Grade 8 Research Lesson Plan - Systems of Equations

Team Members

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Lesson Date:	Instructor:	Grade Level:
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1. Title of Lesson

Refer to Plan | Step 3: Identify the Lesson

Systems of Equations

2. Research Theme

The long-term goals of our students

Refer to Plan | Step 1: Start Planning

We want to help our students approach mathematics as a process of inquiry, rather than a process of answer-getting. Evidence of this development would include: students justifying and critiquing mathematical arguments; asking each other questions to understand different problem-solving strategies; acknowledging errors as learning opportunities; and contributing to a classroom culture that is positive, collaborative, and encouraging for all.

We hope to achieve this goal by improving our practice of teaching mathematics through problem solving, with a focus on student discussion. Student discussion will be fostered by strategic and consistent notebook use, frequent opportunities to justify and critique mathematical strategies both in small and large groups. This will be intentionally taught to ultimately generalize the value of the process of problem solving.

3. Background and Research on the Content

- Why we chose to focus on this topic - for example, what is difficult for our students, what we noticed about student learning
- What resources we studied, and what we learned about the content and about student thinking
- What we learned from studying our own curriculum and other resources

Refer to [Plan | Step 1: Start Planning]

Much of the eighth grade curriculum focuses on algebraic and representational thinking. As we reflected on patterns of student learning from the past, we noticed that many struggle with several aspects of systems of equations, including identifying when a situation suggests the use of solving simultaneous situations; accurately selecting and applying a method for solving; and making sense of this solution in the context of the problem. We have tended to revert to a more traditional I do, we do, you do approach, given that students had such difficulty independently setting up systems in a variety of formats and contexts. Last year, we used the CMP3 curriculum as a framework, and found that the progression of goals and lessons led to confusion and frustration. We want the kids to have a strong conceptual foundation which will allow them to flexibly move between solution strategies and make connections between them.

As part of our approach to introducing systems of equations, we consulted _____ (Proulx, 2009). We learned that understanding this topic involves the related skills of *conceptualizing*, *representing*, *solving*, and *interpreting*. As this research lesson would be the first lesson to introduce the concept of systems of equations by name, *conceptualizing* seemed like the most relevant skill for us to research. In other words, how do we get students to understand what a system of equations *is* and what it *means* to solve one? Proulx writes that students need to understand what it means to have a combination of constraints and what it means to satisfy both equations. Further, students need to be able to identify the constraints within the situation in order to model it using a system of equations.

We also read "Problems Before Procedures: Systems of Equations" (Allen, 2013). This article is an account of a teacher who uses real-life problems to introduce systems of equations, encouraging students to use any methods that make sense to them to find the solution. The author shows how different students used different approaches to solve, such as graphing and algebraic reasoning, and lists questioning that might be used to support students in their work. While we do not plan to emphasize algebraic approaches in our research lesson, we reflected that we could plan similar questions to support students. For example, we might replace the algebraic approach with questions for students who chose to make a table as their main strategy.

One of our main decisions for this lesson was whether to use a problem that lent itself to being represented by equations in standard form or slope-intercept form. As an example of standard form, we researched the lesson used in Mathematics International (MI), Grade 8. The following problem lends itself to equations in standard form:

> *In a basketball game, Miss Shimizu made a total of 10 shots and scored 24 points. How many 2-point shots and how many 3-point shots did she make?*

We also researched the lesson in Open Up (OU) that first introduces systems of equations using a problem that can be represented in slope-intercept form. The following problem is used:

> *Han and Jada both decide to hike from the parking lot to the lake and back, but they start their hikes at different times. At the time that Han reaches the lake and starts to turn back, Jada is 0.6 miles away from the parking lot and hiking at a constant speed of 3.2 miles per hour toward the lake. Han's distance, d, from the parking lot can be expressed as d=-2.4t+4.8, where t represents the time in hours since he left the lake.*
>
> 1. *What is an equation for Jada's distance from the parking lot as she heads toward the lake?*
> 2. *Draw both graphs: one representing Han's equation and one representing Jada's equation. It is important to be very precise.*

We discussed the benefits and drawbacks of each type of problem. For one, the context used in MI uses simple numbers so that students can focus more on the concept of satisfying the two constraints within the problem. Also, the use of standard form equations ($x + y = 10$ and $2x + 3y = 24$) makes it more likely that students will consider both variables as they try to find the values for x and y that make both equations true.

