Towards a Deeper Understanding of Fractions

Here, we offer explanations rooted in conceptual understanding to common questions students (and adults) have about fractions and additional resources to support your fraction learning and teaching journey.

Use Context to Support Conceptual Understanding

Students will likely be grappling with their understanding of fractions as numbers during any discussion. Therefore, the easiest way to support your students is to ground all problems and tasks in context. This will significantly support their understanding by situating the fractions in a familiar context for your students. For example:

- For the expression ½ + ⅓:
    - Kacey went for a long hike. She hiked ½ of the trail and took a break for a snack Then she hiked another ⅓ of the trail before stopping for lunch. How much of the trail did Kacey hike so far?
- For the expression ½ x ¼:
    - Charlie had ½ a bag of potato chips. If each bag contains ¼ lbs of potato chips, how much is left in her bag?
- For the expressions ½ ÷ ¼ OR ¼ ÷ ½:
    - Camilla has ½ of a cup of juice. If she puts them into containers that hold ¼ cup, how many ¼ cup containers will she need?
    - Camilla has ¼ cup of juice. If she puts it into a container that holds a ½ cup, How much of the container will be filled?

The Multiple Meanings of Fractions

But what is a fraction? Often stated as "an equal part of a whole," fractions are typically positioned as a formulaic way of representing shaded pieces of an object or "n parts out of m equal parts." The truth is, fractions are so much more than that! They do not have a single meaning. As Empson and Levi describe in *Extending Children's Mathematics: Fractions and Decimals*, Fractions can stand for:

- amounts of stuff, such as ⅗ of a candy bar
- relationships between amounts, such as 3 candy bars for every 5 children
- processes involving amounts, such as 3 candy bars shared among 5 children
- a point on the number line or rational numbers written in the form $\frac{p}{q}$ with integers p and q and q ≠ 0

The authors go on to caution us, "When children learn to think of fractions only, or mainly, as parts of wholes…it is hard for them to grasp what really makes a fraction what it is: a multiplicative relationship between the numerator and the denominator." But what does this mean? And how do we support children (and ourselves) to see fractions as more than parts of wholes? We recommend beginning by reading the introduction of *Extending Children's Mathematics: Fractions and Decimals* to learn more.

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Common Denominators:

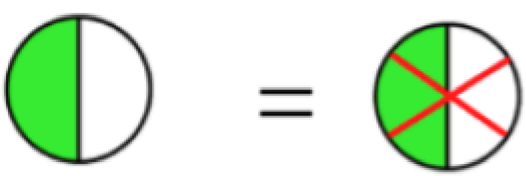
Why do we need to use denominators? The necessity of common denominators in addition and subtraction of fractions stems from the need for uniform units to facilitate calculation. Each denominator represents a unique unit of measurement. Just as we cannot add apples and oranges directly, fractions with different denominators require alignment to a common unit for accurate computation. Pro Tip: I often introduce the denominator as the "unit" we are working with to help students make that connection conceptually.

- For example, we can't add 2 apples and 3 oranges. We'd still just have 2 apples and 3 oranges. However, if we changed their categorization, or units, to pieces of fruit, we can now combine them and say we have 5 pieces of fruit.
- Similarly, we can't add $\frac{1}{2}$ and $\frac{1}{3}$ as is, as they are different units! Therefore, the easiest way to get like units is to "recategorize" them both as sixths.

When seeking common denominators, why do we multiply both numerator and denominator by the same number? This practice arises from the identity property of multiplication, which states that a number retains the same value when multiplied by 1. This property helps us ensure equivalence while establishing common units for ease of calculation.

- For example, in the example of $\frac{1}{2} + \frac{1}{3}$ we need to recategorize $\frac{1}{2}$ and $\frac{1}{3}$ so they have the same unit, (sixths). Follow parts A-C in Figure 1 to better understand how the identity property of multiplication supports us to recategorize fractions. Then, put it to practice! Show and explain how the identity property of multiplication works for recategorizing ⅓ into sixths?

Fig. 1



$$\frac{1}{2} \times \boxed{\frac{3}{3}} = \frac{3}{6}$$

B) If we multiply the numerator by the same number as the denominator, in this case 3, then the value of the fraction ½ will remain the same. ½ = 3/6. But why does that work?

A) If we want to recategorize halves into sixths, we need to change the unit, by multipling the denominator, 2, by the digit 3 because 2 x 3 = 6. But doesn't that change the value of the fraction ½?

C) Three-thirds equals 1 whole. So when we multiply ½ by 3/3 we are multiplying ½ by 1. All we are doing is increasing the amount of equal pieces in the whole. As a result, the value of the original number remains the same (½ = 3/6), even though the whole was partitioned into more pieces, as you can see from the visual above.

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Multiplication and Division:

When grappling with multiplication and division of fractions, it is helpful to think of fractions within a context. It is also always helpful to draw upon familiar principles of arithmetic with whole numbers. Check out the fraction learning progressions below to develop a more robust understanding of fraction multiplication and division.

- Boise State Unit Fractions Progression
- OGAP Fraction Progression
- Eduquate Progression of Models and Strategies for Fractions (draft in progress)

Ready to learn more? Dive into the resources below:

Videos:

- Fraction Meaning, Equivalence, Comparison
- Dividing Fractions

Text: Extending Children's Mathematics: Fractions and Decimals

Website: Achieve the Core

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