**~Review of Algebra II**

- Remember to take the plus/minus after finding the square root of something
- SOAP → same side, opposite side, always positive for CUBIC expressions
- Differences of cubes: $(a-b)(a^2+ab+b^2)$
- Sum of cubes: $(a+b)(a^2-ab+b^2)$
- You have to go slowly… don't forget the negative signs
- You need to put everything into simplest form
  - Radicals… watch out for roots
  - SOAP → DOTS?
- Odd roots = one sign; even roots = +/-
- SOAP can be used for difference and/or sum
  - Difference of Cubes: $(a-b)(a^2+ab+b^2)$
  - Sum of Cubes: $(a+b)(a^2-ab+b^2)$
- Stop making careless errors
  - Leaving out numbers especially when doing SOAP
  - Forgetting "i"
  - Negative exponents go to the other side of fraction
  - When bringing a whole number to the other side of a fraction, it turns into a fraction, not a negative whole number
- Put everything in simplest form

**~Unit 1: Transformations**

- You MUST follow order of operations (PEMDAS)
- Don't forget $(b/2)^2$
- Parent Functions:
  - Linear (constant + slope)
  - Quadratic (squared)
  - Square root
  - Cubed root
  - Cubic
  - Reciprocal (rational)
  - Absolute
- Transformations:
  - Vertical shift up/down
  - Horizontal shift left/right
    - Counterintuitive
  - Vertical stretch/shrink
    - Outside

- Horizontal stretch/shrink
    - Inside
  - Reflections over x and y
- Anything OUTSIDE is for Y
- Anything INSIDE is for X
- Order of operations are extremely important to use for order of transformations
- Double x:
  - One value for horizontal stretch/shrink
  - One value for the horizontal shift
- Cannot have double negations → makes the x value intuitive
- **When you're in vertex form, you need to subtract the c term back to the other side**
- **Even when the y-value is 0 and there is a negation of the y-value, it's still going to be reflected over the x-axis.**
- When finding a line of reflection using the factoring method, you need to set it equal to 0 and find the line.
- **Radicands are greater than or equal to 0 in the NUMERATOR, but radicands are greater than 0 in the DENOMINATOR.**
- **Never forget to use your number lines**
- **Do not simplify unless the question asks**
- When you have composition functions, you cannot magically set them equal to 0.
- Be careful when finding values of composition functions (reading the graph)
- Be careful with complex fractions… remember from Algebra II that you need to get rid of the denominators.
- Remember that when you're finding the domain of composition functions, you must…
  - Find restrictions in the inner function
  - Find restrictions of the composition
  - Use number line
- If the question is asking for NEGATIVE or POSITIVE, you need to reject the other
- READ THE QUESTIONS CAREFULLY
- Make sure you write the statements to conclude whether two functions are inverses.
- INNER FUNCTIONS ALWAYS COME FIRST!!!
- Skills needed:
  - Understanding transformations and parent functions
  - Counterintuitive vs. intuitive
  - Double x values

    - Finding the domain
        - Numerator: Greater than or equal to 0
        - Denominator: Greater than 0
    - Understanding inverse functions
    - Understanding what makes a function one-to-one
    - Finding the range of a function using its inverse
    - Finding domain of composition functions
        - Find the inner function's restrictions
        - Find the outer function with inner function domain restrictions
        - Use the number line
    - Understand the importance of working with number lines and working from the inside out

**~Unit 2: Absolute Value Functions**

**Absolute Value Functions**

- Finding Solutions to One Absolute Function
    - Equation: Isolate → original/negate → solve → reject any extraneous
    - Inequalities: Same procedure as equations, but write solution in INTERVAL NOTATION (use the number line)
- When absolute value equals 0, you only need to use the original expression
- When absolute value equals negative number, there is NO solution.
- Finding Solutions to Absolute Value Functions on BOTH Sides
    - Method 1: Follow the cases
        - Equation: Only need two of the four cases (+/+ and +/-)
        - Inequality: Need ALL four cases
    - Method 2: Square both sides and solve
        - Applies for both equations and inequalities
        - More efficient for inequalities
- Piecewise Functions for SIMPLE Absolute Functions (ex. $f(x) = |x-2|$)
    - Find the root
    - Keep the expression and negate the other
    - The positive function will have x is GREATER than the root and the negated expression will have x is LESS THAN the root
- Piecewise Functions for COMPLEX Absolute Functions (when there are two absolute values on one side)
    - Find the roots
    - Draw number line
    - Figure out what needs to be kept positive or negated

        - *Once the sign changes, it will stay that way (unless you're dealing with quadratics)
    - Add or subtract depending on the expression at hand (BE CAREFUL WHEN DOING THIS)
    - Write piecewise function with the correct intervals
        - Make sure you have equal sign once for each root
- Solving Complex Absolute Value Functions
    - Follow all of the steps above for writing piecewise functions
    - Set each piece of the piecewise functions equal to the number on the right hand side of the equation
    - Solve for each piece and make sure they fall within the interval
        - If they don't, you need to reject
- Checking Procedure
    - Plug a random number into each of the pieces and the original equation, if you get the same number, then you know you're correct.
    - Remember that if there is an additive inverse of one expression, you don't need to check the other one because it will result in the same thing automatically.
    - ALWAYS USE THE NUMBER LINE TO DETERMINE POSITIVE OR NEGATIVE.
- Whenever you're solving for quadratic functions in absolute values, you need to make separate number lines for the final answer.
- Careful with adding/subtracting the columns.
- |x-3|means that we're counting FROM 3 (POSITIVE).
- You put the equal sign once for every number.
- When we're talking about the range in Pre-Calc, it is NOT like the range in statistics… you need to find the middle number and then the distance from there.
- For something like f(x)=|x-2|, you just need to know what that is as a piecewise function without actually graphing it.
    - Original → greater than root
    - Negated → less than root
- When you're squaring fractions with two absolute value functions, find the common denominator to then take the numerators.
- Make sure you read carefully, if it's asking for you to create an absolute value inequality, that is what you need to create.
- Absolute value represents the distance between one point and another.
- Make sure you add and subtract correctly… don't forget negatives.

**Matrices**

- Order: Rows x Columns

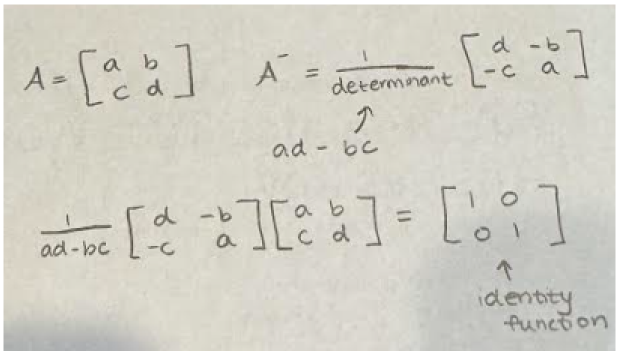
- Defined Matrices:
    - Adding/Subtracting: Orders are the SAME (ex. 2x1 and 2x1)
    - Multiplying/Dividing: First column and second row are the SAME (ex. 2x2 and 2x2 OR 2x3 and 3x2)
- Undefined Matrices:
    - Adding/Subtracting: Orders are NOT the same (ex. 2x1 and 2x2)
    - Multiplying/Dividing: First column and second row are NOT the same (ex. 3x2 and 3x2; Note: Even though they are the SAME order, they will still not work)
- Before performing ANY operations on two matrices, you need to make sure that they are DEFINED first by using the rules above.
- Adding/Subtracting Matrices:
    - Make sure that they are DEFINED
    - Add and subtract corresponding numbers in rows/columns
    - Be careful… sometimes they will throw in multiplication, which you would have to perform first on the matrix before adding/subtracting
        - Ex. 1/2A - B: Multiply each number in matrix A by ½ and then subtract each number from matrix B from the new numbers in matrix A.
- Multiplying Matrices (haven't learned how to divide yet):
    - Make sure that they are DEFINED
    - Figure out the new order of the product matrix by crossing out the first column and second row
        - Ex. Matrix A is 2x3 and Matrix B is 3x5, so the resulting product matrix will be 2x5.
    - Multiply each corresponding numbers in rows/columns and then ADD them all up to get one number
- Determinant: ad - bc
- Finding Inverses:
    - Find the determinant… if it is equal to 0, there is NO inverse because it's undefined
    - If the determinant is NOT 0…
        - Switch the a and d terms
        - Negate the b and c terms
        - Multiply each number by 1/determinant
- Always row x columns
- Whenever you have ½A - B, you need to multiply all values of A by ½ first before subtracting.