Despite these advantages, we were hesitant to use this problem for a few reasons. For one, students will have already explored a similar problem (quarters and dimes) two lessons earlier in the unit. The goal of that lesson (4-10) is for students to "focus on what must be true based on the two facts they know [in the problem]." Thus, students will have already spent some time in this unit informally exploring what it means to satisfy two constraints in a situation. With students already having had this experience, we thought it was no longer as crucial to present a problem using standard form equations in this lesson.

We also decided to go with a problem that uses slope-intercept because, for students that attempt to graph the situations, the slopes will be more meaningful -- they will represent each rate within the situation. Students have a lot of practice doing this with one linear equation, so we thought this lesson might be a good opportunity for students to apply this skill as a bridge to learning something new: what happens when we graph two equations.

Finally, another rationale for sticking with a problem closer to the one found in the OU lesson is simply to test out the progression within the curriculum unit itself and see what effect it has on student learning. For example, we think one reason why equations in slope-intercept are used at this point in the unit is because it leads (in future lessons) to lessons about setting these equations equal to one another to solve for x. We are interested to see how this progression goes moving forward through the unit.

Finally, a note about our changes to the problem itself. While we decided to stick with a rate context, we thought that the OU problem was overly complex for a problem that is intended to officially present systems of equations for the first time. We also wanted the slopes to both be positive to make it easier for students to interpret the graphs, should they or their peers choose to use this method. We decided to create a context using jobs and $/hr because we thought students would find it engaging and also because we thought we could configure the problem to be easily solvable but not immediately obvious upon reading the question.

4. Rationale for the Design of Instruction

- IF we teach this topic by doing X, THEN students will better understand Y.
- IF we structure the lesson as X, THEN we will observe Y. *(Research Theme)*

Refer to ***Plan | Step 8: Final Preparations***
If we carefully design the lesson to foster independence, initiative, student inquiry, multiple perspectives, and communication via notebooks and discussion, then students will be better able to independently set up, solve, and understand situations involving systems of linear equations.

5. Goals of the Unit

Refer to ***Plan | Step 2: Create a Unit***

Long-Term Goals for the unit are for students to:

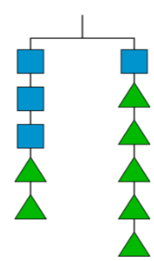
- Understand that two functions with the same two variables can be solved using a system of equations.
- Represent systems in equations, graphs, and tables
- Write systems of equations in two variables given real-world situations, and interpret the variables in the context from which the equations arose
- Solve systems of equations in two variables given real-world situations, and interpret the solutions in the context from which the equations arose.

- Apply graphing, substitution, addition/subtraction, or elimination methods to solve systems of equations.
- Understand and conceptualize what a solution to a system (or infinite solutions, or no solutions) means in context

In other words, we want students to recognize, write, and solve systems of equations when given real-world situations that involve two variables and interpret solutions in tables, graphs, and equations in context.

6. Unit Plan

The lesson sequence of the unit, with the task and learning goal of each lesson. The asterisk (*) shows the research lesson

Lesson	Learning goal(s) and tasks
4.1	Goal: Students represent and solve problems in context. Task: Solve number puzzles using any representation including tape diagrams, number lines, and equations.
4.2	Goal: Understand and explain why you can add, subtract, multiply, and divide an expression from each side of an equation when the expressions involve positive numbers. Task: Answer questions about the values of the squares and triangles balanced on either side of a hanger. 
4.3	Goal: Understand and explain why you can add, subtract, multiply, and divide each side of an equation by an expression involving rational numbers and still have an equivalent equation (as long as you don't divide by 0). Task: Students must match cards (such as shown below) to show a move that will turn the top equation can be turned into the bottom equation. 1 $3x + 7 = 5x$ / $7 = 2x$ Add $-3x$ to each side. B
4.4	Goal: Understand that there can be multiple correct paths to solve a linear equation. Task: Students answer questions comparing two solution methods for the same equation.

