Lehman Lesson Study Research Lesson - First Draft

<u>The Goal of Our Research Lesson:</u> *Students will enjoy math as a conquerable challenge by building on successes and developing a systematic approach to problem-solving.* We want students to see math as a challenge, not as a problem. They will have the confidence to not view struggle as a negative statement about their own ability, but as an expected part of learning. They will use their personal experience and the information given to approach mathematical situations with the spirit of a scientist, with the confidence to move forward and try things that may not work. With each challenge they work through, they will gain confidence in their ability to use a systematic approach as they make sense of unfamiliar situations or problems.

Content Standards: ● Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. (TASC Item Specifications) ● Explain volume formulas and use them to solve problems. (College and Career Readiness Standards for Adult Education)	Mathematical Practices: ● MP. 1 Make sense of problems and persevere in solving them ● MP. 2 Reason abstractly and quantitatively ● MP. 3 Construct viable arguments and critique the reasoning of others ● MP. 5 Use appropriate tools strategically ● MP. 6 Attend to precision ● MP. 7 Look for and make use of structure ● MP. 8 Look for and express regularity in repeated reasoning

Pre-Assessment: A Fair Deal

Steps/Questions in activity	What are students doing? Expected responses/reactions	Teachers' response to student reactions	Goals/Method of Evaluation/Assessment (For each step)
Teacher gives students 15-20 minutes to complete as much of the A Fair Price handout as they can. This will happen in a class prior to the research lesson being observed. After the pre-assessment, look through student work and summarize their difficulties as a list of questions. Students will get their initial assessment with a question/prompt along with a blank assessment to revise their work <u>after</u> the research lesson.	Asking clarifying questions Looking confused Putting their pencils down Being curious Correctly answers all the questions (probably not common) Common Issues: ● Leaves question blank ● Assumes diagrams are accurate representations ● Fails to mention scale ● Focusing on non-mathematical issues ● Makes a technical error ● Simply triples the price of the pizza or doubles the price of a cone of popcorn	If you are not sure how to answer any of the questions, for now, please just try your best to write any ideas you have. During our class on December 29th, we'll be working on a lesson that will help you. After that lesson, you will have a chance to revise your work. Think of this as your first draft.	Though we won't be going over these pre-assessments, students will have the opportunity to revise their work after completing the research lesson. We will return students' work with a question/prompt along with a blank copy of the handout. Does the research lesson impact students' ability to improve their initial response to this activity?[1] Some students will work on both the pre- and post-assessment, some will only work on the post—one thing to consider is... Does working on A Fair Deal before the research lesson impact students' reasoning during the research lesson?

Message to Students before the research lesson:

- A group of teachers have been working together, writing a lesson on some of these ideas. One of our goals is to focus on your learning and become better teachers. This activity will give us a better sense of how we can help you learn some important concepts in math.
- The other teachers in the group will observe the class to see how it goes, since we all planned it together. They are observing to see how well the lesson works, not whether you get the right answer or not.
- We are planning to take notes and record the lesson so that we can meet afterwards to see what worked well and what could be improved.
- Your participation is voluntary. We really appreciate your willingness to help us improve our teaching.

[1] Research questions are highlighted with a dashed border. Observers should look for and document evidence related to the question.

Launch

Steps/Questions in activity	What are students doing? Expected responses/reactions	Teachers' response to student reactions	Goals/Method of Evaluation/Assessment (For each step)

Testing the Validity of Statements
 	
Give out handout to students with the following statements, then read aloud:
 	
If you double the length and width of a rectangle, then you double the perimeter
 	
If you double the length and width of a rectangle, then you double the area
 	
Ask students…
• *What is a rectangle?*
• *What are area and perimeter?*
• *How do we measure the area and perimeter of a rectangle?*
• *What does it mean to double something?*
	Students may not believe that a square is a rectangle.	Square vs. rectangle:
		• These are the characteristics of a rectangle:
		  ○ Opposite sides are equal.
		  ○ 90 degree angles in the corners
		• Does a square fit these requirements?
		  ○ Yes. A square is a special kind of rectangle.
			The rectangle work at the beginning does a few things:
			
			• Lets students know they can use manipulatives and drawings
			• Lets students know they can just try different numbers
			• Models evaluation of statements, and revising a false statement to be true
			• Provides the form of the If/Then sentence.