- Careful, they may also SWITCH the order of the letters and matrices.
- Had a small misconception with undefined matrices… one column and one row number need to be EQUAL.
- Make sure you multiply and add EVERYTHING (when applicable).
  - You are always multiplying row by column.
- 2x2 and 3x3 products for matrices have the SAME concept (go from row to row and multiply by each column).
- Determinant = **ad - bc**
- If the determinant EQUALS 0, then there is NO inverse because it's UNDEFINED.
- Careful with NEGATIVES!
- Make sure you're reading what they're asking for… minus vs. adds
- When you have something like -2A - 1/6B, you need to multiply everything in B by -⅙, but you don't need to go subtracting again.

**~Self-Study: MATRIXES**
- Weak spots:
  - Determine dimensions of the operation result
    - Careful if it's asking for addition or subtraction or multiplication or division
  - Multiplying matrices
    - You need to multiply each row by column and add
    - **PLEASE BE CAREFUL - NO CARELESS MISTAKES**
- Matrix Order: Number of rows x number of columns
- Determine the value of $a_{21}$ = row 2, column 1
  - Undefined = not possible
- A - B → literally just subtracting corresponding numbers in each matrix
  - The same thing applies for any operation!
- Matrixes are defined when one of their rows and columns (from each matrix) is equal when dividing or multiplying.
  - Ex. (2x5)(5x1) → 5 columns = 5 rows → Order is 2x1
  - Ex. (2x1)(2x1) → Although the order is the same, it is NOT EQUAL→ UNDEFINED
- Matrixes are defined when both orders are equal to each other when adding or subtracting.
  - Ex. (2x5)(5x1) →NOT THE SAME→ UNDEFINED
  - Ex. (2x1)(2x1) → SAME → DEFINED

*The PRODUCT is equal to the outer numbers.

- When multiplying matrixes, you find the product order and then multiply and add each one.
- For 2x2 and 3x3 matrices, the steps are the same as above.
- Determinant = a(d) - b(c)
- Inverses:




- Follow the picture above and the check if two matrices are inverses, multiply them out and make sure they equal to the identity function
- Every element within the matrix needs to be multiplied by the flipped determinant
- If the determinant is 0, the matrix will not have an inverse and is considered undefined.

**Unit 3a: Rational Equations and Inequalities**

Summary:

- Solving Rational EQUATIONS:
  - Determine what values of x make the equation undefined (set denominators equal to 0)
  - Find the common denominator (easiest way to do so is through multiplication)
  - Remove the denominators and use only the numerators
  - Solve for x
  - Reject any extraneous solutions
- Solving Rational INEQUALITIES:
  - Move everything onto one side of the inequality (if applicable)
  - Find the common denominator and multiply parts of the numerator that don't already have it
  - Determine what values of x are the zeroes AND undefined values
    - Zeros = set numerators equal to 0
    - Undefined = set denominators equal to 0

    - Make a number line with all of these values and determine the positive/negative parts of it
    - Write your solution in interval notation
- NEVER CROSS MULTIPLY
- Math Jail: Canceling out squared expressions when separated by addition or subtraction signs
    - Can only do it through multiplication
- Whenever something is squared, it will be positive (this will affect your number line).
- Multiplicity will make the number line not skip because they will "bounce back."

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- Undefined values get OPEN circles
- Zeroes get OPEN OR CLOSED circles depending on the question (greater than or greater than AND equal to symbol)
- Rational fractions underneath a radical need to be greater than or equal to 0 or else they would be imaginary.
    - The process of finding their domain is literally the same
- Don't cancel expressions out if they're being separated by addition or minus symbols.
    - This is not okay… it's "math jail"
- You need to reject undefined values in the denominator of the fraction.
- Don't leave out your ( )!
- Make sure you don't leave out the negative signs and find the undefined/zeros correctly.
- For additive inverses: Negate ONE of the terms
- Make sure you know if it's supposed to be greater than or less than 0 for rational inequalities.
- Make sure you know when to use brackets and/or parentheses.
- If there is a square of something, it will always be positive!
- Careful with this question…

$$\frac{x+2}{x+1} - \frac{x}{1-x} = \frac{x}{x-1}$$

- ***(1-x) is already a common factor of (x+1)(x-1) because it's the additive inverse of (x-1), thus you would disregard it in the numerator, negate the existing term, and then multiply by (x+1).

**Unit 3b: Asymptotes and Graphing Rational Functions**

**Summary:**

- Finding… (PUT EVERYTHING INTO LOWEST TERMS)
  - Vertical asymptotes
    - Set the denominator equal to zero
  - Horizontal asymptotes
    - n<m: y=0
    - n>m: none
    - n=m: divide leading coefficient
  - Oblique/Slant asymptotes
    - Divide the ORIGINAL equation using long division
    - Exclude the remainder
  - Holes
    - Sub x back into the lowest term equation to find the y-value
  - X-intercepts
    - Zeros in the numerator
  - Y-intercepts
    - Plug 0 in for x
  - Horizontal/oblique asymptote intercepts
    - HA: Only when n=m and then you can set the entire LOWEST TERM equation equal to the asymptote.
    - OA: Works for any slant asymptote; set the entire equation equal to the LOWEST TERM equation
  - End behaviors
    - As x approaches positive/negative infinite, y approaches horizontal asymptote.
- Rules for Graphing Rational Functions
  - Odd degrees = opposite infinites
  - Even degrees = same infinites
  - You can never make another x-intercept
  - You can cross the horizontal and oblique asymptotes, but you cannot cross the vertical asymptotes because that marks an undefined value
  - When there is a hole, you just straight through it

- **When you have an intercept at an asymptote, you either go straight through if there is no x-axis for another intercept or you have to make a bubble and come back up.**

- Special Cases
  - HA/OA Intercepts - look above
  - Indeterminants - this is just the hole
- Graphing Rational Functions: n<m
  - y=0, you cannot cross
- Graphing Rational Functions: n=m
  - y=#, you can have an intercept
- Graphing Rational Function: n>m
  - Slant asymptote
- Writing Equations of Rational Functions
  - Skeleton: k(x-intercepts)/(vertical asymptotes)
  - If n is not equal to m, then you need to plug in a point and solve for k.

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- Indeterminate = 0/0 (not possible)
  - This is considered the hole
- Whenever you're putting a function into the lowest term, you need to rewrite with a different function (ex. f(x) → g(x))
- n > m = no horizontal asymptote
- A hole is a point on the graph that indicates where it's undefined - occurs when you need to put a function in the lowest term
  - When there is a hole somewhere, it eliminates that line to be a vertical asymptote
- When something is not in lowest terms, you need to find the common factor and cancel things out to 1.
  - Even if variables are attached to a coefficient, you can divide out so long as they're separated by multiplication.
- When finding possible x-intercepts, you need to figure out if there is a hole or else that intercept would not be possible.
- If there is only a number in the numerator, an x-intercept is not possible.
- When finding the possible x-intercepts, you're always using the reduced form!
- After you get the lowest form, you can determine the vertical asymptote AND THEN the hole.
- The degree of n and m are determined by the degree of the leading coefficient (whether expanded or not).

