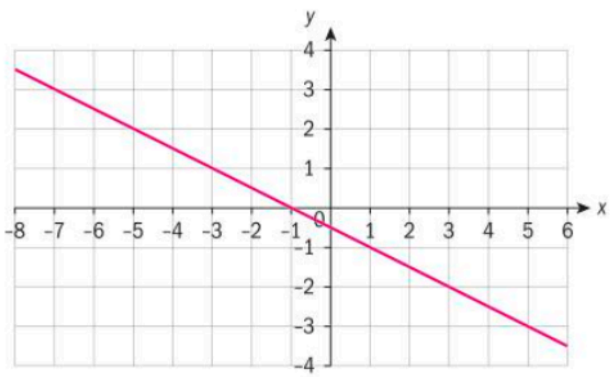
PRACTICE Gradient, Rates, and Linear Functions

* Full, worked solutions can be found in the folder linked on the Course Website ☺

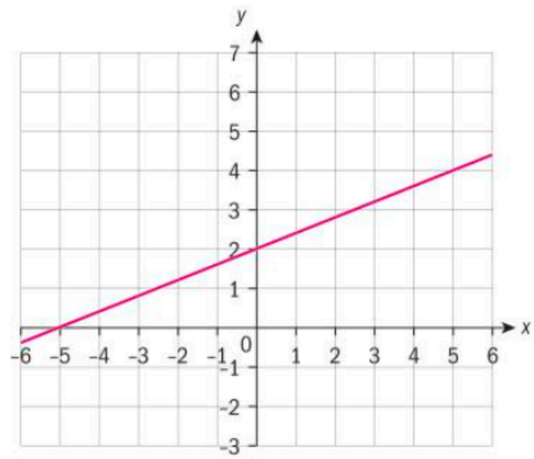
Exercise 3A

1 Find the gradient of each line:

a



b



2 Find the gradient of the line passing through the given points:

a $(4, 8), (8, 11)$ **b** $(-2, 2), (4, -4)$

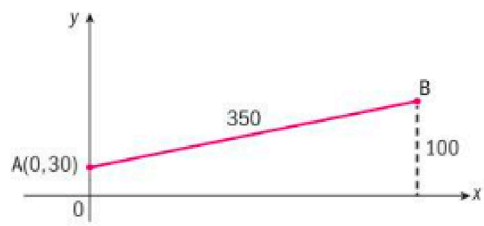
c $(-7, 1), (7, 8)$

3 The height, h, of a burning candle t seconds after it is lit is shown in the table. Find the gradient of the line joining a scatter graph of these points, and explain what the gradient of this line tells you.

t (s)	h (cm)
20	4.3
30	4.15
60	3.7

4 Baldwin Street in Dunedin, New Zealand, is known as the steepest street in the world.

Baldwin Street is a short residential street about 350 metres long. From its base at approximately 30 metres above sea level, it rises to a height of about 100 m above sea level. The segment shown in the following diagram models Baldwin Street.



a Find the coordinates of B, to the nearest hundredth.

b Find the gradient of segment AB, to the nearest hundredth.

The grade of a road is given as a percent.

It is calculated by the formula:

$\frac{\text{rise}}{\text{run}} \times 100$ (or $\frac{\text{vertical change}}{\text{horizontal change}} \times 100$).

c Find the grade of Baldwin Street.

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Exercise 3B

1 Determine whether lines 1 and 2 which pass through the given points are parallel, perpendicular or neither.

a Line 1: $(3, 6)$ and $(6, 11)$
Line 2: $(4, -1)$ and $(9, 2)$

b Line 1: $(5, -1)$ and $(3, 7)$
Line 2: $(-1, 4)$ and $(0, 0)$

2 A line passes through the points $(3, 2)$ and $(x, 5)$ and is perpendicular to a line with gradient $\frac{4}{3}$. Find the value of x.

3 Liam works up to 60 hours each week. His weekly pay, in dollars, depends on the number of hours he works, as shown in the graph.

a Find the gradient for each line segment in the graph.

b Explain the meaning of each gradient in the context of Liam's work.

