See this page in the course material.

Learning Outcomes

- Simplify exponential expressions containing negative exponents
- Simplify exponential expressions containing exponents of 0 and 1

We previously saw that the Quotient Property of Exponents has two forms depending on whether the exponent in the numerator or denominator was larger.

Quotient Property of Exponents

If [latex]a[/latex] is a real number, [latex]a\ne 0[/latex], and [latex]m,n[/latex] are whole numbers, then

[latex]{\Large\frac{{a}^{m}}{{a}^{n}}}={a}^{m-n},m>n\text{ and }{\Large\frac{{a}^{m}}{{a}^{n}}}={\Large\frac{1}{{a}^{n-m}}},n>m[/latex]

Define and use the negative exponent rule

We now propose another question about exponents. Given a quotient like [latex] \displaystyle \frac{{{2}^{m}}}{{{2}^{n}}}[/latex] what happens when *n* is larger than *m*? We will need to use the *negative rule of exponents* to simplify the expression so that it is easier to understand.

Let’s look at an example to clarify this idea. Given the expression:

[latex]\frac{{h}^{3}}{{h}^{5}}[/latex]

Expand the numerator and denominator, all the terms in the numerator will cancel to [latex]1[/latex], leaving two *h*s multiplied in the denominator, and a numerator of [latex]1[/latex].

[latex]\begin{array}{l}
\frac{{h}^{3}}{{h}^{5}}\,\,\,=\,\,\,\frac{h\cdot{h}\cdot{h}}{h\cdot{h}\cdot{h}\cdot{h}\cdot{h}} \\
\,\,\,\,\,\,\,\,\,\,\,=\,\,\,\frac{\cancel{h}\cdot \cancel{h}\cdot \cancel{h}}{\cancel{h}\cdot \cancel{h}\cdot \cancel{h}\cdot {h}\cdot

{h}}\\\,\,\,\,\,\,\,\,\,\,\,=\,\,\,\frac{1}{h\cdot{h}}\\\,\,\,\,\,\,\,\,\,\,=\,\,\,\frac{1}{{h}^{2}} \end{array}[/latex]
We could have also applied the quotient rule from the last section, to obtain the following result:

[latex]\begin{array}{r}\frac{h^{3}}{h^{5}}\,\,\,=\,\,\,h^{3-5}\\\\=\,\,\,h^{-2}\,\,\end{array}[/latex]

Putting the answers together, we have [latex]{h}^{-2}=\frac{1}{{h}^{2}}[/latex]. This is true when *h*, or any variable, is a real number and is not zero.

The Negative Rule of Exponents

For any nonzero real number [latex]a[/latex] and natural number [latex]n[/latex], the negative rule of exponents states that

[latex]{a}^{-n}=\frac{1}{{a}^{n}}[/latex]
Now that we have defined negative exponents, the Quotient Property of Exponents needs only one form, [latex]{\Large\frac{{a}^{m}}{{a}^{n}}}={a}^{m-n}[/latex], where [latex]a\ne 0[/latex] and *m* and *n* are integers.

When the exponent in the denominator is larger than the exponent in the numerator, the exponent of the quotient will be negative. If the result gives us a negative exponent, the negative exponent tells us to re-write the expression by taking the reciprocal of the base and then changing the sign of the exponent. We rewrite it by using the definition of negative exponents, [latex]{a}^{-n}={\Large\frac{1}{{a}^{n}}}[/latex]. Any expression that has negative exponents is not considered to be in simplest form, so we will use the definition of a negative exponent and other properties of exponents to write an expression with only positive exponents.

Example

Evaluate the expression [latex]{4}^{-3}[/latex].

Show Solution

First, write the expression with positive exponents by putting the term with the negative exponent in the denominator.

[latex]{4}^{-3} = \frac{1}{{4}^{3}} = \frac{1}{4\cdot4\cdot4}[/latex]

Now that we have an expression that looks somewhat familiar.