- Nothing can cross the vertical asymptote because it will be undefined, but it can cross the horizontal asymptote.
- When you want to find the highest degree in the rational function, try simplifying it all out if needed.
  - Especially when they're equal and you need to divide.
- Tips for graphing:
  - The degree of the x-intercept is determined by the degree of the numerator.
  - When you have multiple expressions in ( ) in the denominator, their degrees are separate (do not try to add them).
  - You determine whether or not you go up or down by looking at how many x-intercepts you have (or none) - you cannot create MORE
  - Odd degrees are opposite and even degrees are same
  - **For determining what direction the graph goes after the y-intercept, you have to find the sign of the x-intercept OR look if there are any x-intercepts.**
  - When in doubt, literally just look for the x-intercepts.
- If there is something like 3x and x in the numerator and denominator, they can be cancelled out.
- **DOTS not SOTS**
- **No careless mistakes when factoring please**
- When expressions with ( ) are set equal to 0, you can make a t chart.
- Asymptotes can be crossed except vertical
- **YOU CAN NEVER MAKE NEW X-INTERCEPTS**
- Even if there is a common factor of one variable, you can have a hole.
- *****If there is a hole on the graph, the x-value of the hole needs to be included in both the numerator and denominator.**
- **Whether the graph bounces back up or through the asymptote intersection is dependent on the degree.**
- **Multiply by the entire equation of the OA when finding if there are points of intersection.**
- Get terms with denominators to figure out asymptotes.
- **When you're finding the intersections at the asymptote, make sure you use the LOWEST TERMS!!!**

**~Unit 4: Partial Function Decomposition**

**Summary:**

Steps (ALWAYS):

- Split
  - Dependent on what type of expressions there are: linear, non-linear, irreducible, repeating, non-repeating, etc
- Clear the denominators
  - Multiply everything out
- Find the roots
  - x=#
- Sub in for the letters
  - Expand them out and move to the other side
  - If you don't want to deal with fractions, make sure you multiply everything out to eliminate the fraction
- Equate coefficients through factoring OR expanding
  - Equating coefficients is more efficient, especially for the more advanced problems
  - Sometimes, you can use what is on the right side to factor out the left side, thus cancelling terms out on the other side

There are four different scenarios…
- Linear non-repeating denominators
  - Ex. x+1
  - Only need A and B (one letter)
- Linear repeating denominators
  - Ex. $x^2$
  - Only need A and B (one letter)
- Irreducible non-repeating quadratic factors
  - Ex. $x^2+4$
  - Need Ax+B to match the format of the quadratic FACTOR, but also one degree less
- Irreducible repeating quadratic factors
  - Ex. $(x^2+2)^2$
  - Need Ax+B to match the format of the quadratic FACTOR, but also one degree less
- Additional: Irreducible AND linear factors
  - Ex. $(x^2+2)^2(x-1)$
  - Need A and Bx+C because you have a combination of two

**Potential Blast from the Past Topics:**
- Finding asymptotes
  - Vertical asymptotes: Make sure there is NO hole; set denominator equal to 0

- Horizontal asymptotes…
    - n<m: y=0
    - n=m: y=#
    - n>m: NONE
  - Oblique asymptote: Only if n>m by ONE degree
    - Long division!
- Solving for rational functions
  - Undefined values are from the denominator and need an OPEN circle
  - Zeros are from the numerator and need a CLOSED circle IF it's less than/greater than or EQUAL to 0
  - Make a number line and figure out which sign (positive or negative) corresponds with each region

*If there is JUST less than or greater than, 0 is NOT included, so you do not need closed circles!

- Functions dividing by h
  - Sub everything in and divide by h at the end
  - You can check by looking at the terms at the end and in the beginning, two of them should correspond/be the same
- Composition functions
  - Careful… f(x) can equal to $x^2$, especially when we have "sinx"
    - Ex. $f(x) = x^2$ $g(x) = \sin x$ $h(x) = 5x$
  - When reading compositions: go from right to left
  - Work from the inner and then outer
- Solving inequalities with absolute values
  - If AV is only on one side…
    - Isolate the AV
    - Split into two: One regular and one negated without the AV
    - Solve!
  - If AV is on both sides…
    - Method 1: Square both sides of the inequality and then set them equal to 0 to solve
    - Method 2: Use all four cases to determine the interval notation solution set
- Finding the domain of a function
  - Radical in the denominator = greater than 0
  - Radical in the numerator = greater than OR equal to 0
  - Expression in the denominator = not equal to 0
- Finding the inverse of a function

    - Sub y in for x and x in for y and then isolate y
- POTENTIALLY: Graphing rational fractions
    - Find all asymptotes (look above)
    - Find the hole
    - Make sure you know which one's are even and odd degrees for opposite and even direction of the graph
    - Connect!

***MAKE SURE TO REJECT EXTRANEOUS SOLUTIONS**
***FOLLOW PEMDAS IN BLAST FROM THE PAST QUESTIONS**
***PLS MULTIPLY CORRECTLY**

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- Careful with multiplying everything out (expansion)
- There can always be a way to factor to cut down the amount that you need to expand
- Look at the right side of the equation to find a potential factor when factoring the left side in efforts to cancel out terms
- Careful when performing all four operations
- FACTOR CORRECTLY PLEASE
- Avoid careless mistakes pls…

**~Unit 5: Logarithmic Functions**
**Summary:**
**Characteristics of Exponents**
- Parent Function: $y=b^x$ (b>0, but cannot equal 1)
- Exponential Growth: b>1
- Exponential Decay: 0>b>1 (aka, a fraction)
- Horizontal asymptote (y=#) - this is found by +/- K
- There are no restrictions to the domain of exponential functions
- The y-intercept is ALWAYS (0,1) UNLESS there has been a shift upward/downward.

**Characteristics of Logarithms**
- Inverses of exponential equations
- Parent Function: $y=\log_b(x)$ (b>0, but cannot equal 1; x>0)
- When b is greater than 0, the log is the inverse of an exponential growth function
- When b is less than 1 and greater than 0, the log just looks like exponential decay
- Vertical asymptote (x=#) - this is found by setting the argument equal to 0
- The domain of the log is found by setting the argument GREATER THAN 0
- The x-intercept is ALWAYS (1,0) UNLESS there has been a shift to the left or right

**Transformations (examples will be using the number 8)**

- f(x+c) →horizontal shift c units to the left
- f(x-c) → horizontal shift c units to the right
- f(x)+c → vertical shift upward c units
- f(x)-c → vertical shift downward c units
- -f(x) → reflection over the x-axis
- f(-x) → reflection over the y-axis
- af(x), a>1 →vertical stretch by a scale factor of a
- af(x), 0<a<1 → vertical shrink by a scale factor of a
- f(ax), a>1 → horizontal shrink by a scale factor of a
- f(ax), 0<a<1 → horizontal stretch by a scale factor of a

**Graphing Exponentials and Logs**

- Determine parent function
    - Exponential growth vs. decay (increasing or decreasing)
    - Parent log functions:
        - When b>1 = inverse of exponential growth function
        - When 0<b<1 = exponential decay
- Determine if there are any transformations (look at the list above) and then perform them on the graph
- Find the domain and range of the function
    - Exponentials: No domain restrictions
    - Logarithms: No range restrictions
- Find the x and/or y intercepts
    - Sub in 0 for either x and y
    - If NO TRANSFORMATIONS are applied…
        - Exponential functions will always have a y-intercept of (0,1)
        - Log functions will always have an x-intercept of (1,0)
        - Remember that they are inverses
- Find the asymptotes
    - Exponentials will have horizontal asymptotes: look at +/- K
    - Logs will have vertical asymptotes: look inside of the argument and set it equal to 0
- State the end behavior (if needed)
    - As x approaches something, y approaches something
    - Intermediate behavior is when x is approaching something other than infinite (whether negative or positive), which is something to be concerned about for log functions only

**Solving Exponential Equations**

- Two Strategies:

    - Find the common base and set the exponents equal to each other
    - Take the log or natural log of each side of the equation and solve for x through division
        - Remember that if you do it this way, there is a possibility that you will need to distribute the log to each term and then pull the x out as the GCF
- You may need to use the change of base formula if asked to put the answer into $\log_b(a)$ form
    - Change of base formula: log(a)/log(b)
        - Works for natural logs as well
- Try to evaluate wherever you can
- Never reject x-values because there are no domain restrictions, unless the entire exponential equation equals a negative number (which is not possible).
- Sometimes you will need to make an exponent (ex. $3^x$) equal to a variable (ex. $3^x=y$) and then solve using factoring and then subbing the original back in.

