Document No. EDIPR/VI/12/19-001

ENGINEERING DESIGN AND INNOVATION PROJECT REPORT

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Abstract

In the present article, a mathematical theory for the flow field within a Tesla disc turbine has been formulated in the appropriate cylindrical co-ordinate system. The basis of the theory is the Navier–Stokes equations simplified by a systematic order of magnitude analysis. The presented theory can compute three-dimensional variation of the radial velocity, tangential velocity and pressure of the fluid in the flow passages within the rotating discs. Differential equations as well as closed-form analytical relations are derived. The present mathematical theory can predict torque, power output and efficiency over a wide range of rotational speed of the rotor, in good agreement with recently published experimental data. The performance of the turbine is characterized by conceptualizing the variation of load through the non-dimensional ratio of the absolute tangential velocity of the jet and the peripheral speed of the rotor. The mathematical model developed here is a simple but effective method of predicting the performance of a Tesla disc turbine along with the three-dimensional flow field within its range of applicability. A hypothesis is also presented that it may be possible to exploit the effects of intelligently designed and manufactured surface roughness elements to enhance the performance of a Tesla disc turbine.

Keywords

Tesla turbine, turbine efficiency, power, torque, analytical theory, three-dimensional flow field

Introduction

Tesla turbine, a bladeless turbine, was patented by the famous scientist Nikola Tesla (1856–1943) in 1913. Up to now, a major drawback in its commercial use has been its low efficiency and certain other operational difficulties. However, there has been research interest in this type of turbines because they have several advantages and hence may be appropriately developed and used in certain application areas. In this article, an analytical theory has been developed for predicting the performance of Tesla turbines. The Tesla turbine is also known as disc turbine because the rotor of this turbine is formed by a series of flat, parallel, co-rotating discs, which are closely spaced and attached to a central shaft. The working fluid is injected nearly tangentially to the rotor by means of inlet nozzle. The injected fluid, which passes through the narrow gaps between the discs, approaches spirally towards the exhaust port located at the centre of each disc. The viscous drag force, produced due to the relative velocity between the rotor and the working fluid, causes the rotor to rotate. There is a housing surrounding the rotor, with a small radial and axial clearance. Tesla turbine has several important advantages: it is easy to manufacture, maintain and balance the turbine, and it has high power to weight ratio, low cost, significant reduction in emissions and noise level, a simple configuration which means an inexpensive motor. Tesla turbine can generate power for a variety of working fluids like Newtonian fluids, non Newtonian fluids and mixed fluids. This turbine has self-cleaning nature due the centrifugal force field. This makes it possible to operate the turbine in case of non-conventional fuels like biomass which produce solid particles. It also suggests that this bladeless turbine can be well suited to generate power in geothermal power stations. Further research and modification of Tesla turbine were temporarily suppressed after the invention of gas turbine which was much more efficient than Tesla turbine. From 1950 onwards both theoretical and experimental research on Tesla turbine has been regenerated. Quite a number of analytical models for the conventional configuration of Tesla turbine have been developed. Currently the field of micro-turbine is an active research area; the bladeless Tesla turbine because of its simplicity and robustness of structure, low cost and comparatively better operation at high rpm may become a suitable candidate for this application. For this to happen the efficiency of the Tesla turbine, however, has to be improved. Researchers are attempting to achieve this by modification of the configuration of the conventional Tesla turbines. The objective of the present work is to formulate a mathematical theory for a Tesla turbine, developed in the appropriate cylindrical co-ordinate system. The scope of the project is to develop an analytical model for a more realistic case considering three-dimensional flow and consequences of the viscous drag force. The model can compute the three-dimensional variation of the radial velocity, tangential velocity and pressure of the fluid in the flow passages within the rotating discs. Differential equations as well as closed-form analytical relations have been derived. The present mathematical model can predict torque, power output and efficiency over a wide range of rotational speed of the rotor.

Pre-Analysis:

The analysis begins with the Navier-Stokes equations in the cylindrical co-ordinates.

