Key Equations

One-to-one property for exponential functions	For any algebraic expressions S and T and any positive real number b , where [latex]b>0,\text{b\ne 1, {b}^{S}={b}^{T}[/latex] if and only if $S = T$.
Definition of a logarithm	For any algebraic expression <i>S</i> and positive real numbers <i>b</i> and <i>c</i> , where [latex]b\ne 1[/latex], [latex]{\mathrm{log}}_{b}\left(S\right)=c[/latex] if and only if [latex]{b}^{c}=S[/latex].
One-to-one property for logarithmic functions	For any algebraic expressions S and T and any positive real number b , where [latex]b\ne 1[/latex], [latex]{\mathrm{log}}_{b}S={\mathrm{log}}_{b}T[/latex] if and only if $S = T$.

Key Concepts

- We can solve many exponential equations by using the rules of exponents to rewrite each side as a power with the same base. Then we use the fact that exponential functions are one-to-one to set the exponents equal to one another and solve for the unknown.
- When we are given an exponential equation where the bases are explicitly shown as being equal, set the exponents equal to one another and solve for the unknown.
- When we are given an exponential equation where the bases are *not* explicitly shown as being equal, rewrite each side of the equation as powers of the same base, then set the exponents equal to one another and solve for the unknown.
- When an exponential equation cannot be rewritten with a common base, solve by taking the logarithm of each side.
- We can solve exponential equations with base e by applying the natural logarithm to both sides because exponential and logarithmic functions are inverses of each other.
- After solving an exponential equation, check each solution in the original equation to find and eliminate any extraneous solutions.
- When given an equation of the form [latex]{\mathrm{log}}_{b}\left(S\right)=c[/latex], where S is an algebraic expression, we can use the definition of a logarithm to rewrite the equation as the equivalent exponential equation [latex]{b}^{c}=S[/latex] and solve for the unknown.
- We can also use graphing to solve equations of the form
 [latex]{\mathrm{log}}_{b}\left(S\right)=c[/latex]. We graph both equations
 [latex]y={\mathrm{log}}_{b}\left(S\right)[/latex] and [latex]y=c[/latex] on the same
 coordinate plane and identify the solution as the x-value of the point of intersecting.
- When given an equation of the form [latex]{\mathrm{log}}_{b}T[/latex], where S and T are algebraic

- expressions, we can use the one-to-one property of logarithms to solve the equation S = T for the unknown.
- Combining the skills learned in this and previous sections, we can solve equations that
 model real world situations whether the unknown is in an exponent or in the argument of a
 logarithm.

Glossary

extraneous solution a solution introduced while solving an equation that does not satisfy the conditions of the original equation

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