**Properties of Logs**

- Product Rule: Addition = multiplication
- Quotient Rule: Subtraction = division
    - <u>CAREFUL:</u> log2 - log8 - log9 would require the log2 to be divided by the product of log8 and log9 because it's technically log2 - (log8 + log9).
- Power Rule: Coefficient = exponent
- Change of Base Formula: log(a)/log(b)
- Condensing is multiplication and division while expanding is addition and subtraction

**Solving Logarithmic Equations**

- Set the arguments equal to each other if the bases are the same and then solve for x
- Convert into an exponential equation if there is only one log and then solve for x
- If there are NO common bases…
    - Use the power rule (if applicable)
    - Use the change of base formula for every log you see
    - The third step varies…
        - Evaluate and then raise everything to the common denominator (usually a number)
        - Raise everything to the common denominator (usually a log) and then evaluate
        - Get the logs to have the same denominator by multiplying by a fraction equal to 1 (this number raised to what equals the common

denominator?) and then raise everything to the common denominator
    - Product rule (condensing)
    - Solve for x
- Always convert the smaller numbers into the larger numbers

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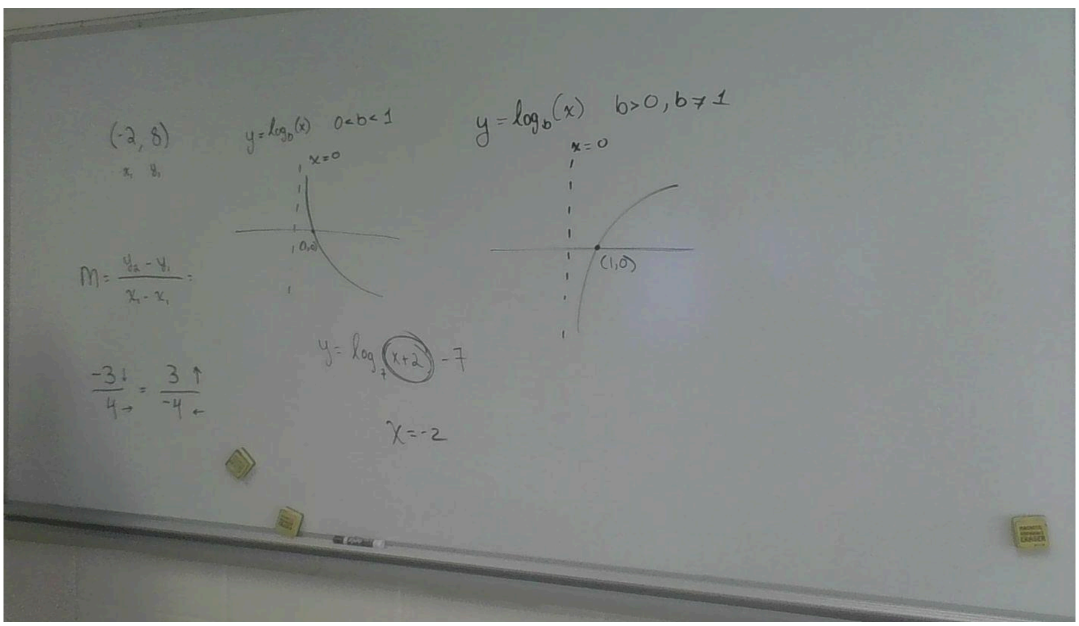
- In order to find the domain of a log function, you need to set the argument GREATER THAN 0.
- Whenever you have a horizontal shift to the left or right, you need to sub in 0 for the x value to find the new y-intercept.
- The +d is the equivalent of your horizontal asymptote for EXPONENTIAL FUNCTIONS.
- The x +/- # in ( ) is the equivalent of your vertical asymptote for LOGARITHMIC FUNCTIONS.
- Whenever you're finding the x-values for exponential functions, you don't need to reject any x-value!
    - Review why?...
- Whenever we have two subtraction signs in logs, you're multiplying the last two because of the negation (distribution).
- When you cannot set the arguments equal to each other, pull out the "ln" as a GCF and distribute afterwards.
    - Move x's to one side afterwards, divide, and use change of base formula.
- Change of base formula needs condensing and then putting a/b to convert to $\log_b a$.
- Always use natural logs (ln) first and then use the change of base formula to convert.
- Exponents can never equal 0 or a negative number (NOT POSSIBLE).
    - Thus reject if you get either one for an answer when solving for "x."
- Log properties apply to natural logs as well.
- There cannot be negative arguments for logs.
- Don't be fooled: Logs always have vertical asymptotes and exponents always have horizontal asymptotes.
- Power/root!!
- **( ) go over the ENTIRE TERM when rewriting logarithmic equations into exponential ones**
- **Evaluate whenever you can**
- **Remember that anything raised to the 0 power is 1**
- **Whenever you are doing the intermediate behavior, you need to have a plus sign as an exponent when x→#.**

- **REMEMBER: Keep → change → FLIP (TAKE THE RECIPROCAL)**
- Please read all of the transformations and end behaviors correctly…

From Delta Math…

- Logs and exponents with the same base will cancel each other out (in terms of the log being the exponent and the base being the same there and the exponent.
- An exponent applies to every term within the expression (if all in ( ) )
- Vertical asymptotes are x=#!
- When raising the bases to a power, you need ( ) over the ENTIRE THING
- **Reject solutions if they make the arguments negative!**
- When asked to rewrite an exponential function into a logarithmic function (or vice versa), don't overthink because they're just inverses.
- Go back to…
    - Logs and Exponentials as Inverses (graphically)
        - You just replace y with x and x with y and then solve for y.
        - When dividing by a negative, the exponent itself does not get affected.
    - Any transformations




From the picture…

- Know that these are the parent log functions
- The x-intercept will always be (1,0)
- There are vertical asymptotes so there wouldn't be a crossing at the y-axis
- When finding the vertical asymptote, completely disregard everything and only in the ( ).