	Here is an equation, and then all the steps Clare wrote to solve it: $14x - 2x + 3 = 3(5x + 9)$ $12x + 3 = 3(5x + 9)$ $3(4x + 1) = 3(5x + 9)$ $4x + 1 = 5x + 9$ $1 = x + 9$ $-8 = x$ Here is the same equation, and the steps Lin wrote to solve it: $14x - 2x + 3 = 3(5x + 9)$ $12x + 3 = 3(5x + 9)$ $12x + 3 = 15x + 27$ $12x = 15x + 24$ $-3x = 24$ $x = -8$
4.5	Goal: Solve more complex linear equations. (Fluency Practice) Task: Students are given equations and work in partners to describe the steps they take and how each step maintains the equality of the statement.
4.6	Goal: Understand strategies for solving linear equations. Task: Without solving, identify whether each equation has a solution that is positive, negative, or zero.
4.7	Goals: 1) Understand that some equations have no solution and some have an infinite number of solutions. 2) Use structure to reason about the number of solutions an equation has. Task: Students sort equations into two types: true for all values and true for no values, e.g., n = n and 2r + 6 = 2(r + 3)
4.8	Goal: Solve a variety of linear equations with one solution, no solution, and an infinite number of solutions. (Fluency Practice) Task: Students sort equations on cards and then sort them into categories of their own choosing (e.g., Card B has an equation with the same number of x's on both sides).
4.9	Goal: Use linear equations to solve problems. Task: Students analyze a table that shows the amount of water in two tanks every 5 minutes. They describe what is happening in each tank and estimate when the tanks will have the same amount. They set two given equations equal to each other to find a solution.
4.10	Goals: 1) Given descriptions of two linear relationships, interpret ordered pairs in contexts, focusing on when or where the same ordered pair makes each relationship true. 2) Make connections between ordered pairs in tables and graphs. Task: Jada told Noah that she has $2 worth of quarters and dimes in her pocket and 17 coins altogether. Guess how many of each type of coin she has. Students complete a table and analyze the ordered pairs on a graph.
4.11	Goals: 1) Understand that some relationships share the same set of ordered pairs. 2) Make connections between tables, graphs, and contexts given two linear relationships, focusing on when or where the same ordered pair makes each relationship true.

	Task: “An ant and ladybug are a certain distance apart, and they start walking toward each other.” Students are given a graph showing the ladybug’s distance from the starting point over time and the point where the two meet. Students are also given the ant’s walking rate and must write an equation to show the relationship between the ant’s distance from the ladybug’s starting point and the amount of time that has passed.
4.12 *	Goals: 1) Students will extend their understanding of using equations to represent real-life contexts to now understand that some real-life contexts must be represented by more than one equation and that these equations can share unknowns. 2) Students will be able to identify these equations that “work together” as a “system of equations.” 3) Students will understand, through making tables and/or graphing, that a solution to the system of equations is found when the values in an ordered pair satisfy all equations within the system. Task: Over the summer Jose and Evelyn have started lawn care companies. Jose makes $7.50 for each lawn he cuts. Evelyn makes $5 for each lawn she cuts and already has $35 in her bank account. 1. Will Jose and Evelyn ever have the same amount of money? How do you know? 2. If so, at what point will they have the same amount and how much will they have? How could we demonstrate this? How could we prove it? 3. Are there different ways we could represent this situation? Could you write it algebraically? Evelyn's Excellent Lawncare   Jose's A+ Landscaping
4.13	Goals: 1) Start solving systems algebraically. 2) Connect features of the graphs to the number of solutions the system has. 3) Understand that a system of equations can have no solutions, one solution, or infinitely many solutions.