Exercise 3C

1 Write down the gradient and y-intercept for the following lines:

a line 1: $y = 3x - 7$

b line 2: $y = -\frac{2}{3}x + 4$

c line 3: $y = -2$

2 Write down the equation of the line with gradient $\frac{1}{5}$ passing through $(0,1)$.

3 Find the equation, in gradient-intercept form, of the following lines:

a the line that passes through the point $(0,-1)$ and is parallel to the line $y = 4x - 3$

b the line that passes through the points $(-3,-2)$ and $(1,10)$

4 Write down the following:

a the equation of the vertical line that passes through $(8,-1)$

b the equation of the horizontal line that passes through the point $(-3,-10)$

c the equation of the line perpendicular to $y = 1$ that passes through $(9,5)$

d the point of intersection of the lines $x = -2$ and $y = 7$

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Exercise 3D

1 Draw the graph of the line with the given equation:

a $y = -3x$

b $y - 2 = \frac{1}{3}(x + 4)$

c $y = \frac{1}{2}$

d $y = -\frac{3}{4}x + 5$

2 Find the equation, in point-gradient form, of the line with gradient -3 that passes through $(2,6)$.

3 Consider the line pas
$(-3,-4)$ and $(-5,2)$.

a Find the gradient of the line.

b Write down two different equations for the line in point-gradient form.

c Verify that the two equations represent the same line.

Exercise 3E

1 Write the equation of each of these lines in the general form $ax + by + d = 0$ where a, b and d are integers.

a $y = \frac{1}{6}x - 3$

b The line with gradient $-\frac{2}{3}$ and y-intercept $(0, 4)$.

c The line with gradient -1 that passes through $(-3, 2)$.

2 Change the general form of the equation of each line to gradient-intercept form, $y = mx + c$.

a $3x + y - 5 = 0$

b $2x - 4y + 8 = 0$

c $5x + 2y + 7 = 0$

3 Sketch the graph of the line and label the coordinates of the axial intercepts.

a $x + 2y + 6 = 0$

b $2x - 6y + 8 = 0$

Exercise 3F

1 Use your GDC to find the point of intersection for each pair of lines:

a $y = 2x - 1$ and $y = 3x + 1$

b $y = 2x + 1$ and $4x + 2y = 8$

c $y = 4.3x + 7.2$ and $y = 0.5x - 6.4$

d $2x - 3y = -1$ and $y = -\frac{3}{4}x + 2$

2 Solve each equation by using your GDC to find a point of intersection:

a $3x - 4 = 0.5x - 1.75$

b $6.28x + 15.3 = 2.29x - 4.85$

3 The following equations give the weekly salary an employee can earn at two different sales jobs, where x is the amount of sales in euros and y is the weekly salary in euros. Find the amount of sales for which the weekly salaries would be equal.

$$y = 0.16x + 200$$
$$y = 0.10x + 300$$

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Exercise 3G

1 Consider the functions $f(x) = -x + 5$, $g(x) = 2x + 3$ and $h(x) = \frac{1}{3}x - 4$. Find the following:

a $f(3)$ **b** $g(0)$

c $h(6) - g(1)$ **d** $f(2) + g(-1)$

e $(f \circ g)(4)$ **f**

g $(f \circ g)(x)$ **h**

2 Give the domain and function:

a $f(x) = 3x + 8$ **b** $h(x) = x - 6$

3 Sketch a graph of the following:

a a linear function with range $\{6\}$

b a line that is not a function.