The continuity equations, the momentum equation and the boundary conditions are written in terms of relative velocity

For this purpose the following relations are used:

$$U_r = V_r$$
$$U_z = V_z$$
$$U_\theta = V_\theta + \Omega r$$

The conservation equations take the following simplified form

- Continuity Equation:

$$\frac{\partial V_r}{\partial r} + \frac{V_r}{r} = 0$$

- Theta-Momentum equation:

$$Vr\frac{\partial V_\theta}{\partial r} + \frac{V_r V_\theta}{r} + 2\Omega V_r = \nu \frac{\partial^2 V_\theta}{\partial z^2}$$

- r- Momentum Equation:

$$Vr\frac{\partial V_r}{\partial r} - \Omega^2 r - 2\Omega V_\theta - \frac{V_\theta^{\,2}}{r} = -\frac{1}{\rho}\frac{dp}{dr} + \nu \frac{\partial^2 V_r}{\partial z^2}$$

- z-Momentum equation:

$$\frac{\partial P}{\partial z} = 0$$

Given equations are solved for following Boundary Conditions:

- At z=0

$$V_r = 0$$
$$V_\theta = 0$$

- At z=b

$$V_r = 0$$
$$V_\theta = 0$$

- At z=b/2

$$\frac{\partial V_r}{\partial z} = \frac{\partial V_\theta}{\partial z} = 0$$

Within the boundary layer the developed between flat solid discs, relative radial and tangential velocity at any radius between r_1 and r_2 can be modelled as:

$$R = \frac{r}{r_2} \qquad \zeta(R) = \frac{\overline{V_\theta}(r)}{\overline{V_{\theta 2}}}$$

$$\xi(R) = \frac{\overline{V_r}(r)}{\overline{V_{r2}}}$$

$$G(z) = \frac{V_\theta(r,z)}{\overline{V_\theta}(r)} \qquad H(z) = \frac{V_r(r,z)}{\overline{V_r}(r)}$$

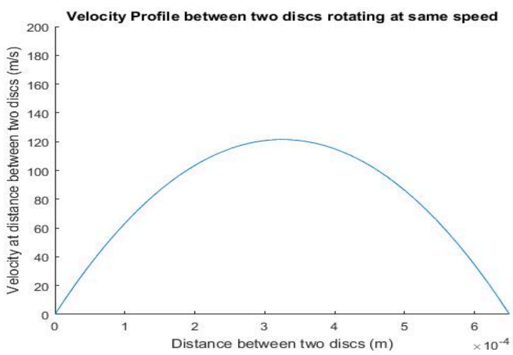
Velocity variation between discs:

G and H are respectively z variation of tangential and radial velocity within boundary layer. Here we assume velocity of fully developed flow is parabolic.

Accordingly, G and H are:

$$G = 6 * \frac{z}{b}\left(1 - \frac{z}{b}\right) \qquad H = 6 * \frac{z}{b}\left(1 - \frac{z}{b}\right)$$

Where, b is gap between two discs.



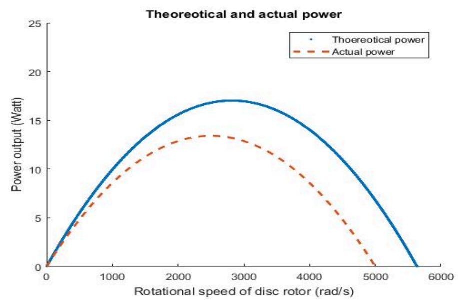
Streamline visualization:

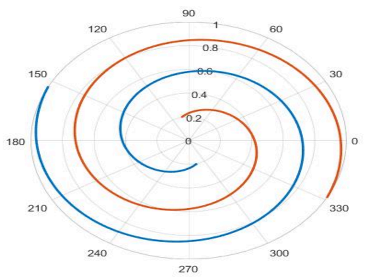
For the visualization of streamlines following equation can be integrated and plotted with help of MATLAB:

$$\left(\frac{d\theta}{dR}\right)_{st} = -\frac{R+\phi_2}{\gamma}$$

For

$$\phi_2 = 0.678 \qquad \gamma = 0.119$$





Calculation of Power and Efficiency:

The torque on single side of the disc:

$$\mathrm{T} = \left(\frac{12\pi\mu\overline{V_{\theta 2}}r_2^{\,3}}{b}\right)\left[C_3(R_2^{\,2}-R_1^{\,2}) - \frac{C_4}{C_1}\times\left[\exp\left(-\frac{C_1R_2^{\,2}}{2}\right)\right.\right.$$

Where,

$$C_1 = \frac{10\nu}{\phi_2\Omega b^2} \qquad C_2 = \frac{-10}{6(\gamma-1)}$$

$$C_3 = \frac{C_2}{C_1} \qquad C_4 = (1-C_3)\exp\left(\frac{C_1}{2}\right)$$

For simple theoretical considerations Hoya and Guha show that:

$$T = T_0 - c\Omega$$

Therefore,

$$\dot{P}_{theo} = T\Omega = T_0\Omega - c\Omega^2$$

The frictional work is given by Munson et al as:

$$W_f = d\Omega$$

Therefore,

$$\dot{P}_{act} = T\Omega - W_f = T_0\Omega - c\Omega^2 - d\Omega$$

Where

T_0= 0.012085181

c= 0.000002145

d= 1.360181x10^{-3}

MATLAB Code:

1. **Plot of output power v/s rotational speed:**

```
1. Gam=1.5;
2.   phi2=(84/1000);
3.   c1=((10*1.47*10^-5)/(phi2*1000*(.46*10^-3)
     ^2));
4.   c2=-10/(6*(gam-1))
5.   c3=c2/c1
6.   c4=(1-c3)*exp(c1/2)
7.
8.   r2=0.025;
9.   Vra=-11.25*(1);
```

```
dVra=Vra*10^-6;
Vta=84*(c3./1)+((c4*exp((-c1*1.^2)./20))./1);
r=r2-dVra;
R=r/r2;

hold on
nd=8;
w=1:1:5634.117;
axis([0 6000 0 25]);
T=0.012085181-0.000002145*w;
Twl=0.012085181-0.000002145*w-1.360181*
10^-3;
Pt=T.*w;
Pact=Twl.*w;
plot(w,Pt,'.');
plot(w,Pact,'--','LineWidth',2);
title('Theoretical and actual power');
xlabel('Rotational speed of disc rotor (rad/s)');
ylabel('Power output (Watt)');
hold off
```

2. Visualization of streamline between two plates:

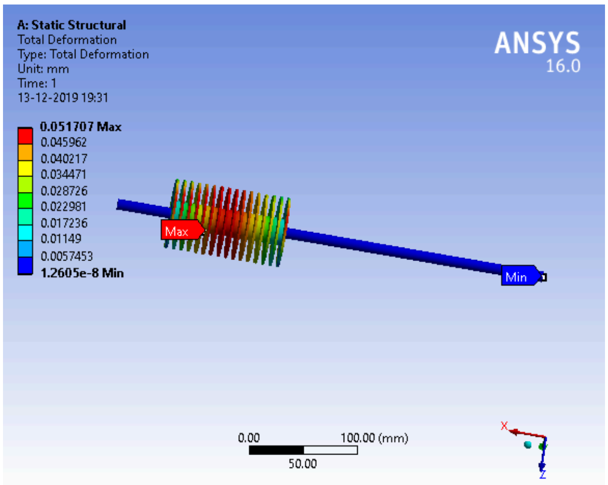
```
Wo=0.678;
Vro=0.119;
syms R
K=-((R+Wo)/(Vro));
ht=int(K)
R=0.2:0.0001:1;
fx=-(2*R.*(250.*R + 339))/119;
polarplot(fx,1);
hold all
polarplot(fx,R,'LineWidth',2);
polarplot((fx+3.112),R,'LineWidth',2);

hold off
```

