String Length vs Oscillation Time

(Return) Chloe Farris

Introduction

In elementary school, while others played tag, soccer, wallball, or story games at recess, I was on the swings. Whether it was alone or with others, there was always entertainment with the swings. Competitions to see who could swing the highest, who could make the goofiest swinging pattern, or who could spin the swing the fastest was what flowed through my mind. The swings were always a space for thought, intrigued by its workings, and how someone would have produced it.

The swing everyone knows today was created in the early 1900s by a man named Charles Wicksteed[1]. The creation of playgrounds allowed for children to play somewhere other than the highways, and for promised enjoyment to be located[2]. A swing is a pendulum; a mass that hangs from an overhead bar by a pair of chains. The swing must move from its equilibrium position to begin swinging back and forth. As the position moves outwards and upwards gravity will then pull the swing towards the initial position. However, the speed found at the equilibrium point causes the swing to overshoot its intended destination[3]. This causes a repeat of gravity in the opposite direction. In the case of a spinning swing, it would emulate a similar pattern.

A swing has multiple ways to represent an oscillating system. The swing can swing like a pendulum, or it can spin around its center of mass. A simple pendulum can be described as a mass attached to massless rod and is only able to move along the arch formed from the pivotal point of the rod[4]. A simple pendulum can also rotate about its center of mass, instead of swinging like a playground swing. This rotation done about the central mass of the object, can resemble an oscillating system. It can adjust to an amplitude, trying to return to its equilibrium, and swing past it to an apposing amplitude. This exhibits simple harmonic motion, as the swing oscillates around its equilibrium position. Simple harmonic motion is repetitive, using T as the period of time, it measures the time it takes an object to complete a singular cycle and return to its starting position. The inverse of this period is frequency ($f = \frac{1}{T}$). The angular frequency can be defined as $\omega = \frac{2\pi}{T}$, measures in radians per second, we use this to find the rotation rate of a mass in periodic motion. With this information we can infer that $f = \frac{1}{T} = \frac{2\pi}{T}$, which would give us the total number of oscillations per time unit. This is measured using Hertz ($1\ Hz = \frac{1}{s}$). Within simple harmonic motion, acceleration ($a = -\omega^2 x$) is proportional to the displacement just in the opposite

[1] (“When Were Swings Invented | History of Playgrounds”)
[2] (Winder)
[3] (McCormick)
[4] (Chasnov)

direction. If an object shows signs of simple harmonic motion, it must have a force acting on it. This force can be represented as $F = ma = -m\omega^2 x$, which obeys Hooke's Law[5]. Hooke's Law states that the strain of a material is proportional to the stress applied within the materials elastic limit[6], represented by the formula $F = -kx$, and k can be written as $k = m\omega^2$. As an object attached to a string is moved away from its lowest point, the string will exert a force on the object that is proportional to the displacement of the string. As an object is displaced/turned to a position away from its equilibrium, the string will become stressed or compressed, with a reaction to exert a force on the object that is proportional to its displacement. When this force is exerted, it will cause the object to accelerate as it is trying to return to its equilibrium position. In its attempts to return to the position it will be at its maximum speed as it crosses equilibrium. After crossing equilibrium it will begin decelerating, once again trying to return to its equilibrium position. Repeating the motion in effort to return to the equilibrium position, creating an oscillating system as it repeats back and forth. In regard to that information, the equation to find the angular frequency would be $\omega = \sqrt{\frac{k}{m}}$, the equation to find the period would be $T = 2\pi\sqrt{\frac{m}{k}}$, and the equation to find the frequency would be $F = \frac{1}{2\pi} \times \sqrt{\frac{k}{m}}$.[7] If an object suspended on strings is displaced away from its equilibrium position, it will presume to have a similar period many times over. A question within this pendulum system can be found. What would happen to a pendulum if it were twisted, instead of swung? Instead of a pendulum exhibiting behaviors of a playground swing, it would exhibit behaviors of twisting and rotating about its center of mass.

The purpose of this investigation is to determine if there is a relationship between the length of a pendulum's strings and the time it takes to complete 10 rotations. Different string lengths are used to help determine the effect rotating a pendulum has on the period of time. Control variables include the mass of the rod, the angle of the systems displacement and the string separation.