[latex]\frac{1}{4\cdot4\cdot4} = \frac{1}{64}[/latex]

Answer

[latex]\frac{1}{64}[/latex]

example

Simplify:

1. [latex]{4}^{-2}[/latex]
2. [latex]{x}^{-3}[/latex]

Solution

1.	
	[latex]{4}^{-2}[/latex]
Use the definition of a negative exponent, [latex]{a}^{-n}={\Large\frac{1}{{a}^{n}}}[/latex].	[latex]{\Large\frac{1}{{4}^{2}}}[/latex]
Simplify.	[latex]{\Large\frac{1}{16}}[/latex]

2.	
	[latex]{x}^{-3}[/latex]
Use the definition of a negative exponent, [latex]{a}^{-n}={\Large\frac{1}{{a}^{n}}}[/latex].	[latex]{\Large\frac{1}{{x}^{3}}}[/latex]

try it

See this interactive in the course material.

try it

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When simplifying any expression with exponents, we must be careful to correctly identify the base that is raised to each exponent.

example

Simplify:

1. [latex]{\left(-3\right)}^{-2}[/latex]

2 [latex]{-3}^{-2}[/latex]

Show Solution

Solution

The negative in the exponent does not affect the sign of the base.

1.	
The exponent applies to the base, [latex]-3[/latex] .	[latex]{\left(-3\right)}^{-2}[/latex]
Take the reciprocal of the base and change the sign of the exponent.	[latex]{\Large\frac{1}{{\left(-3\right)}^{2}}}[/latex]
Simplify.	[latex]{\Large\frac{1}{9}}[/latex]

2.	
The expression [latex]-{3}^{-2}[/latex] means: find the opposite of [latex]{3}^{-2}[/latex] The exponent applies only to the base, [latex]3[/latex].	[latex]-{3}^{-2}[/latex]
Rewrite as a product with [latex]−1[/latex].	[latex]-1\cdot {3}^{-2}[/latex]
Take the reciprocal of the base and change the sign of the exponent.	[latex]-1\cdot {\Large\frac{1}{{3}^{2}}}[/latex]
Simplify.	[latex]-{\Large\frac{1}{9}}[/latex]

try it

See this interactive in the course material.

We must be careful to follow the order of operations. In the next example, parts 1 and 2 look similar, but we get different results.

example

Simplify:

1. [latex]4\cdot {2}^{-1}[/latex]

2. [latex]{\left(4\cdot 2\right)}^{-1}[/latex]

Show Solution

Solution

Remember to always follow the order of operations.

1.	
Do exponents before multiplication.	[latex]4\cdot {2}^{-1}[/latex]
Use [latex]{a}^{-n}={\Large\frac{1}{{a}^{n}}}[/latex].	[latex]4\cdot {\Large\frac{1}{{2}^{1}}}[/latex]
Simplify.	[latex]2[/latex]

2.	[latex]{\left(4\cdot 2\right)}^{-1}[/latex]
Simplify inside the parentheses first.	[latex]{\left(8\right)}^{-1}[/latex]

Use [latex]{a}^{-n}={\Large\frac{1}{{a}^{n}}}[/latex].	[latex]{\Large\frac{1}{{8}^{1}}}[/latex]
Simplify.	[latex]{\Large\frac{1}{8}}[/latex]

try it

See this interactive in the course material.

When there is a product and an exponent we have to be careful to apply the exponent to the correct quantity. According to the order of operations, expressions in parentheses are simplified before exponents are applied. We'll see how this works in the next example.

example

Simplify:

1. [latex]5{y}^{-1}[/latex]
2. [latex]{\left(5y\right)}^{-1}[/latex]
3. [latex]{\left(-5y\right)}^{-1}[/latex]

Show Solution

Solution

1.	
Notice the exponent applies to just the base [latex]y[/latex] .	[latex]5{y}^{-1}[/latex]
Take the reciprocal of [latex]y[/latex] and change the sign of the exponent.	[latex]5\cdot {\Large\frac{1}{{y}^{1}}}[/latex]
Simplify.	[latex]{\Large\frac{5}{y}}[/latex]

2.	
Here the parentheses make the exponent apply to the base [latex]5y[/latex] .	[latex]{\left(5y\right)}^{-1}[/latex]
Take the reciprocal of [latex]5y[/latex] and change the sign of the exponent.	[latex]{\Large\frac{1}{{\left(5y\right)}^{1}}}[/latex]
Simplify.	[latex]{\Large\frac{1}{5y}}[/latex]

3.	
	[latex]{\left(-5y\right)}^{-1}[/latex]
The base is [latex]-5y[/latex] . Take the reciprocal of [latex]-5y[/latex] and change the sign of the exponent.	[latex]{\Large\frac{1}{{\left(-5y\right)}^{1}}}[/latex]
Simplify.	[latex]{\Large\frac{1}{-5y}}[/latex]
Use [latex]{\Large\frac{a}{-b}}=-{\Large\frac{a}{b}}[/latex]	[latex]-{\Large\frac{1}{5y}}[/latex]

try it

See this interactive in the course material.