3. Plot of velocity profile between discs:

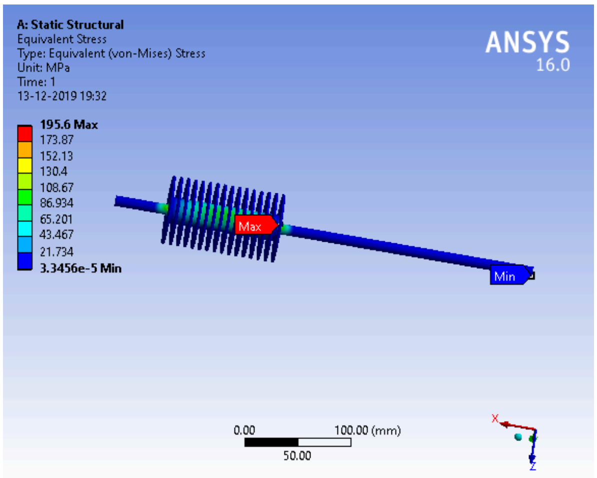
```
hold on
syms z
axis([0 0.65*10^-3 0 200]);
K=81*6*(z/(0.65*10^-3))*(1-(z/(0.65*10^-3)));
vT=matlabFunction(K);
fplot(@(z) vT(z));
title('Velocity Profile between two discs rotating at same speed');
xlabel('Distance between two discs (m)');
ylabel('Velocity at distance between two discs (m/s)');
hold off
```

CAD Model Design:

Analysis in ANSYS:



Total Deformation Disk



Equivalent Stress in Disk

Manufacturing of prototype:

Processes involved in manufacturing the prototype are as follows:

1. Laser Cutting (For Discs):
   Laser cutting is mainly a thermal process in which a focused laser beam is used to melt material in a localised area. A co-axial gas jet is used to eject the molten material and create a kerf. A continuous cut is produced by moving the laser beam or workpiece under CNC control. There are three major varieties of laser cutting: fusion cutting, flame cutting and remote cutting. In fusion laser cutting, an inert gas (typically nitrogen) is used to expel molten material out of the kerf. Nitrogen gas does not exothermically react with the molten material and thus does not contribute to the energy input. In flame laser cutting, oxygen is used as the assist gas. In addition to exerting mechanical force on the molten material, this creates an exothermic reaction which increases the energy input to the process. In remote laser cutting, the material is partially evaporated (ablated) by a high-intensity laser beam, allowing thin sheets to be cut with no assist gas. The laser cutting process lends itself to automation with offline CAD/CAM systems controlling either three-axis flatbed systems or six-axis robots for three-dimensional laser cutting. Improvements in accuracy, edge squareness and heat input control mean that laser cutting is increasingly replacing other profiling cutting techniques, such as plasma and oxy-fuel.

2. Welding (For assembly):
   Welding is a fabrication or sculptural process that joins materials, usually metals or thermoplastics, by using high heat to melt the parts together and allowing them to cool causing fusion. Welding is distinct from lower temperature metal-joining techniques such as brazing and soldering, which do not melt the base metal. In addition to melting the base metal, a filler material is typically added to the joint to form a pool of molten material (the weld pool) that cools to form a joint that, based on weld configuration (butt, full penetration, fillet, etc.), can be stronger than the base material (parent metal). Pressure may also be used in conjunction with heat, or by itself, to produce a weld. Welding also requires a form of shield to protect the filler metals or melted metals from being contaminated or oxidized. Many different energy sources can be used for welding, including a gas flame (chemical), an electric arc (electrical), a laser, an electron beam, friction, and ultrasound. While often an industrial process, welding may be performed in many different environments, including in open air, under water, and in outer space. Welding is a hazardous undertaking and precautions are required to avoid burns, electric shock, vision damage, inhalation of poisonous gases and fumes, and exposure to intense ultraviolet radiation.

3. Drilling (For Casing):
   Drilling is a cutting process that uses a drill bit to cut a hole of circular cross-section in solid materials. The drill bit is usually a rotary cutting tool, often multi-point. The bit is pressed against the work-piece and rotated at rates from hundreds to thousands of revolutions per minute. This forces the cutting edge against the work-piece, cutting off chips (swarf) from the hole as it is drilled. In rock drilling, the hole is usually not made through a circular cutting motion, though the bit is usually rotated. Instead, the hole is usually made by hammering a drill bit into the hole with quickly repeated short movements. The hammering action can be performed from outside the hole (top-hammer drill) or within the hole (down-the-hole drill, DTH). Drills used for horizontal drilling are called drifter drills. In rare cases, specially-shaped bits are used to cut holes of non-circular cross-section; a square cross-section is possible.