I believe that when the string length is shortened, the rotational period will shorten. The shorter the string length the less time it takes to return to equilibrium. Similar to a spring when it is compressed more, it will return with more force.

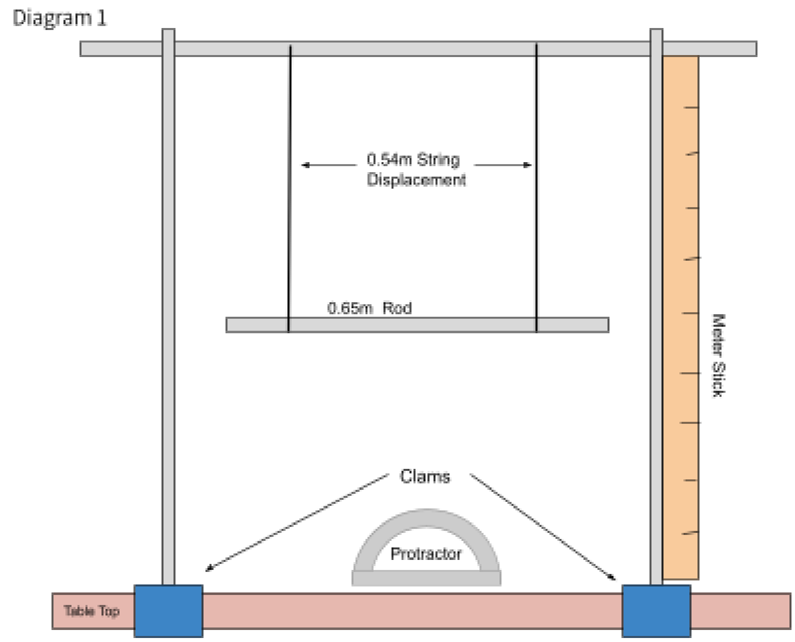
Method

For this investigation, the equipment used includes:

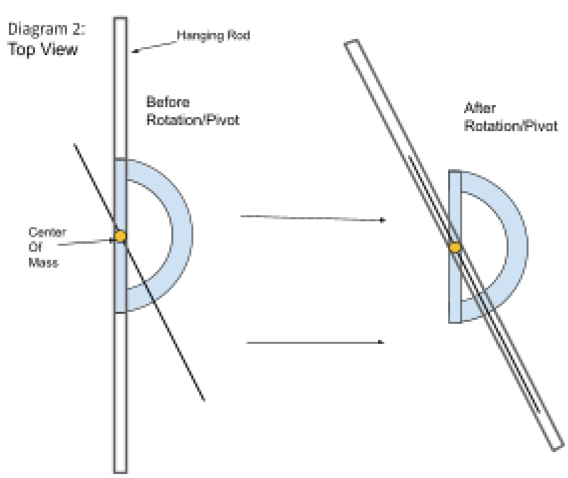
- Metal rod < 1 Meter in length
- Two 1.2 meter long strings
- 3 Metal rods > 1 Meter in Length
- Two table top clamps
- Two clamps that can clamp the rods to the pendulum rod, they look like little S's(I call them S clamps)
- Protractor

[5] (Chasnov)
[6] (The Editors of Encyclopedia Britannica)
[7] ("Harmonic Motion")

- Meter Stick
- Stopwatch
- Tape



In order to set this up, first you will need to attached the clamps to the main support rods(the vertical rods in diagram 1). One S clamp on each vertical rod is used to attach the horizontal rod at the top. Using table top clamps, the vertical rods to the table or surface that you will be conducting the experiment using. One vertical rod will have the meter stick attached. The meter stick will be attached with 0 at the top and 1 meter marker at the bottom. After finishing that you will attach one end of each string to the smaller rod, giving close to 0.54m of space between them. It is more important that the string is evenly spaced away from the pendulum rod edges than from each other (reference diagram 1 if confused). Tie a knot, and then tape over the knot to secure the string in place. Once the string is in place, you will set the rod on the table below the support rod. Extending the strings to tie them onto the supporting rod at an equal distance to the spacing on the hanging rod. Once tied onto the supporting rod, spin the knot around the rod, until both strings are pulled to a similar strain. Then you will tape over the string at this spot to maintain the tension on the strings. Once the string is taped in place you will loosen the clamps holding the supporting rod, as you will then spin the supporting rod to raise the height of the hanging rod to the 0.85 meter label on the meter stick. Once this part is set up, you will use the protractor to identify an angle that you wish the center rod to turn to. The angle personally chosen was 30°, and once the angle is selected, you will mark this on a piece of paper, or on the protractor itself. This will be your identifier as to how far to turn the rod every time. Place the protractor/paper with the angle, directly below the rod (lower the rod if necessary to line up where the protractor is with the rod). Finally, make sure your stopwatch/time taking device is on standby for data collection.