**~Unit 6: Trigonometry**

**Summary of the Unit:**

**Trigonometry Refreshers (SOHCAHTOA, Special Angles, Unit Circle)**

- SOHCAHTOA
  - $\sin\theta$ = opp/hyp
  - $\cos\theta$ = adj/hyp
  - $\tan\theta$ = opp/adj
  - $\csc\theta = 1/\sin\theta$ or hyp/opp
  - $\sec\theta = 1/\cos\theta$ or hyp/adj
  - $\cot\theta = 1/\tan\theta$ or adj/opp
- Special Angles
  - $\sin(\pi/6) = 1/2$
  - $\cos(\pi/6) = \sqrt{3}/2$
  - $\tan(\pi/6) = \sqrt{3}/3$
  - $\sin(\pi/4) = \sqrt{2}/2$
  - $\cos(\pi/4) = \sqrt{2}/2$
  - $\tan(\pi/4) = 1$
  - $\sin(\pi/3) = \sqrt{3}/2$
  - $\cos(\pi/3) = 1/2$
  - $\tan(\pi/3) = \sqrt{3}$
- Unit Circle
  - Points: (0,1), (0,-1), (1,0), (-1,0)
  - Hypotenuse (aka radius) length is 1
  - Origin is the center
- Positive or Negative Trigs in Quadrants
  - All Students Take Calc
  - Trick: Start in quadrant I with the special angle (ex. $\pi/6$) and then work around in a counterclockwise order to fill in the other quadrants with numerators that have no association with the denominator/cannot be simplified (ex. next term would be $5\pi/6$).
- Finding the Exact Value
  - QRS - Quadrant, reference angle, and sign
  - If NEGATIVE…
    - Use coterminal angles and then QRS <u>OR</u> just go in a clockwise motion on the coordinate plane.
  - If RECIPROCAL…
    - Evaluate with QRS as if it was NOT a reciprocal
    - Put the exact value under one and then find the new value

    - If QUADRANTAL…
        - Use the unit circle OR the trick with sin, cos, and tan graphs that Ambrose taught last year

**Characteristics of Trig Graphs**

- Parent functions:
    - y=Asin[B(x-C/B)]+D
    - y=Acos[B(x-C/B)]+D
- Characteristics:
    - A = vertical stretch/shrink
    - |A| = amplitude (ALWAYS POSITIVE)
        - Distance between max or min and mid
    - B or |B| = frequency
        - Number of cycles within 2π
    - C/B = phase shift
        - Counter-intuitive shift to the right or left on the domain UNLESS B is negative
    - D = vertical shift or midline
    - Period: 2π/B
    - Finding B: Set 2π/B equal to the period and solve!

**Graphing Sine and Cosine Functions**

- **Sine**
    - Find all characteristics of the function
    - Graph in this order: mid → max → mid → min → mid
    - IF A IS NEGATIVE… mid → min → mid → max → mid
- **Cosine**
    - Find all characteristics of the function
    - Graph in this order: max → mid → min → mid → max
    - IF A IS NEGATIVE… min → mid → max → mid → min

***Look for any interval restrictions.**

***Factor out the B term if needed.**

**Graphing with Phase Shifts**

- Everything remains the same, except you would just factor out the B value to find the phase shift to perform the transformation.
- If the phase shift is EQUAL to the period, then you just keep the graph as is.
- If the phase shift is not equal to the x-scale, then you make the phase shift happen and increase/decrease based on the x-scale.

**Writing Sine and Cosine Equations**

- Find all of the characteristics and write the equation for the most obvious one (based on the y-axis).
- Write the OTHER equation by creating a phase shift (preferably to the right to make life easier).

***Do not overlook the amplitude… negative or positive.**

---

- **The reciprocal of 0 is undefined.**
- Don't forget to determine the B value when creating the equations of sine or cosine functions.
- Don't forget to factor out the B value in any situation.
- Make sure you draw the first graph without a phase shift and then a second one with it.
- Simplify the fractions when finding the exact value.
- The amplitude of the sine and cosine functions MAY NOT BE THE SAME depending on how you look at it from the FIRST AXIS.
    - Ex. There can be -2sin(x), but then 2cos(x+pi/2).
- READ THE AXIS SCALE CAREFULLY PLEASE!
- Check: Sub in ONE COORDINATE point that contains either a max or min to see if the equation checks out (is equal on both sides).
- Frequency = how many cycles in $2\pi$
- Absolute value of A is amplitude
- 9 points in 2 cycles, not 10
- Make sure you're using the actual phase shift, not the x-scale.
- Amplitude and frequency are both positive
- Make sure you know if it's a face shift to the left or right.
- Read the interval restriction carefully in the directions.
- You use the fraction given to you first when determining which quadrant it's in (NOT the reference angle).
- **When finding the face shift, you need to look at the MIDLINE when writing cosine or sine functions.**
- Take the square after evaluating the trigonometric function.
- **PLEASE DO NOT OVERLOOK THE PHASE SHIFTS!**
- When writing phase shift functions…
    - Look for the midline for sine functions
    - Look for the min or max for cosine functions
- If the period equals the phase shift, then there is NO difference in terms of the way you graph the function.
- **READ THE INTERVALS GIVEN IN THE DIRECTIONS.**

- **FIGURE OUT WHETHER POSITIVE OR NEGATIVE AMPLITUDE PLS.**
- **It's vertical stretch or shrink… not SHIFTING FOR THE VALUE.**

From Amplify Activity:

- Sine functions start from the midline
- Cosine functions start from a maximum, but if it starts at a minimum, you need to negate it
- The A value before the trig function sign is determined by finding the distance between the max/min and midline

Redo Delta Prompts:

- Equivalent Functions and Radian Angles
- Trig Values with Order of Operations
- Trig Graph Discovery (sin/cos) L5

**~Unit 7: Trigonometric Equations**

**Summary:**

**Logistics:**

- **Special Angles**
    - $\sin(\pi/6) = 1/2$
    - $\cos(\pi/6) = \sqrt{3}/2$
    - $\tan(\pi/6) = \sqrt{3}/3$
    - $\sin(\pi/4) = \sqrt{2}/2$
    - $\cos(\pi/4) = \sqrt{2}/2$
    - $\tan(\pi/4) = 1$
    - $\sin(\pi/3) = \sqrt{3}/2$
    - $\cos(\pi/3) = 1/2$
    - $\tan(\pi/3) = \sqrt{3}$
- **Quadrantals (unit circle)**
    - Sine
        - $\sin(0) = 0$
        - $\sin(\pi/2) = 1$
        - $\sin(\pi) = 0$
        - $\sin(3\pi/2) = -1$
    - Cosine
        - $\cos(0) = 1$
        - $\cos(\pi/2) = 0$
        - $\cos(\pi) = -1$
        - $\cos(3\pi/2) = 0$
    - Tangent

        - $\tan(0) = 0$
        - $\tan(\pi/2)$ = undefined
        - $\tan(\pi) = 0$
        - $\tan(3\pi/2)$ = undefined
- **All Students Take Calc**
- **Reciprocals**
    - $1/\sin\theta = \csc\theta$
    - $1/\cos\theta = \sec\theta$
    - $1/\tan\theta = \cot\theta$
- **Quotient Identities**
    - $\tan\theta = \sin\theta/\cos\theta$
    - $\cot\theta = \cos\theta/\sin\theta$
- **Pythagorean Identities**
    - $\sin^2\theta + \cos^2\theta = 1$
    - $1 + \cot^2\theta = \csc^2\theta$
    - $\tan^2\theta + 1 = \sec^2\theta$

*c's go together and t + s go together

**General Solution**

- Infinite solutions written in solution + $2\pi k$ where k is an integer (negative, positive, or 0)

**Steps for Solving Trig Equations (no identities or factoring)**

1) Isolate the trig function
2) Take the reciprocal if needed
3) QRS
4) If asking for ALL solutions and more can fit within the interval, add $2\pi$ (find coterminal angles) or REJECT solutions if outside the interval
5) Write solution set

**Steps for Solving Trig Equations with Let Statements**

1) Write a let statement for whatever is within the ( )
2) Solve using the normal procedure
3) Set the solution equal to what was originally in the ( ) (let statement)
4) Add/subtract $2\pi$ as many times as the frequency indicates WITHOUT BEING OUTSIDE OF THE INTERVAL - must add/subtract to ALL solutions
5) Solve for the unknown variable(s)
6) Reject an extraneous solutions if needed and write solution set

**Steps for Solving Trig Functions (identities or factoring)**

1) Either sub in an equivalent expression using an identity for the LEAST COMMON TRIG FUNCTION in the equation OR factor through trinomial factoring, DOTS, or grouping

    a) If you use DOTS, you don't need to care for extraneous solutions because you have essentially found the "hole"
2) Use normal solving procedure

**When do you reject extraneous solutions?**

1) Whenever there may be a FRACTION
    a) Multiply both sides of the equation with the same factor
    b) Hidden fractions in reciprocals and tangent… solutions cannot result in undefined answers
2) Whenever you SQUARE both sides of the equation because it will create MORE solutions (so sub back into the ORIGINAL equation and check)

**Inverses (not tested but need to know for midterms)**

*EVERYTHING MUST BE IN THE INTERVAL!!! (there can be more than 2 solutions if the interval is large enough).