4 Find $f^{-1}(x)$ for each of the following linear functions. Give your answers in the form $f^{-1}(x) = mx + c$.

a $f(x) = \frac{1}{2}x + 4$ **b** $f(x) = -3x + 9$

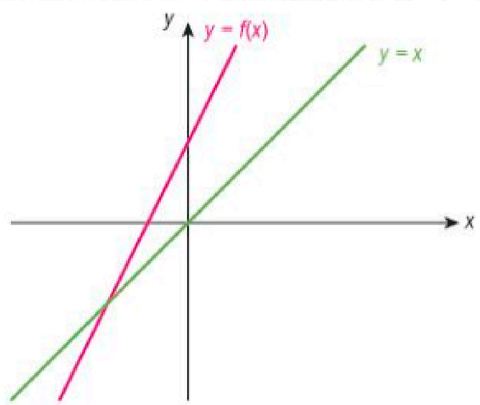
Exercise 3H

1 Find $f^{-1}(x)$ for each of the following linear functions. Give your answers in the form $f^{-1}(x) = mx + c$.

a $f(x) = 4x - 5$ **b** $f(x) = -\frac{1}{6}x + 3$

c $f(x) = 0.25x + 1.75$

2 The graph of a linear function $y = f(x)$ and the line $y = x$ is shown below. Copy the graphs and then add a sketch of the graph of $y = f^{-1}(x)$.



3 A t-shirt company imprints logos on t-shirts. The company charges a one-time set-up fee of $65 and $10 per shirt. The total cost of x shirts, in CAD, is given by $f(x) = 10x + 65$.

a Find the total cost for 55 t-shirts.

b Find $f^{-1}(x)$ and tell what x and $f^{-1}(x)$ represent in this function.

c Find the number of t-shirts in an order with a total cost of $5065.

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Exercise 3I

1 The force applied to a spring and the extension of the spring are connected by a linear relationship.

When a spring holds no mass, its extension is zero. When a force of 160 newtons (N) is applied, the extension of the spring is 5 cm.

a Find a linear model for the extension of the spring in terms of the force applied. Make sure you state clearly the variables you use to represent each quantity.

b Find the extension of the spring when a force of 370 N is applied.

2 Frank is a salesman. He is paid a basic weekly salary, and he also earns a percentage of commission on every sale he makes.

In a certain week, Frank makes sales totalling £1500. His total pay for that week is £600.

In another week, Frank makes sales totalling £2000. His total pay for that week is £680.

a Find, in gradient-intercept form, an equation that relates Frank's total weekly pay, £y, with his total sales revenue, £x.

b Explain the meaning of both the gradient and y-intercept in your model.

c Find Frank's total weekly pay when his weekly sales total £900.

3 A new fitness gym is offering two membership plans.

Plan A: A one-off enrollment fee of \$79.99, and a further monthly fee of \$9.99 per month

Plan B: no enrollment fee, and monthly fees of \$20.00 per month

a Find a linear model for each plan, where total cost is a function of number of months. Identify the variables you use.

After a certain number of months, Plan A becomes more cost-effective than Plan B.

b Use the models from part **a** to determine how many months a person needs to be a member before Plan A becomes more cost-effective than Plan B.

4 Liam works up to 60 hours each week. His weekly pay, £p, depends on the number of hours, h, he works. This information is presented in the graph shown.

a Find the equations for the piecewise function.

b Find the amount Liam is paid in a week that he works:

i 22 hours

ii 47 hours

5 Office Resource has 3000 printers available to sell in a certain month. On average, sales drop by 6.5 printers for each €1 increase in price. This is modelled by the demand function, $q = -6.5p + 3000$, where €p is the sales price and q is the number of printers sold during the month.

a Find the number of printers the model predicts Office Resource sells in a month, if the selling price is €200.

b Explain how raising the sales price by €20 affects the sales.

The manufacturer supplying the printers to Office Resource controls supply according to the function $q = 48p - 1600$, where q is the number of printers supplied during the month and €p is the price Office Resource must pay the manufacturer for each printer they supply. This function is known as the supply function.

c Find the price per printer Office Resource must pay to be supplied with 2000 printers a month.

d Graph the supply and demand functions on your GDC and then sketch the graphs on your paper.

e When the quantity supplied equals the quantity demanded the market is said to be in equilibrium. Find the equilibrium price and the equilibrium demand.