Point out manipulatives and grid paper. Encourage students to use these materials when they evaluate the statements. Say: *I would like you to decide whether each of these two statements is true or false. For each statement, write a*	Some students may: • struggle to understand what they're being asked to do. • not want to "write on the board", and prefer that the teacher do the writing. • not volunteer their answers	For students who struggle to get started: • Suggest they draw a picture of a rectangle • Ask the student to assume side lengths for the rectangle. What could the length and width be? See what	This introduction provides students with a model of how they should work during the collaborative task. Will students use manipulatives to see the change in perimeter and area when doubling length and width of a rectangle?

convincing explanation. If you think a statement is false then replace it with a correct statement. Students should work independently for 3 minutes before sharing with a partner. Look for the following solution methods for evaluating the rectangle T/F statements: Used tiles or other manipulatives to build two rectangles with appropriate length and width. Drew two separate rectangles with appropriate length and width. Compared the perimeter and area of two rectangles using numeric values for length and width. Drew two rectangles, the original inside the larger rectangle, so that the original is seen as ¼ of the enlarged rectangle. After 5-7 minutes, ask two or three students for their answers. Order presentations from tiles to diagrams to numeric values. Don't present algebraic explanations yet. After the presentations, ask the rest of the class: *How do we know for sure that these answers are correct for all rectangles?* Conclude and write on the board:	• not even try to accept the challenge • guess (correctly or incorrectly) instead of reasoning/experimenting • express that they "can't draw" • not be able to explain their answer • not be able to "replace" a false statement • confuse area and perimeter • may double length but not width, or vice-versa. Questions students may ask: • Can I/we use a calculator? • What size is the rectangle? • Will this be on the test? • How much time do we have? • Can I have more time? • Is this right/correct? • Do we have to do the math for each one?	happens. For students who struggle to explain their reasoning: • Can you draw a picture to convince me? • Can you give some examples to convince me? For students who finish early: Can you cre	Will students make drawings to see the change in perimeter and area when doubling length and width of a rectangle? Will students try different numbers (like scientists!) to see the change in perimeter and area when doubling length and width of a rectangle?

If you double the length and width of a rectangle, then its perimeter is multiplied by 2. *If you double the length and width of a rectangle, then its area is multiplied by 4.*			

Problem Posing

Steps/Questions in activity	What are students doing? Expected responses/reactions	Teachers' response to student reactions	Goals/Method of Evaluation/Assessment (For each step)
Admin Organize students in groups of two or three. Hand out calculators. **Shapes and Their Characteristics** Display sample objects (shoebox, ball, coffee can, water cup/ice cream cone (cone). Give out handout with pictures of 3D shapes (rectangular prism, sphere, cylinder, cone). Ask students to write down anything they can about each shape. Put up pictures of shapes on the board. *What is the name of each of these shapes? What can you tell me about … (each shape)? Where do we see examples of this shape in real-life?* Record student ideas next to each picture. *What are different ways we can measure these shapes?* Focus on the can of beans: • What part of the can could I measure using length? • What part of the can could I measure	Students may not be able to identify each shape with correct mathematical term. Students suggest the following measurements: • *height* • *length* • *width* • *perimeter* • *area* • *volume* Students might be confused between the difference between a rectangle and rectangular prism, circle and sphere.	Share the correct mathematical term for each object. Ask about the following additional measurements, if they don't come up: • *radius* • *diameter* • *circumference* • *surface area* • *curved surface area* What is the difference between a rectangle and rectangular prism?	All students should be able to name each of the shapes. Students should understand that three-dimensional shapes can be measured in terms of length, area and volume. Students should understand the elements in all of the geometric formulas to be used during the problem-solving time. Will students be able to identify different measurements in the real objects in front of them?

using area? • What part of the can could I measure using volume? Ask students to add measurement words to their handout. Ask for volunteers to describe these measurements using the sample objects. Demonstrate location of measurements on 3D shapes. Possible calculator instructions: Say: We're going to be using pi (π) in some of our calculations today. Q: Does anyone know this is? A: It's got to do with circles. It's 3.14. There are more decimals. It keeps going. (Or however students respond.) For the activity today, we will be using 3.14. Give out Problem-Solving Instructions. Read instructions aloud. What am I asking you to do? Who can put this in their own words? Give each group a set of cut-up True or False cards, the formula sheet, a piece of chart paper and tape or a glue stick.			