Let’s looks at some examples of how this rule applies under different circumstances.

Example

Write [latex]\frac{{\left({t}^{3}\right)}}{{\left({t}^{8}\right)}}[/latex] with positive exponents.

Show Solution

Use the quotient rule to subtract the exponents of terms with like bases.

[latex]\begin{array}{r}\frac{{\left({t}^{3}\right)}}{{\left({t}^{8}\right)}}={t}^{3-8}\\={t}^{-5}\,\,\end{array}[/latex]

Write the expression with positive exponents by putting the term with the negative exponent in the denominator.

[latex]=\frac{1}{{t}^{5}}[/latex]

Answer

[latex]\frac{1}{{t}^{5}}[/latex]

Example

Simplify [latex]{\left(\frac{1}{3}\right)}^{-2}[/latex].

Show Solution

Apply the power property of exponents.

[latex]\frac{{1}^{-2}}{{3}^{-2}}[/latex]

Write each term with a positive exponent, the numerator will go to the denominator and the denominator will go to the numerator.

[latex]\frac{{3}^{2}}{{1}^{2}}{ = }\frac{{3}\cdot{3}}{{1}\cdot{1}}[/latex]

Simplify.

[latex]\frac{{3}\cdot{3}}{{1}\cdot{1}}{ = }\frac{9}{1}{ = }{9}[/latex]

Answer

[latex]9[/latex]

Example

Simplify.[latex]\frac{1}{4^{-2}}[/latex] Write your answer using positive exponents.

Show Solution

Write each term with a positive exponent, the denominator will go to the numerator.

[latex]\frac{1}{4^{-2}}=1\cdot\frac{4^{2}}{1}=\frac{16}{1}=16[/latex]

Answer

[latex]16[/latex]

In the following video you will see examples of simplifying expressions with negative exponents.

[Video Link](#)

Simplify Expressions with Zero Exponents

A special case of the Quotient Property is when the exponents of the numerator and denominator are equal, such as an expression like [latex]\Large\frac{{a}^{m}}{{a}^{m}}[/latex]. From earlier work with fractions, we know that

[latex]\Large\frac{2}{2}\normalsize =\Large\frac{17}{17}\normalsize =\Large\frac{-43}{-43}\normalsize =1[/latex]

In words, a number divided by itself is [latex]1[/latex]. So [latex]\Large\frac{x}{x}\normalsize =1[/latex], for any [latex]x[/latex] ([latex]x\ne 0[/latex]), since any number divided by itself is [latex]1[/latex].

The Quotient Property of Exponents shows us how to simplify [latex]\Large\frac{{a}^{m}}{{a}^{n}}[/latex] by subtracting exponents. What if [latex]m=n[/latex] ?

Now we will simplify [latex]\Large\frac{{a}^{m}}{{a}^{m}}[/latex] in two ways to lead us to the definition of the zero exponent.

Consider first [latex]\Large\frac{8}{8}[/latex], which we know is [latex]1[/latex].

	[latex]\Large\frac{8}{8}\normalsize =1[/latex]
Write [latex]8[/latex] as [latex]{2}^{3}[/latex] .	[latex]\Large\frac{{2}^{3}}{{2}^{3}}\normalsize =1[/latex]

Subtract exponents.	[latex]{2}^{3 - 3}=1[/latex]
Simplify.	[latex]{2}^{0}=1[/latex]

We see [latex]\Large\frac{{a}^{m}}{{a}^{m}}[/latex] simplifies to a [latex]{a}^{0}[/latex] and to [latex]1[/latex] . So [latex]{a}^{0}=1[/latex]. At this time, we will define both the exponent of 0 and the exponent of 1.