	Task: Students are asked to match systems of equations to their graphs, solve algebraically (by setting equations equal to each other), and checking the point intersection on the graph.
4.14	Goals: 1) See and describe structure in systems of equations. 2) Solve system of equations problems where the two equations have different structures. Task: Sort systems of equations according to difficulty and then choose four to solve. Nudge students toward seeing why substitution can be used in cases such as: $\begin{cases} y = 7 \\ x = 3y - 4 \end{cases}$
4.15	Goals: 1) Write a system of equations given a real-world situation. 2) Solve system of equations with non-integer coefficients. Task: Given a variety of scenarios, students write systems of equations and interpret what the solutions would represent in the context.
4.16	Goal: Write and solve a system of equations to solve problems. Task: Solve problems using systems of equations. Create a new problem and solve.

7. Relationship of the Unit to the Standards

- How the learning in the unit relates to the grade-level standards.
- How the learning in the unit relates to prior standards and future standards.

Prior learning standards that unit builds on	Learning standards for this unit	Later standards for which this unit is a foundation
7.EE.A.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. 7.EE.4 Solve real-life and mathematical problems using numerical and algebraic expressions and equations. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and	8.EE.C.7 Solve linear equations in one variable 8.EE.C.8 Analyze and solve pairs of simultaneous linear equations.	HSF.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

inequalities to solve problems by reasoning about the quantities.		

8. Goals of the Research Lesson

Refer to ***Plan | Step 3: Identify the Lesson***

Students will extend their understanding of using equations to represent real-life contexts to now understand that some real-life contexts must be represented by more than one equation and that these equations can share unknowns.

Students will be able to identify these equations that "work together" as a "system of equations."

Students will understand, through making tables and/or graphing, that a solution to the system of equations is found when the values in an ordered pair satisfy all equations within the system.

9. Research Lesson Plan

Learning task and activities, anticipated student responses, key questions or comparisons that will build insights	Teacher support	Assessment (Points to Notice)
Do Now Student A bought bottles of water and bottles of soda. Water is $1, soda is $2. How many combinations could they get if they had $12 to spend? How many of each would they have bought if they bought 9 drinks? (Did they get 9	T:*what two things are we keeping track of that are changing?* *What situation do we start with?* *--The money has to add up to $12*	What methods are students using to find the solution? Are students aware that they have to satisfy the

waters? 7 waters and 2 sodas? etc.) How could you find this information?	*What new situation is added?* *--The number of drinks has to total 9.*	two things they know about the situation, similar to the coins problem from 4-10?
Introduction **Review and objective** SW: Find a solution that satisfies two parts of a real world problem.	T: What is a solution in a mathematical sense? T: What are some strategies we have/can use to help us find solutions? (Tables, graphs, equations)	Do students know what two parts (two facts) means in this sentence? Are they thinking of the two parts of the coin problem or water bottle problem, for example?
Posing the Task Over the summer Jose and Evelyn have started lawn care companies. Jose makes $7.50 for each lawn he cuts. Evelyn makes $5 for each lawn she cuts and already has $35 in her bank account. 1. Will Jose and Evelyn ever have the same amount of money? How do you know? 2. If so, at what point will they have the same amount and how much will they have? How could we demonstrate this? How could we prove it? 3. Are there different ways we could represent this situation? Could you write it algebraically? Students read and think about the situation. T asks students to summarize the context.	T: Clear up any misunderstandings before work gets started -What are the variables (what is changing that we want to keep track of)? What values are we looking for?	What do students see as "new" about this type of problem? Are students excited to solve the problem?

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<td>What is new or different about this problem than what we have already done?

Students are given 3 minutes to work before coming back to their shoulder partners to discuss their methods. Then students have another 8 - 10 minutes to solve with their partners.</td>
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<td>Anticipated Student Responses

1. Guess and check
2. Make a table without graphing
3. Graph by plotting ordered pairs from a table
4. Graph by using slope-intercept form
5. Use the difference of slopes (e.g., “Angel makes up $2.50 more each job. It will take _35/2.5_ jobs to have as much as Evelyn…”)
6. Solves the equations by setting them equal</td>
<td>T: If students cannot start T will ask them to consider ways in which we could represent how much money each has after given quantities.

T: Note how students are graphing (using intercept then slope vs. graphing ordered pair solutions)

T: Remind students that a table can help them organize their data

For students who are stuck:
- Encourage students to use two tables to represent the situation
- Ask students about the “35” -- what does it mean in the situation</td>
<td>What is the distribution of solution methods (tables, graphing, guess and check)?