Adjusting the rod so it is hanging at the 0.85m marker to start, you will then look downward on the system, to see the rod and the angle marked below it. Displacing one end of the rod, as to make it cover the marked angle below is how to line up the rod to its amplitude (see diagram 2). Once the rod is rotated or displaced to this amplitude, you will let go of the rod and start your stopwatch at the same time. Count 10 times that the rod oscillates, stopping the timer when you get to the end of the tenth oscillation. Repeat this recording of oscillations for 5 total trials at this length, once you have finished these you will loosen the clamps and turn the support rod until the hanging rod is at 0.8m, repeating the process again. You will do this for 10 total string lengths at the lengths of 0.85m, 0.80m, 0.75m, 0.70m, 0.65m, 0.60m, 0.55m, 0.50m, 0.45m, 0.40m, and 0.35m. The choice of these lengths was purely based on set up space, and the equipment available. Had there been more vertical space to conduct more variations, there would be more to analyze. The ability to control consistency the closer the hanging rod came to the support rod was difficult, as the string length can only be shorten so far to still maintain the angle of displacement. The time taken by one individual, with consistency in what they determined the completion of an oscillation to be, is how the dependent variable remained consistent. The mass of the rod remained constant as no additional mass was added or removed. The angle displaced remained consistent as the person adjusting the rod adjusted similarly every time, and to ensure that the human error was to affect the results less, an average was calculated for each length of string. The constant of string separation remained evenly spaced throughout, as the string rolled over itself, it consistently rolled on the right side of the central tie, making the upper separation consistent, while never changing the lower separation.

Results

Data Link

String Length	Time(s)					Average	Reaction time Uncertainty
Meters							(+/-) 0.29
0.80	11.87	12.01	11.63	11.80	11.60	11.78	0.29
0.75	11.33	11.55	11.32	11.38	11.31	11.38	0.29
0.70	11.16	11.12	11.15	11.09	11.08	11.12	0.29
0.65	10.8	10.81	10.58	10.71	10.68	10.72	0.29
0.60	10.22	10.27	10.34	10.33	10.22	10.28	0.29
0.55	9.88	10.06	9.95	9.87	9.84	9.92	0.29
0.50	9.65	9.42	9.55	9.68	9.64	9.59	0.29
0.45	9.05	9.39	9.09	9.14	9.02	9.14	0.29
0.40	8.54	8.67	8.69	8.53	8.57	8.60	0.29
0.35	8.04	8.21	8.14	8.14	8.35	8.18	0.29