Student Problem Solving

Steps/Questions in activity	What are students doing? Expected responses/reactions	Teachers' response to student reactions	Goals/Method of Evaluation/Assessment (For each step)
During small group work and problem-solving: ● Take notes of student approaches to the task ● Support students without doing the thinking for them. **Look for the following solution methods:** ● Builds rectangular prisms with *manipulatives* to see what happens when different dimensions are doubled. ● Draws *diagrams* of objects to see what happens when different dimensions are doubled. ● Uses *formula with sample dimensions* for original object before scaling. Calculates measurements using 3.14 or other approximate, e.g. $\pi(4)^2 = 50.24$ and $\pi(8)^2 = 200.96$. Compares original and scaled measurement to find scale factor. ● *Retains* π *in calculations*, e.g. compares 16π and 64π. Compares original and scaled number multiplied by π to find scale factor.	Faster students might work past their group members Some students might struggle to get started.	Encourage students to engage with each other's explanations and take responsibility for each other's understanding. ● *Judith, why do you think this statement is true/false?* ● *James, do you agree with Judith? Can you put her explanation into your own words?* If students are struggling to get started on the task: ● Encourage students to group shapes together in the same group ● Have them draw a picture of the figure and label the parts of the shape. Ask "How can you test this statement?" We want to invoke the answer "By using numbers." If students are struggling midway: ● *What formula can you use to check if the statement is correct? What values can you put into this formula?* ● *If it is not twice as big, by what factor has the area/volume increased? How do you know?* At first, you may want to focus your questioning on the cards about rectangular	Will students remember what these measurements mean when they use formulas? Will students draw rectangular prisms in order to visualize the effect of doubling dimensions? Will students retain pi for their measurements or will they substitute an equivalent like 3.14? Will students disregard numbers to the right of the decimal point when comparing sizes of measurements, e.g. area of two circles: (3.14159 * $(4)^2$ = 50.26544 and (3.14159 * $(8)^2$ = 201.06176? Do we need an activity like the following to help students prepare for dealing with decimals? **How to Prepare for Issues with Student Calculations** (In order to highlight usefulness of disregarding decimals): Draw a rectangle with a width of 4.1 and a height of 2.2. If you double the length and the width, what happens to the perimeter and the area?

• *Algebraically compares* two versions of formula to see the effect of scaling, e.g. πr^2 and $\pi(2r)^2 \rightarrow \pi 4r^2$. Finds the scale factor in the second equation. When a group finishes their poster, ask them to flip the Problem-Solving Instruction Sheet. On the back are the questions for comparing their poster with the work of a neighboring group.	Students might not understand doubling. Or they may double only one measurement when there are multiple measurements to double (i.e.- the student may see the word *double* and think "times 2" for each one). Some students might move through the task more quickly than others.	prisms: • *Are any of these statements true? What is it about the formula that makes the statement true?* • *What has the volume of the rectangular prism increased by for this statement? How does this increase relate to the formula?* Focus student attention on all the words on the slip. Ask the student one of the following questions: • What does it means to double something? • What measurement did you double? • What should be the total number of doublings you need to do for this one? How does it differ from others you have doubled? Encourage students who work through the task more quickly to think about how they can explain the scaling in general terms. They may use algebra in their explanation, or simply highlight the properties of a formula that determine the scaling. • *Can you use algebra to show you are correct? If the radius has a length of n, what is double its length? How can you use this in the formula?* • *Can you figure out if the statement is true or not just by looking at the formula? Why? Why not?* • *What happens if the phrase 'multiply*	Will students understand why it's okay to retain π in their calculations?

	What other challenges will students have during their group problem solving? Students might not work well as a group--there might be tension within the group.	*by 3' replaces the word 'double.'* Support questions: ● What is the difference between perimeter, area and volume? ● Which of your answers are similar? In what way? Why do you think this is so? ● Can the statements be proven using just the formulas? ● How could you use this information in real life? ● If you group the shapes, which ones "go together"? Why? Get them to work together by posing questions such as above to get started or assign them each tasks such as one person draw, the other person choosing the formula needed and deciding how to incorporate the formula.	

Whole Class Discussion

Steps/Questions in activity	What are students doing? Expected responses/reactions	Teachers' response to student reactions	Goals/Method of Evaluation/Assessment (For each step)
Order presentations from different students or groups (possible): ● manipulatives ● diagrams/drawings ● numeric examples ● algebra After the selected groups present:	Students present their posters in the order indicated by the teacher. Some students in the audience listen. Others may still be thinking about their poster and will be distracted.	Ask the class, "What do you like about this group's work?"	How well do students listen to each other? Are students able to demonstrate understanding of other students' reasoning?
Refer students back to the board notes from the problem-posing section: *If I double the length and width of a rectangle, then the area is multiplied by 4.* Tell students: We are going to look at a chart that follows the same structure of this sentence. It includes all the statements on the cards that you've been working on. Share the handout, ***What happens when we double dimensions of different shapes?*** Ask students: What do you notice? Take a few minutes on your own to write things you notice. Pair/share for a couple minutes. After five minutes, record student responses on the board.	Students will notice a range of things in the chart. Some students may be overwhelmed by the amount of information in the chart. During sharing of noticing, some students might share an inference.	Discuss as a class how the structure of a formula determines the increase. For students who are overwhelmed: ● *How does this chart work? Can you read the first few lines for me?* ● *What patterns do you see?* ● *Look at all the times when something is multiplied by 2 or by 4 or by 8? What do those situations have in common?* ● *(pick a row) Which dimensions changed in the first column? Circle those dimensions in the rewritten formula column. Do that for a couple and see if you notice any patterns,* Ask students who share an inference to explain what they say in the chart that helped them to notice that.	