Exponents of 0 or 1

Any number or variable raised to a power of [latex]1[/latex] is the number itself.

[latex]n^{1}=n[/latex]

Any non-zero number or variable raised to a power of [latex]0[/latex] is equal to [latex]1[/latex]

[latex]n^{0}=1[/latex]

The quantity [latex]0^{0}[/latex] is undefined.

In this text, we assume any variable that we raise to the zero power is not zero. The sole exception is the expression [latex]{0}^{0}[/latex]. This appears later in more advanced courses, but for now, we will consider the value to be undefined, or DNE (Does Not Exist).

example

Simplify:

1. [latex]{12}^{0}[/latex]

2. [latex]{y}^{0}[/latex]

Show Solution

Solution

The definition says any non-zero number raised to the zero power is [latex]1[/latex].

1.	
	[latex]{12}^{0}[/latex]
Use the definition of the zero exponent.	[latex]1[/latex]

2.	
	[latex]{y}^{0}[/latex]
Use the definition of the zero exponent.	[latex]1[/latex]

try it

See this interactive in the course material.

See this interactive in the course material.

Now that we have defined the zero exponent, we can expand all the Properties of Exponents to include whole number exponents.

What about raising an expression to the zero power? Let’s look at [latex]{\left(2x\right)}^{0}[/latex]. We can use the product to a power rule to rewrite this expression.

	[latex]{\left(2x\right)}^{0}[/latex]
Use the Product to a Power Rule.	[latex]{2}^{0}{x}^{0}[/latex]
Use the Zero Exponent Property.	[latex]1\cdot 1[/latex]
Simplify.	[latex]1[/latex]

This tells us that any non-zero expression raised to the zero power is one.

example

Simplify: [latex]{\left(7z\right)}^{0}[/latex].

Show Solution

Solution

	[latex]{\left(7z\right)}^{0}[/latex]
Use the definition of the zero exponent.	[latex]1[/latex]

try it

See this interactive in the course material.

Now let’s compare the difference between the previous example, where the entire expression was raised to a zero exponent, and what happens when only one factor is raised to a zero exponent.

example

Simplify:

1. [latex]{\left(-3{x}^{2}y\right)}^{0}[/latex]

2. [latex]-3{x}^{2}{y}^{0}[/latex]

Show Solution

Solution

1.	
The product is raised to the zero power.	[latex]{\left(-3{x}^{2}y\right)}^{0}[/latex]
Use the definition of the zero exponent.	[latex]1[/latex]

2.	
Notice that only the variable [latex]y[/latex] is being raised to the zero power.	[latex]{-3{x}^{2}y}^{0}[/latex]
Use the definition of the zero exponent.	[latex]-3{x}^{2}\cdot 1[/latex]
Simplify.	[latex]-3{x}^{2}[/latex]

Now you can try a similar problem to make sure you see the difference between raising an entire expression to a zero power and having only one factor raised to a zero power.

try it

See this interactive in the course material.

See this interactive in the course material.

In the next video we show some different examples of how you can apply the zero exponent rule.

Video Link

As done previously, to evaluate expressions containing exponents of [latex]0[/latex] or [latex]1[/latex], substitute the value of the variable into the expression and simplify.

Example

Evaluate [latex]2x^{0}[/latex] if [latex]x=9[/latex]

Show Solution

Substitute 9 for the variable *x*.

[latex]2\cdot9^{0}[/latex]

Evaluate [latex]9^{0}[/latex]. Multiply.

[latex]2\cdot1=2[/latex]

Answer

[latex]2x^{0}=2[/latex], if [latex]x=9[/latex]

Example

Simplify [latex]\frac{{c}^{3}}{{c}^{3}}[/latex].

Show Solution

Use the quotient and zero exponent rules to simplify the expression.

[latex]\begin{array}\text{ }\frac{c^{3}}{c^{3}} \,\,\,= \,\,\,c^{3-3} \\ \,\,\,\,\,\,\,\,\,\,\,\,=\,\,\,c^{0} \\\,\,\,\,\,\,\,\,\,\,\,\,=\,\,\,1\end{array}[/latex]

Answer

[latex]1[/latex]

In the following video there is an example of evaluating an expression with an exponent of zero, as well as simplifying when you get a result of a zero exponent.

Video Link

Video Link

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