Mass of hanging rod

0.2154 kilograms

Displacement Distance from Equilibrium

$$X_o = (\frac{0.65}{2}) \times sin(30) = 0.16\, m$$

Averages Calculation

$$(\frac{11.87+12.01+11.63+11.80+11.60}{5}) = 11.78\, s$$

Log Calculations

$$ln(0.8) = -0.22$$

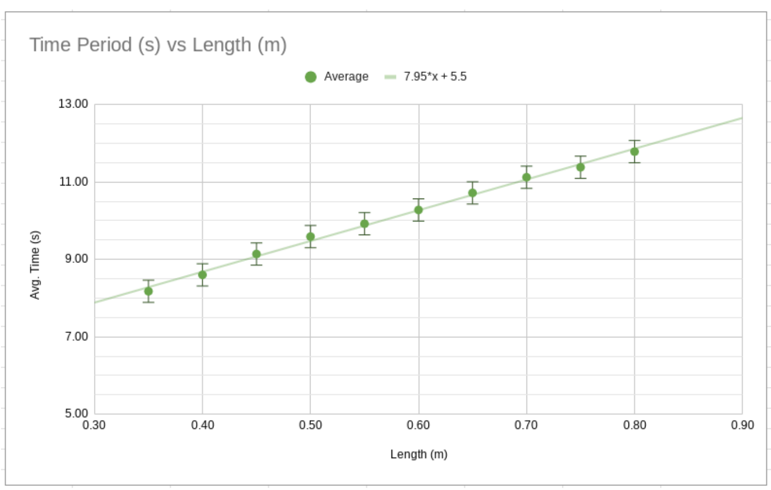
Power Function Calculations

$$y = Ax^n$$

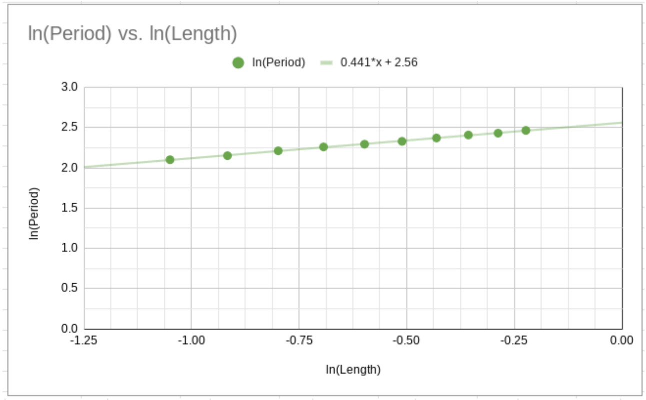
$$log(y) = nlog(x) + log(A)$$

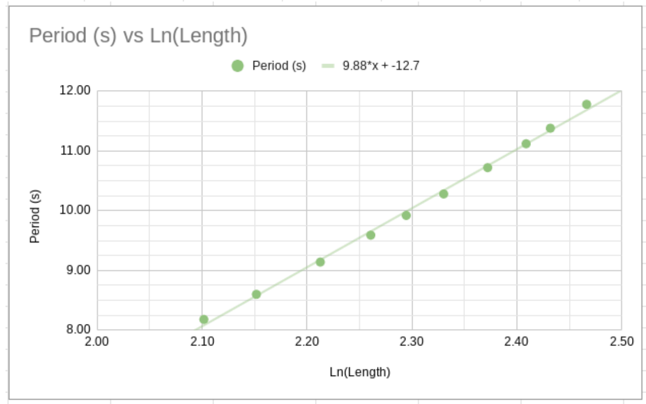
$$A = e^{2.56} = 12.94$$

$$n = 0.441$$



Length (m)	Period (s)	ln(Length)	ln(Period)
0.80	11.78	-0.22	2.47
0.75	11.38	-0.29	2.43
0.70	11.12	-0.36	2.41
0.65	10.72	-0.43	2.37
0.60	10.28	-0.51	2.33
0.55	9.92	-0.60	2.29
0.50	9.59	-0.69	2.26
0.45	9.14	-0.80	2.21
0.40	8.60	-0.92	2.15
0.35	8.18	-1.05	2.10





Conclusion

The results of this experiment show that the relationship between the length of a pendulums swing, and the time period of its rotations have a relationship shown with a downwardly concave trend. This trend can be seen in the *Time Period vs Length* graph, the concave trend led to the use of the natural log graph. The use of the log graph is to help define the coefficient and the exponent that would be affecting the independent variable. The coefficient of the function was found to be 12.94, and the exponent was found to be 0.441. The exponent being close to 0.5, which could be rewritten as a square root. The period of a pendulum function is defined as $T = 2\pi\sqrt{\frac{L}{g}}$ [8], which would mean that the exponent deduced within this experiment matches that of a pendulum's period function. The function for the time period of a pendulum is nearly the same as the function for the time period of an oscillating system, due to the fact they do roughly the same thing. This relation shows that the relationship between the length of the strings has a correlation with the time period. This conclusion leads to proving my hypothesis, that the shorter the string length the shorter the rotational period. When the string length increases, the period of time will increase due to the string's length and the inertia created by the acceleration of gravity. This can be explained by Hooke's Law, the strain of a material is proportional to the stress applied within the materials elastic limit[9]. While this experiment is centered around a pendulum

[8] (Murray)
[9] (The Editors of Encyclopedia Britannica)

swinging about its center of mass, Hooke's Law helps explain that string length aids to the concave data exhibited within the *Time Period vs Length* graph. The last graph displayed is the linearization graph, showing what the data would look like with the input value to the exponent found from the natural log graph.