---

- Taking the square root of something results in +/-
- Be cautious with the interval given (sometimes you will need to convert into fractions to see what solutions need to be rejected)
- Create let statements to make life easier
- Difference between $3\theta$ and $3x$
    - $3\theta$ means that there are three TIMES the solution
    - $3x$ means that you just need to create a let statement and solve for x by subbing in the solution into y afterwards
        - If asking to find ALL solutions, then you need to $+2\pi$
- **The reciprocal of 0 is UNDEFINED**
- **Never forget All Students Take Calc and Special Angles Chart - PLS DO NOT SCREW UP THE COSINE ONE**
    - **Remember that $\tan\theta=0$ is on the UNIT CIRCLE while $\tan\theta=1$ is on SPECIAL ANGLES**
- Make sure that you are within the interval and have ALL possible values included
- **EVERYTHING MUST BE WITHIN THE INTERVALS**
- **Tangent = sin/cos and you need to make sure cos is not 0 or else it's undefined!!!**
    - **Tangent has a HIDDEN fraction.**
    - **Also be aware of the reciprocal functions.**
- Squaring both sides creates extra solutions where you need to reject - but this should be the last resort.

From the Delta Math:

- When finding the general solution with different frequencies, you need to set the entire solution + $2\pi k$ equal to the original frequency and solve for x.
- REMEMBER THE SPECIAL ANGLES CHART PLS
- **When you take the square root of something = +/-**
- **When you are finding more solutions within the interval, you need to add/subtract 2pi from BOTH sides (not just one; interchanging).**

**Unit 8: Sum, Difference, Double, and Half Angle Identities**

- Try to pick things that do not have a fraction in the special angles chart
- EVERYTHING MUST BE RATIONALIZED
- **PLEASE REMEMBER THE SPECIAL ANGLES CHART WITH NEGATIVES AND POSITIVES!!!**
- NO BASIC MATH CALC ERRORS PLS
- When figuring out the identities, you need to think outside of the box…
    - Use pythagorean identities to simplify
    - Multiply by the fractions
- THERE HAS BEEN A MISCONCEPTION: $\cos 2\theta = \cos^2\theta - \sin^2\theta$
    - NOT $\sin^2\theta - \cos^2\theta$

Reminder to self on lesson 8.3:

- Procedure for half angle identities:
    - If it's just finding the exact value…
        - Determine the quadrant →determines the sign of your trig function
        - Divide denominator by ½ and that's the new theta
        - Plug in new theta everywhere
        - Rationalize/flip if needed
    - If it's finding the exact value WITH coordinate plane…
        - Divide by ½
        - Determine the quadrant → factors in what sign your trig will be
        - Plug everything
        - Rationalize/flip if needed

**Summary:**

**Sum and Difference Identities**

- sin(a+B) = sin(a)cos(B) + cos(a)sin(B)
- sin(a-B) = sin(a)cos(B) - cos(a)sin(B)
- cos(a+B) = cos(a)cos(B) - sin(a)sin(B)
- cos(a-B) = cos(a)cos(B) + sin(a)sin(B)
- tan(a+B) = tan(a)+tan(B)/1-tan(a)tan(B)

- $\tan(a-B) = \tan(a)-\tan(B)/1+\tan(a)\tan(B)$

**Double Identities**

- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\cos(2\theta) = \cos^2\theta - \sin^2\theta$
- $\cos(2\theta) = 2\cos^2\theta - 1$
- $\cos(2\theta) = 1 - 2\sin^2\theta$
- $\tan(2\theta) = 2\tan\theta/1-\tan^2\theta$

**Half Identities**

- $\sin(\theta/2)$ = +/- square root of $(1-\cos\theta)/2$
- $\cos(\theta/2)$ = +/- square root of $(1+\cos\theta)/2$
- $\tan(\theta/2)$ = +/- square root of $(1-\cos\theta)/(1+\cos\theta)$
- $\tan(\theta/2) = \sin\theta/1+\cos\theta$
- $\tan(\theta/2) = (1-\cos\theta)/\sin\theta$

**Procedure for Finding the Exact Values**

- Sum and Differences
  - Expand the addition/subtraction inside
  - Simplify
  - Expand with the identity
  - Evaluate
  - Flip if needed and rationalize
- Double Angles
  - Substitute in the entire equation for double identity
  - Expand, evaluate, and rationalize
- Half Identities
  - Figure out the quadrant and sign of the trig function with either the reference triangle or placement of the fraction
  - Plug in everything for theta
    - Divide denominator by 2 if needed when you have $\theta/2$ when it's only a problem like tan(5pi/8) without the need for reference triangles.
  - Evaluate, flip if needed, and rationalize
- Reference Triangles
  - When it's just $\theta$…
    - Draw in the reference triangle in the respective quadrant
    - Use the pythagorean theorem to figure out missing side
    - Figure out the different ratios and signs for trig functions
  - When it's $\theta/2$…
    - Divide everything by 2

        - Place triangle in respective quadrant
        - Use the pythagorean theorem to figure out the missing side
        - Determine the ratios and signs for the trig functions

*$\theta$ only applies for signs with $\theta$ and $\theta/2$ only applies for signs with $\theta/2$.

- Equating Identities

**Other Important Stuff:**

- Use the $\sin\theta/1+\cos\theta$ for the half identity for tangent because you don't need to worry about the sign of the angle.
- Make sure you do all of your calculations correctly…
- You need the special angles chart memorized
- Make sure you don't allow any undefined values from coming in
- Remember sine keeps the sign and cosine changes the signs.
- Never forget to take the reciprocal if needed
- You may need different signs throughout the entire problem (not one slated)

**Unit 9: Polar Coordinates and Graphing**

- Make sure you look at the interval that the question is asking for an answer in
- Keep same and +/- 2pi <u>OR</u> negate and +/- pi
- When finding the polar points…
    - You may not be getting the answer directly… **you get the reference angle but must apply it correctly based on the quadrants.**
    - We usually start with +r (when using the formula) and if we need it to be negated, then we will negate it.
- When converting questions…
    - Polar → rectangular: $x^2 + y^2 = \dots$
    - Rectangular → polar: $r = \dots$
    - **NEVER FORGET TO COMPLETE THE SQUARE FOR POLAR INTO RECTANGULAR**
- When you need to convert $\theta$=5pi/6 into rectangular…
    - $\tan\theta$=tan(5pi/6)
    - y/x=-rad 3
    - y=-x(rad 3)
- When you need to convert y=-1/2x into polar…
    - y/x=-½
    - $\tan(\theta)$=-½
    - $\theta$=5pi/6 OR $\theta$=11pi

Whatever you wrote here is not possible

- $r=\sqrt{x^2+y^2}$, NOT x+y

- $r^2=x^2+y^2$
- You don't always need to square the r's…
  - $r\cos\theta=x$
  - $r\sin\theta=y$
  - $\cos\theta=x/r$
  - $\sin\theta=y/r$
  - $r\sec\theta=1/x$
  - $r\csc\theta=1/y$
  - $\sec\theta=r/x$
  - $\csc\theta=r/y$

*Normal is variable on top
*Reciprocal is r on top

- You need to remember the trig special angles chart
- When converting polar equations into rectangular equations, they must be in…
  - x=something or y=something
  - $x^2+y^2$ = something
- When converting equations into polar equations, they must be in…
  - r=something or θ=something
- Please do not commit any silly graphing mistakes
- Honestly, when in doubt, just square everything to make it an even equation and then everything is fine