Summing Up

Steps/Questions in activity	What are students doing? Expected responses/reactions	Teachers' response to student reactions	Goals/Method of Evaluation/Assessment (For each step)
Students do a pair share: *One thing that helped me figure out if the statements were true or false was…* Ask a few volunteers to share their answers and record them on the board. Looking back at the work you did today, we can see that you have calculated the perimeters, areas and volumes of different shapes after scaling them to larger sizes. You have demonstrated that you know how to use the formulas for a rectangular prism, circle, sphere, cylinder, and cone. These are important skills for the TASC exam. You've developed strategies for figuring out if a mathematical statement is true/false. One important take-away is that doubling one or more dimensions of an object doesn't necessarily double the perimeter, area, or volume (measurements) of that object. What's an example where doubling one or more dimensions doesn't double the perimeter, area, or volume? (If necessary, point to student examples.) For	Sharing and listening		

example, we saw that doubling the radius of a circle makes the area 4 times the original area. Doubling the radius of a sphere makes the volume 8 times the original volume.			

Post-Assessment

Steps/Questions in activity	What are students doing? Expected responses/reactions	Teachers' response to student reactions	Goals/Method of Evaluation/Assessment (For each step)
Return to students their original responses to the *A Fair Price* assessment task as well as a second blank copy of the task. If you have not added questions to individual pieces of work then write your list of questions on the board. Students are to select from this list only the questions they think are appropriate to their own work. *Look at your original responses and the questions (on the board/written on your script).* *Answer these questions and revise your response using what you have learned to improve your work.*	Students may want to take the assignment home or complete it in the next class.		Will there be time to complete the post-assessment during the class period?

Reflection

Hand out the final reflection: ● *Today, I am proud of how I….* ● *Today, I felt like a mathematician when I ….* ● *I feel more______ working with geometry formulas, because I _______*			

Boardwork

Pictures of shapes and characteristics Testing Validity of Statements (problem-posing) Statements to be evaluated	Student Work *If I double the <u>length and width</u> of a <u>rectangle</u>, then the <u>area</u> is multiplied by <u>4</u>.*	Student Problem-Solving Newsprints moved to front ● manipulatives ● diagrams/drawings ● numeric examples ● algebra	What do we notice? - Chart

Materials

- Three-dimensional objects - shoebox, can of beans with paper label, ball, water cup (cone)
- Newsprint (one for each group)
- Markers
- Color tiles
- Graph paper
- Glue sticks
- Calculators
- Rulers
- **Handouts:**
    - A Fair Price (a clean copy and a copy of each student's initial assessment)
    - Picture of 3D shapes for Problem Posing
    - Launch Two Statements on Enlarging Rectangles
    - Group instructions for problem-solving task / Instructions for Looking at Other Group Work
    - The formula sheet (one for each student)
    - True/False cards (cut up, one set per group)
    - Whole Group Discussion: Chart - What happens when we double dimensions of different shapes? (one for each student)
    - Learning Reflection

What happens when we double dimensions of different shapes?

If I double the...	Of a...	Then the...	Is multiplied by...	Formula	Formula (rewritten)
length and width	rectangle	perimeter	2	$P = 2l + 2w$	$P = 2 \cdot l + 2 \cdot w$
length and width	rectangle	area	4	$A = lw$	$A = l \cdot w$
width	rectangular prism	volume	2	$V = lwh$	$V = l \cdot w \cdot h$
width and height	rectangular prism	volume	4	$V = lwh$	$V = l \cdot w \cdot h$
width, height, and length	rectangular prism	volume	8	$V = lwh$	$V = l \cdot w \cdot h$
radius	circle	circumference	2	$C = 2\pi r$	$C = 2 \cdot \pi \cdot r$
radius	circle	area	4	$A = \pi r^2$	$A = \pi \cdot r \cdot r$
radius	sphere	surface area	4	$SA = 4\pi r^2$	$SA = 4 \cdot \pi \cdot r \cdot r$
radius	sphere	volume	8	$V = 4/3\pi r^3$	$V = 4/3 \cdot \pi \cdot r \cdot r \cdot r$
radius	cylinder	curved surface area	2	$CSA = 2\pi rh$	$CSA = 2 \cdot \pi \cdot r \cdot h$
height	cylinder	volume	2	$V = \pi r^2h$	$V = \pi \cdot r \cdot r \cdot h$
radius and height	cylinder	volume	8	$V = \pi r^2h$	$V = \pi \cdot r \cdot r \cdot h$
base radius	cone	volume	4	$V = 1/3\pi r^2h$	$V = 1/3 \cdot \pi \cdot r \cdot r \cdot h$
height and base radius	cone	volume	8	$V = 1/3\pi r^2h$	$V = 1/3 \cdot \pi \cdot r \cdot r \cdot h$

What do you notice?

What happens when you double the length and width of a rectangle?