For students who make tables, are they looking for the set of values that is the same in both tables?

Which students create equations? How do they use these to solve?</td>
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	and how would it be accounted for in a table or graph? - Asking students how much each would have after zero jobs, one job?	
Comparing and Discussing, including Teacher Key Questions Select responses 2, 3, and 4 ((2 Table)) 1. What was your method and thinking behind how you organized your table? 2. Where in this table do we see that Angel eventually has the same amount of money as Evelyn? 3. How can we tell exactly after how many lawns that happens? 4. Was it difficult using the tables to find the point at which they had the same amount of money? ((3 Graph w/ ordered pairs)) 1. How did you graph it? What information did you use to create your graph? 2. Where do we see that represented in your table? 3. Where is the point that Angel and Evelyn have the same amount of money? 4. How can we find that on a graph? ((4 Graphed using slope intercept and has	When each student presents their method, give the class a chance to ask clarifying questions.	Do students listen to each other's ideas? Do they ask questions when they are confused or to learn more?

created equations for both.)) 1. Do the graphs in #2 and #3 look similar? 2. What was the difference in how they chose to make their graph? 3. How can we use these two equations created by C to prove when we think Angel and Evelyn will have the same amount of money? DISCUSSION Possible questions: What is it again that we were trying to figure out? - How did a table help us do that? - How did a graph help us do that? Notice today that we had two different equations… Can anyone highlight which method best solves this task But maybe the graph is not precise? Ask students how they could check that the coordinates they found with the graph are correct. One strategy would be to represent the situation with equations: $y = 7.5x$ $y = 5x + 35$ Introducing this as a system of equations. When we have two or more equations that use the same variables and represent different facts about a situation, this is called a system of equations. Solving a system of	Focus the discussion on the fact that: - there are two equations (or tables, lines, etc) that are both needed to represent this situation. - whether students used tables, or graphed, or guessed and checked, they needed to find values for x and y that made both equations true Help students see that representing the problem using equations is powerful because the values can be checked (and found, eventually) by	Are students building off of each other's ideas? Do students make connections between different strategies and solutions? Will students prioritize the solutions posed by the class? Do students see the benefits of using equations as a representation of the problem?

equations means to find the values of the variables that make both equations true at the same time. How could we prove that we did this correctly? Will the ordered pair work as a solution to both of these equations (make these equations true)? How do you know?	working with the equations themselves.	
Summing Up - What is a system of equations? - What does the solution to a system of equations look like in a table or on a graph? - What is the meaning of (14, 105) in this situation?		What do we learn about students' understanding of systems of equations from what they write? Do students make sense of (14,105) and have an idea of the importance of these values in the problem?

10. Points to notice (Assessment)

Prompts to focus observation and data collection.

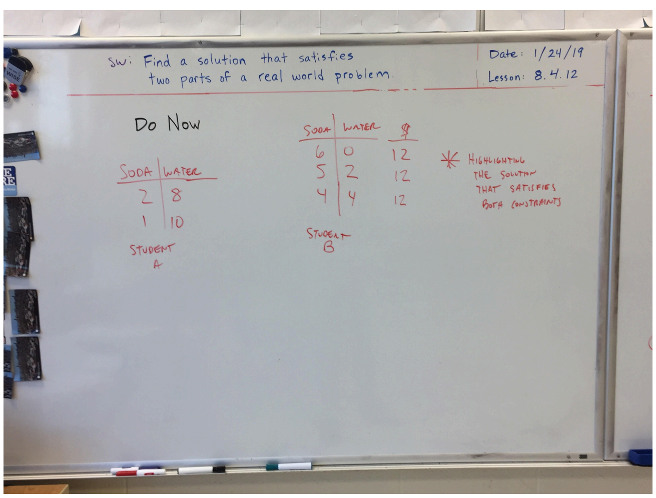
<u>Considerations for the Lesson</u> Is there evidence of students making connections between the various ways solutions to this problem can be represented? (table, graph, and equations) How is this connection made and documented?