4. Lathe operations (For Bearing bush):

Lathes are used in woodturning, metalworking, metal spinning, thermal spraying, parts reclamation, and glass-working. Lathes can be used to shape pottery, the best-known design being the Potter's wheel. Most suitably equipped metalworking lathes can also be used to produce most solids of revolution, plane surfaces and screw threads or helices. Ornamental lathes can produce three-dimensional solids of incredible complexity. The workpiece is usually held in place by either one or two centres, at least one of which can typically be moved horizontally to accommodate varying workpiece lengths. Other work-holding methods include clamping the work about the axis of rotation using a chuck or collet, or to a faceplate, using clamps or dog clutch. Examples of objects that can be produced on a lathe include screws, candlesticks, gun barrels, cue sticks, table legs, bowls, baseball bats, musical instruments (especially woodwind instruments), crankshafts and much more.

Production Cost:

Sr. No	Material	Quantity	Cost
1	Disc	14	300
2	Shaft	1	70
3	Washer	50	10
4	Bearing	2	70
5	Labor cost		250

Conclusion:

A mathematical theory for the performance of a Tesla disc turbine has been formulated here. The basis of the theory is the Navier–Stokes equations simplified by a systematic order of magnitude analysis resulting in the present fundamental set of coupled differential equations (1) to (3) that govern the flow-field within a Tesla disc turbine. The theoretical model can compute the three-dimensional variation of the radial velocity, tangential velocity and pressure of the fluid in the flow passages within the rotating discs. The partial differential equations can be converted to ODEs by suitable assumptions regarding non-dimensional velocity profiles; the coupled set of ODEs (equations (17) and (18)) can be integrated by simple numerical schemes (section 'Integration of the r momentum equations'). Explicit, closed-form analytical results have also been derived, giving Vr, and p as functions of two co-ordinates r and z. The theoretical model can predict torque, power output and efficiency, and compares well with experimental results. A hypothesis is proposed here that it may be possible to exploit the effects of intelligently designed and manufactured surface roughness elements to enhance the performance of a Tesla disc turbine.

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Appendix 1

Notation

b	gap between two consecutive discs
k	isentropic index of fluid
$\dot{m}$	mass flow rate
p	pressure
P	modified pressure $= p - \rho g_z z$
p'	non-dimensional pressure $= \frac{p - p_2}{\rho \Omega^2 r_2^2}$
Q	volume flow rate
r	radial coordinate
R	non-dimensional radius (i.e. radius ratio) $= \frac{r}{r_2}$
U	absolute velocity of fluid
V	relative velocity of fluid
$\dot{W}_{th}$	theoretical ideal power output
$\dot{W}_{loss}$	overall loss in Tesla turbine
$\dot{W}_{act}$	theoretical power output with loss
z	axial coordinate
γ	tangential speed ratio $= \frac{\overline{U}_{\theta_2}}{\Omega r_2}$
Δp_{tc}	pressure drop between inlet and central exit of the rotor
ζ	non-dimensional average relative tangential velocity $= \frac{\overline{V}_\theta(r)}{\overline{V}_{\theta_2}}$
η	efficiency of the turbine
θ	Azimuthal direction in cylindrical co-ordinate system
μ	viscosity of the working fluid
ν	kinematic viscosity of working fluid (here the fluid is air)
ξ	non-dimensional average relative radial velocity $= \frac{\overline{V}_r(r)}{\overline{V}_{r_2}}$
ρ	density of the working fluid
τ_w	wall shear stress on one side of a single disc
ϕ_2	$= \frac{\overline{V}_{r_2}}{\Omega r_2}$
Ω	rotational speed of the disc
$\Im$	torque on one side of a single disc
$\Im_{tot}$	total torque

Subscripts

r	component along the r-direction
z	component along the z-direction