**Summary of Unit 9:**

- Plotting Polar Points
  - +r = right
  - -r = left
  - +θ = counterclockwise
  - -θ = clockwise
  - (r, θ+/-2pi)
  - (-r, θ+/-pi)
- Converting Points
  - Polar → rectangular
    - Graph first
    - $x=r\cos\theta$
    - $y=r\sin\theta$
  - Rectangular → polar
    - Graph first
    - r=square root of $x^2+y^2$
      - Doesn't matter if you use positive or negative

        - $\tan^{-1}(y/x)=\theta$
- Converting Equations
    - Polar → rectangular
        - Steps:
            - Either multiply both sides by r OR square both sides OR sub in using the following substitutions and proceed to solve
            - Place in standard form (complete the square if needed)
                - Can be in circle/quadratic/linear equations
        - Sub $x^2+y^2$ for $r^2$
        - Sub square root of $x^2+y^2$ for r
        - Sub x for $r\cos\theta$
        - Sub y for $r\sin\theta$
    - Rectangular → polar
        - Do the reverse from above
        - If the question is asking for $\theta=5\pi/3$…
            - $\tan\theta=\tan(5\pi/3)$
            - y/x=-rad 3
            - y=-x(rad3)
- Other Substitutions:
    - $\cos\theta = x/r$
    - $\sin\theta = y/r$
    - $\sec\theta = r/x$
    - $\csc\theta = r/y$

**Summary of Unit 9b:**

**~Summary**

- ETIS
    - Extremes
        - Sine or cosine equals -1 and 1
        - When you have a number in front of theta, you need to multiply and divide by that many times
    - Tangents
        - r=0
        - These will be dashed lines and they can be crossed (sometimes)
    - Intercepts
        - These are points of intersection when θ=0, pi/2, pi, or 3pi/2
        - If you don't see the points already on the extremes or tangents, you need to plug in to figure it out
    - Symmetry

        - Sine: θ=pi/2
        - Cosine: θ=0
        - Leminscates (ONLY): POLE (for either trig function)
- Circles: Self-explanatory
- Cardioids: Flattened heart shape
    - r=a+bsinθ
    - |a|=|b|
    - Curved and flattened at the least extreme
- Limacons without loop: Dented circle
    - r=a+bsinθ
    - |a|>|b|
    - There is no intersection at the pole
    - There is a small dimple at the least extreme
- Limacons with loop: Looped circle
    - r=a+bsinθ
    - |a|<|b|
    - There is an intersection at the pole
    - Loops around between the tangents and curves from the pole
- Rose Curves: Flower shape
    - r=asin(nθ)
    - Even "n" = double
    - Odd "n" = same
    - Skip over the tangents
- Leminscates: Helicopter top shape
    - $r^2=a^2\sin(2\theta)$
    - Do not forget about the +/- when solving for r
    - Symmetric over the pole ALWAYS regardless of the trig function

**~Important Points**

- When graphing limacons WITH a pole, you need to hump twice before coming back up from the pole.
- When graphing when the extremes are also the tangents, you just label with tangents (dashed lines).
- Leminscates ALWAYS have symmetry with the pole, nothing else!

**Unit 10: Limits**

- Whenever you are multiplying by the conjugate or LCD, do not forget to multiply by the denominator too
- Do not forget your notation… extremely important

    - “Therefore” statements
- READ CAREFULLY → NO DUMB ERRORS
- BE CAREFUL WITH THE ARTHIMETICS

**Summary of the Unit:**

- **Determining the Limit**
    - Read the graph
    - As x approaches the value, the intended y-value is…
    - If both sides do not reach the same place, then the limit is DNE.
    - If the graph does not cross a point or there is an open circle, the function is undefined.
- **Limit Notation**
    - lim(f(x))=# (with x→a^+/- below the lim)
    - You must keep this notation until you sub in a value
    - When solving for piecewise functions, you need to set the limits equal to each other after finding their values and then write a statement (with the three dots).
- **Finding the Limit Algebraically**
    - Procedure:
        - Sub in → Write answer if there is a number or undefined
        - If indeterminate…
            - Factor out to find the common factor and divide out
            - Multiply by the LCD or conjugate (opposite sign)

*Whatever the x value is approaching, the common factor should contain it in some form

*The hole is the common factor; the VA is in the denominator

- **Limits with Piecewise Functions**
    - Very self-explanatory
    - Set them equal to each other with limit notation
- **Limits with Absolute Values**
    - You switch the sign depending on if the x-value is approaching from left and right AND if it’s making the expression inside the absolute values positive or negative (THIS will determine if you negate or not)

**Blast from the past topics (just in case):**

- $\cos 2\theta = 1 - 2\sin^2\theta$ OR $2\cos^2\theta - 1$
- $\sin 2\theta = 2\sin\theta\cos\theta$
- $\sin^2\theta + \cos^2\theta = 1$
- Common denominator when solving

**Unit 10b: Limits**

- When checking for limits, you need to check on both sides and sub in to determine if it's negative/positive infinite(s) or DNE.
- **You always have to check for indeterminants first**
- Pls do not make silly calculation errors
- When you have absolute values, you want to negate it to make it positive (if applicable)
- Procedure for finding the limit:
  - Always sub in first
  - Split the fraction into an equivalent form
  - Make sure the trig trick applies
- In the Squeeze Theorem, it needs to be that the outer two have to be equal to each other to determine the middle function.
- Tanx → sinx/cosx (DO NOT FORGET)
- REMEMBER SPECIAL ANGLE CHART
- Do not forget about ASTC
- When x approaches (+/-) INFINITY → THREE CASES
- When direct substitution of x is UNDEFINED → intended y-value is (+/-) INFINITY OR FACTOR
- Whenever you get a question with xsin(fraction), use the squeeze theorem to determine the limit (number or DNE).
- Differentiation:
  - When x approaches number
    - Factoring
    - Intended y-value = +/- infinite (if undefined)
  - When x approaches 0
    - TRIG TRICKS
  - When x approaches infinite
    - Three case of exponents
    - EPL (regular unless floating negative)
  - When x approaches negative infinite
    - Sub in -infinite
    - EPL (determine top or bottom heavy first)
- Procedure for Limit Properties:
  - If lim(f(x))=L and lim(g(x))=G, then you can perform all four operations on it
  - If limits do not exist for the functions, then…

        - Figure out the direction they are coming from by finding the limit from either left or right first and seeing if they are the same, then that's your answer
- Procedure for Composition:
    - Find inner function limit
    - If x=L for f(x) is continuous…
        - Regular substitution
    - If x=L for f(x) is NOT continuous…
        - Determine direction from L to the approaching x value
            - From top = right
            - From bottom = left
        - Evaluate limit when x is approaching L from the left/right

## *EVERYTHING IS WITH THE L

From the Delta Math:

- As x approaches infinite and you have $e^{-x}$, then the limit approaches 0 because you continuously divide by a larger number.
- Finding HA:
    - Find the limit when x approaches both negative and positive infinites
    - If it's a real number, that's the HA
    - If there is no real number, then there is no HA
- When solving for trig functions, you may need to move some stuff around

**Summation of the Unit:**

Undefined Limits: For any direct substitution

- If direct substitution causes an undefined limit without factoring or radicals/LCDs to multiply by…
    - Sub in number depending on the x value that is being approached (ex. $5^-$ would be 4.9999)
    - Determine if it's negative or positive infinite
    - ***If the x-value that is being approached does not have the direction that it's coming from, you have to test in both directions and MAY get DNE**

Three Cases: For when the x-value approaches the infinities (positive or negative)

- Case 1: n<m
    - Limit is 0
- Case 2: n=m
    - Limit is the ratio between the leading coefficients
- Case 3: n>m
    - Limit is either negative or positive infinite after substitution (of either negative or positive infinite)

EPL: For when the x-value is approaching positive infinity

- Exponential > polynomial > logarithmic
- Top heavy = positive infinite
- Bottom heavy = 0
- Same growth rate = ratio between the leading coefficients (or look at the degrees and use the cases)
- Be cautious if there is a floating negative (negate the end product)
- If the approaching x-value is negative infinity, then you need to determine if it's top or bottom heavy first

Trig Tricks: For when the x-value is approaching 0

- sin(x)/x=1
- (1-cos(x))/x=0
- Procedure:
    - If the approaching x-value is 0 and original substitution is indeterminate…
    - Break up the function into an equivalent form with one of the trig tricks
    - Perform operations to find the limit

**Squeeze Limit Theorem (not tested)**

- If h(x)<f(x)<g(x) and if lim(h(x))=lim(g(x))=L, then lim(f(x))=L.