A challenge found while collecting data was that of limited working space to acquire different variations. All the variations are close together, this is due to lack of vertical space to play around with, as the space used for this experiment had a clearance of a meter. There was space to acquire string lengths of 0.85 m but after that the space would have become too close to the table that the rods were clamped to. The risk of getting faulty data left less window for more variations. The same problem arose with the opposite side of the variations, it would have been extremely difficult to go to a shorter string length at the angle used for the experiment. Had the variations gone under 0.25 m, it likely wouldn't have yielded results. Consistent displacement likely played a factor within the accuracy of the data, as the displacement was eyeballed each time, matching it up with a line below it, so the accuracy of the data could have been affected by the human error of eyesight.

If the chance were given to do this experiment over again, things to change would include workspace, data collection techniques, and a more accurate displacement strategy. To use a larger rod, and to have a larger space for different variations would aid it more impressive results. The use of a video analysis to acquire the times would yield exact results with little to no variations between trials. To measure an angle and have the rod perfectly displaced to that angle would need to have some sort of block or wall to line up the displacement to, this would allow for it to be perfectly displaced each time. In addition, during the setup of the experiment, the use of a level to make sure it was perfectly straight across would have helped in the accuracy of the string length for each time it was measured.

The results of this experiment sparked an inquiry, what if instead of adjusting both lengths of the string to different heights, what if you only adjusted one of the strings to a different length and then took the time period for those oscillations? How would that data compare to that of this experiment?

<u>Bibliography</u>

Chasnov, Jeffrey R. “10: The Simple Pendulum - Mathematics LibreTexts.” *Math LibreTexts*, https://math.libretexts.org/Bookshelves/Scientific_Computing_Simulations_and_Modeling/Scientific_Computing_(Chasnov)/II:_Dynamical_Systems_and_Chaos/10:_The_Simple_Pendulum. Accessed 04 January 2024.

The Editors of Encyclopedia Britannica. “Hooke's law | Description & Equation.” *Britannica*, https://www.britannica.com/science/Hookes-law. Accessed 04 January 2024.

“Harmonic Motion.” *Harmonic motion*, https://labs.phys.utk.edu/mbreinig/phys221core/modules/m11/harmonic_motion.html. Accessed 04 January 2024.

McCormick, Katie. “Physicists unlock the secret of a child’s swing.” *Science*, 7 April 2023, https://www.science.org/content/article/physicists-unlock-secret-childs-swing. Accessed 04 January 2024.

Murray, Christopher. “HTPIB11C Simple Pendulums.” *YouTube: Home*, 1 May 2020, https://youtu.be/IonWVvd9MGk?si=6aWKF2qTP3Pi8PsL. Accessed 20 February 2024.

“When Were Swings Invented | History of Playgrounds.” *Miracle Recreation*, https://www.miracle-recreation.com/blog/history-of-playground-equipment/. Accessed 04 January 2024.

Winder, Jon. “Revisiting the playground: Charles Wicksteed, play equipment and public spaces for children in early twentieth-century Britain.” *Cambridge University Press*, 23 October 2021, https://www.cambridge.org/core/journals/urban-history/article/revisiting-the-playground-charles-wicksteed-play-equipment-and-public-spaces-for-children-in-early-twentiethcentury-britain/70F9AFEC81414900489B57E895546C62. Accessed 04 January 2024.

Related Links

PhET lab Wave on String

This is a simulation of an oscillating string. This simulation goes through and shows amplitude, waves, frequency, and more.

PhET lab Pendulum

This simulation discusses periods, simple harmonic motion, conservation of energy and more. This lab is similar to this experiment, but it doesn't do any spinning.

PhET lab Hooke's Law

This simulation focuses on Hooke's law involving elasticity, This equation and concept was used within this experiment, so this is similarly related.

Pendulum Simulation

This is another pendulum simulation that allows you to adjust more variables in the simulation than the phet above.

Spark Notes Harmonic Motion

Simple notes and websites for definitions is what I used when developing my project.