than the 6m number.(The smaller numbers than the 6m numbers are connected by those numbers because of reasons mentioned in the 5th part of the solution)

6m, where m=1; 4; 7; 10; 13…, connects to the second number and every fourth number after the second in the nowhere group line when connecting to a larger number than the 6m number. After every third 6m number the 2m increases by 6 ($2 \times 3 = 6$), therefore skipping 1 more number further forward in the line of the nowhere group numbers.(The larger numbers than the 6m numbers are connected by those numbers because of reasons mentioned in the 5th part of the solution)

Every fourth number is skipped by the 6m numbers.

Every fourth number in the line of nowhere group numbers is connected to by (6m-3-1)/4, where m=4; 8; 12; 16…, because every odd number connects to odd numbers that are 4× larger +1 than them.(As shown in the 4th part of my solution)

6m-3-1, where m=4; 8; 12; … is divisible by 4, because -3-1=-4 which divides by 4 and 6 when multiplied by numbers that can be divided by 4 (4; 8; 12; 16…) can also be divided by 4.

Nowhere group is fully connected without any skipped numbers, because 3/4 of them are connected by 6m numbers, where m=1; 4; 7; 10; 13…, and 1/4 of them are connected by the numbers that are 4× smaller -1 than the numbers of the nowhere group that they connect to.

1.7. **Every number can be connected by only 1 other number.**

2 different numbers can't connect to the same number by doubling. $a \neq b \Rightarrow 2a \neq 2b$

2 different numbers can't connect to the same number by -1)/3. $a \neq b \Rightarrow (a - 1)/3 \neq (b - 1)/3$

2 different numbers can't connect to the same number by 1 of them doubling and the other -1)/3. From doubling we get an even number and from -1)/3 we get an odd number. There are no natural numbers both even and odd at the same time.

Those are the only possible scenarios while following the flipped rules.

2. **There are no numbers reached by infinity.(No numbers diverging into infinity in regular collatz)**

For a number to be connected by infinity, there would have to be a string of backwards and/or forward group numbers that are reached by bigger numbers than them. At times there…

Working on this part

3. **There isn't a different number that creates a loop.**

Working on this part

4. **For the Collatz conjecture to be true there can't be any number that increases to infinity and there cannot be any loops. The conjecture can't be disproven by anything else.**

Working on this part

The Collatz Conjecture is true - every natural number leads to 1 after repeating the steps of the conjecture.