**Limit Properties:** As x approaches any value

- If lim(f(x))=L and lim(g(x))=G, then all four operations can be performed to find the new limit
- However, if one of the limits is DNE for either function…
    - Determine if there is an existing limit by using directions
    - Find the limit from the left and right sides (separately) and if they're equal, then that's your answer.
        - If they're not equal, then it's DNE

**Limit Composition:** As x approaches any value

- Find the limit of the inner function first
    - If it's DNE, then the entire answer will be DNE
- Determine whether or not the outer function is continuous at x=L
- If the outer function IS continuous, then you can find the limit by using the composition.
- If the outer function IS NOT continuous…
    - Go back to the inner function → find the direction of the limit (L) to the approaching x-value in the original question
        - If it's top to bottom, then the limit is coming from the right side
        - If it's bottom to top, then the limit is coming from the left side

    - Go back to the outer function and find the lim(f(x)) as x is approaching the L from the inner function limit

*EVERYTHING REVOLVES AROUND L AFTERWARDS

**Other Important Stuff:**

- No silly mistakes
- Hole = non-continuous (no matter what)
- If the approaching x-value does not indicate what direction the limit is coming from, you must substitute both sides if the original function is undefined.
- You are always subbing in first

**Unit 11: Continuity**

- Always simplify to eliminate the hole in every case possible.

**Summation:**

- Types of Continuity:
    - Continuity: For every x-value, there is an existing y-value in a given interval.
    - Removable Discontinuity (Point): When there is only a hole in existence
        - Found as the hole in rational functions
    - Non-Removable Discontinuity (Asymptotic or Jump)
        - Asymptotic: Vertical asymptote exists (found if there is a denominator)
        - Jump: When there is a separate, filled point (found if f(x) doesn't equal the limit)

*There can be different discontinuities in different parts of the graph

- CRUCIAL FORMULA FOR CONTINUITY:
    - f(x) = lim f(x) as x→# (general)
    - f(x) = lim f(x) as $x \to n^-$ = lim f(x) as $x \to n^+$ (for piecewise usually)
- Solving for k
    - Set the entire equation equal to each other and solve
- Intermediate Value Theory: If f(x) is continuous over interval [a,b] and f(x)=L, then there is an existing value for x where a<x<b for all values of x over the interval [a,b].
    - **Less complicated: By IVT, there exists a value for c where f(c)=L.**
- Proving IVT applies:
    - Prove continuity - all polynomials are continuous
    - Sub in the intervals for f(x)
    - Write statement (summarized above)

**Unit 12a (12.1): Derivatives**

- Find the root for synthetic division by plugging in 0 for the x value
- No careless errors

**Summation:**

- Limit Definition: Constant change in slope or the slope of the tangent line between two points where h is approaching 0.
  - h = distance between two points on a function
- FORMULA FOR LIMIT DEFINITION:
  - $f'(x) = \lim(h \to 0)\ f(x+h) - f(x)\ /\ h$
- Procedure:
  - Using the formula above, sub in everything into the original function
    - (x+h) must be subbed into x for the first time you write the equation
  - Expand and divide everything out
  - Sub in 0 for h
  - Whatever you have remaining is the answer (write: f'(x)=(answer))
- Trick for Checking: Multiply the exponent by the leading coefficient, drop the degree one down, and then only take the coefficient of the 2nd term → this is your answer

**Unit 12b: Derivatives:**

- The constant rule applies to anything with pi
- Do not make stupid arithmetic mistakes
  - Do not forget about the radicals (+/- too) and negative signs
  - Just expand everything on your paper
- The limit definition of a function should be as h approaches 0… not infinite
- Make sure you remember to take the negative sign of the trig derivatives (for everything that starts with "c"
- Take the reciprocal of the inverse function BEFORE multiplying everything out
- Whenever you have an exponent with trig function, you will ALWAYS use chain rule
- Whenever there is a polynomial (or something other than "x" alone in the ( )), then you need chain rule
- Make sure you differentiate between the quotient and product rules…
- **CAREFUL WITH THE SIGNS OF THE SLOPES (+/-)**
- Unless you are solving for the value of x, you must take the positive value of the radical.
- Always simplify before expanding
- Make sure you can identify the product rule (ex. y=xsinx)

    - Whenever there is a variable attached to the constant, Y
    - YOU MUST USE PRODUCT RULE!
- DO NOT FORGET THE EXPONENTS
- DO NOT FORGET TO TAKE THE RECIPROCAL

**Summary of the Unit:**

**Part 1:**

- Limit Definition of a Derivative: lim(f(x)) as h→0=f(x+h)-f(x)/h
    - Sub in 0 for h after fully simplified
    - If asked what the answer is, you can take a short cut by using the power rule to find f′(x) based on the filled in equation
- Finding the Derivative (slope of the tangent line OR instantaneous rate of change)
    - Power Rule: Multiply the exponent and leading coefficient and then subtract 1 from the exponent
    - Sum and Difference Rule: Can do all at one shot using the power rule
    - Product Rule: h′(x)=f′(x)g(x)+g′(x)f(x)
        - Only use if expanding is extremely difficult
    - Quotient Rule: $h'(x)=(f'(x)g(x)-g'(x)f(x))/(g(x))^2$
        - Longest of them all; only use when there is a polynomial in the denominator
- Equation of the Tangent Line: y-y1=m(tan)(x-x1)
    - Essentially point-slope form
- Equation of the Normal Line: y-y1=-1/m(tan)(x-x1)
- Standard Form: Ax + By = C
- Y-intercept Form: y=mx+b
- Finding m(tan) through equations…
    - Sub in the x-value into the original equation to find y1 if not given
    - Find the derivative of f(x)
    - Sub in the x-value to find m(tan)
    - Write the equation of either the tangent or normal (perpendicular to tangent) line
- Finding points for horizontal tangents to the derivative…
    - Find the derivative of f(x) and set equal to 0
    - Solve to find the roots
    - Sub the x-values back into the ORIGINAL equation to find the y-values or the horizontal tangent line (y=#)
- Constant Rule = Derivative of a constant is always 0 even if it’s pi
- When simplifying…

    - Rewrite after using power/root (in radical form)
    - Negative exponents are brought to the opposite side of the fraction

*If the question is find the limit for ((1+h)^2+1^20)/h, it's going to be the limit of $x^{20}$

**Part 2:**

- Chain Rule:
    - $f(g(x)) \rightarrow f'(g(x))(g'(x))$
    - If there is product or quotient rule involved, you would take the derivative in the $(g'(x))$ and then factor OR expand everything out
- Derivative of Trig Functions:
    - $\sin(x) = \cos(x)$
    - $\cos(x) = -\sin(x)$
    - $\tan(x) = \sec^2(x)$
    - $\csc(x) -\csc(x)\cot(x)$
    - $\sec(x) = \sec(x)\tan(x)$
    - $\cot(x) = -\csc^2(x)$
- When taking the derivative of trig functions…
    - Rewrite the functions if there are complex fractions (simplify) or exponents
    - May use the chain rule TWICE if there are 2+ ( ) or [ ].