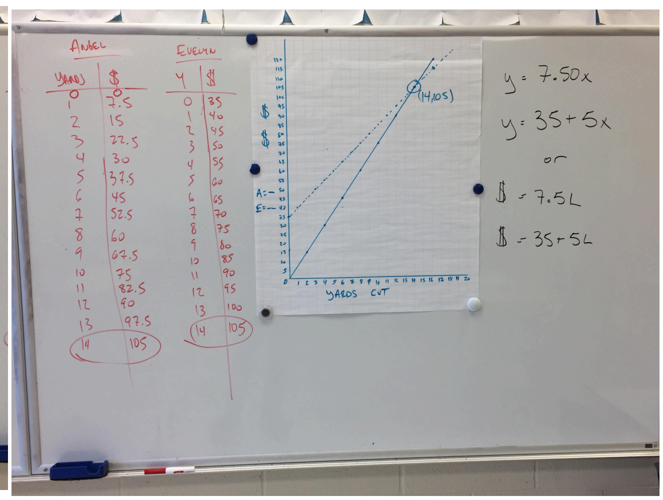
<u>Broader Questions for us to Consider</u>
What is the effect of student discussion and the overall structure of the lesson on student learning?

How are students able to make connections throughout the series of lessons towards a unit learning goal?

11. Board Plan

Highlight in order:
1) Table
2) Plotting ordered pairs from table
3) Graphing using knowledge of slope-intercept





12. End of Cycle Reflection

What Did We Learn? (to be filled out after the post-lesson discussion)

Refer to [Reflect | Step 4: Consolidate Your Learning]()

Upon reflection, we are considering whether using equations in standard form would have been more appropriate than using equations in y-intercept form. In this context, we asked students to find the value for *x* such that *y* would be true in both equations. Or, graphically, we were asking students to find the coordinate pair where the two lines meet. We reflected that this is really just a constraint that is imposed by the teacher through the context (e.g., "How many lawns do they have to cut so they both have the same amount of money?") With a context that uses two standard form equations, on the other hand, the constraint is clear because it is inherent in the equations themselves. For the situation where 10 shots are made and 24 points are scored ($x + y = 10$ and $2x + 3y = 24$) the situation itself is defined by the two constraints.

This also relates to the clarity of the task we gave. Some students thought they had found the answer when they came across the first instance of two *y* values being the same in each table. For example, Jose and Evelyn both make $75 so the answer is y = 75. These students did not attend to the fact that Jose made $75 after cutting 10 lawns, while Evelyn made $75 after cutting 8 lawns. With the context we used, the question is unclear. These students could have really thought they answered the question, thinking "They both make $75 when Jose cuts 10 lawns and Evelyn cuts 8 lawns." This is not the answer to the system of equations, but it's easy to see how students could think this is sufficient as an answer to the problem we gave.

We also discussed the use of tables by the students during the lesson. Regarding students that didn't write the *x* values in their tables, we wondered if these students were really thinking about the relationship between *x* and *y* or if they were just listing *y* values until they found ones that matched. We are not sure if some students may have still been implicitly thinking about *x* values even when not writing them. Nonetheless, this tension between writing tables showing one values for one variable or both variables was not something we anticipated when planning. So this represented some new learning for us.

Finally, we all thought that this process was a good opportunity to see a lesson in practice and see how students compared responses and discussed their mathematical thinking. Moving forward we are considering how we can help students steer more of the thinking and argument making. For one, we want to be careful to not pack too much into a single lesson. We also want to be continue to work on making sure the task is clear and aligned with the lesson goals. We are all thinking about Jonathan, who, at the end of the lesson, asked, "Do we have to do this [make a table and a graph] every time?" We think this is actually a really good mathematical question that gets at a deeper curiosity about which methods might be most applicable to and useful for a given mathematical task. We realize that he asked this question because our team didn't help the students make sense for why one might choose to graph, for example, in this situation. Perhaps graphing should not have even been part of the learning for the day. However, the question shows that students are thinking in this way. Good lessons incorporating problem solving can make use of this type of reasoning to build deeper student understanding.