financialalgebra

1.7 Piecewise Functions
Student Activity Packet
UNIT: TAXES & FUNDAMENTALS OF ALGEBRA

Name:

IN THIS LESSON, YOU WILL:

- Write equations for piecewise functions
- Read the graphs of piecewise functions
- Identify whether functions are continuous or discontinuous
- Represent stepped tax rates using piecewise functions
- Use piecewise functions to deepen understanding of marginal and effective tax rates for income or payroll taxes

INTRO

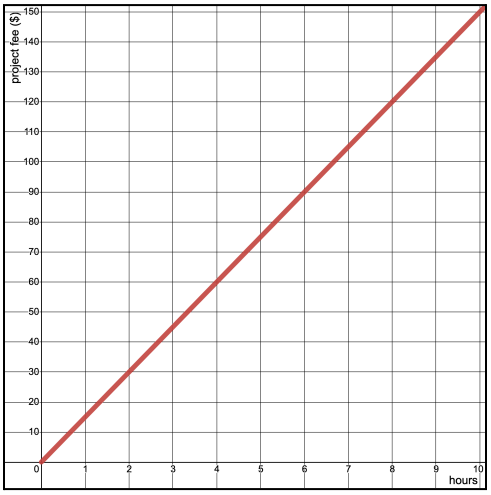
ANALYZE: Which Graph Fits?

1. **Tyrese is a freelance artist who charges a flat fee of $60 for any project that takes 4 hours or less. If a project takes him longer than 4 hours, he charges $15 per hour.**
    a. How much does Tyrese charge for a project that takes him 3 hours?

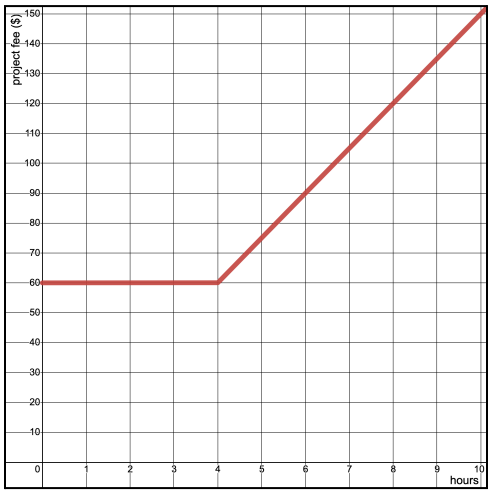
    b. How much does Tyrese charge for a project that takes him 8 hours?

2. Which of the following graph accurately represents Tyrese's pricing? How do you know?

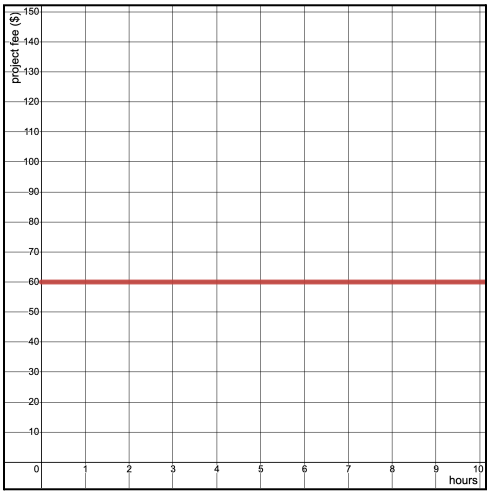
Graph A



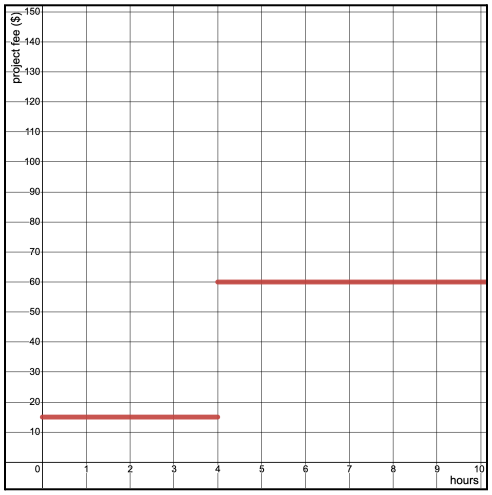
Graph B



Graph C



Graph D



LEARN IT

Part I: Introducing Piecewise Functions

A piecewise function has multiple "pieces" that follow different rules, depending on the domain (x values).

1. **In your own words, explain why Tyrese's pricing was a piecewise function.**

2. **What other real-life situation(s) might be represented by a piecewise function?**

Part II: Writing Equations for Piecewise Functions

We can use case notation to write an equation for piecewise functions. This tells us what rule to apply for each "case" or "piece" of the function.

Here is the equation for Tyrese's project fees:

Equation	What it Means	How to Read It
$f(x) = \begin{cases} 60 & 0 < x \leq 4 \\ 15x & x > 4 \end{cases}$	f(x) = 60 for these x values: 0 < x ≤ 4 f(x) = 15x for these x values: x > 4	*"F of x equals 60 when x is greater than 0 and less than or equal to 4. F of x equals 15x when x is greater than 4."*

3. **Tyrese's business is booming, so he decides to increase his prices. Now, he charges $100 for any project that takes five hours or less. He charges $20 per hour for projects that take longer than 5 hours. Write the new equation for his pricing.**

$$f(x) = \begin{cases} \_\_\_\_ & \_\_\_\_\_ \\ \_\_\_\_ & \_\_\_\_\_ \end{cases}$$

4. Marshall is renting a bike for the day. It costs $13 for up to one hour. After one hour, the price increases to $20. After three hours, the price increases again to $50. The maximum time he can rent the bike is 10 hours total.

a. Fill in the blanks to write an equation that represents this situation.

$$f(x) = \begin{cases} \_\_\_\_ & \_\_\_\_ < x \leq \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ < x \leq \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ < x \leq \_\_\_\_ \end{cases}$$

b. What is f(4)?

c. What is f(1)?

d. For which x values does f(x) = 20?

e. What is the domain of this function?

PRACTICE IT

5-Point Practice: Writing Equations for Piecewise Functions

Complete any number of problems that add up to 5 points. For each problem, write an equation to model the situation.

1 point each

An airline charges for in-flight internet. It costs $7 for 1 hour or less of internet. It costs $19 for more than 1 hour, with a maximum of 24 hours total.

Equation:

$$f(x) = \begin{cases} \_\_\_\_ & 0 < x \leq \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ < x \leq \_\_\_\_ \end{cases}$$

Bagels are $1 each if you buy 12 or fewer bagels. If you buy more than 12 bages, they cost $0.75 each.

Equation:

$$f(x) = \begin{cases} 1x & \_\_\_\_\_ \\ 0.75x & \_\_\_\_\_ \end{cases}$$

A parking lot charges $3 to park for anytime up to 2 hours. After 2 hours, they charge $1.50 per hour

Equation:

$$f(x) = \begin{cases} \_\_\_\_ & 0 < x \leq \_\_\_\_ \\ \_\_\_\_ & x > \_\_\_\_ \end{cases}$$

2 points each

Kaustabh is a DJ who charges $200 for any event under 2 hours. For events 2 hours or longer, he charges $100 per hour.

Equation:

Apples cost $3 per pound if you buy less than 5 pounds. If you buy 5 pounds or more, they cost $1.25 per pound.

Equation:

Sara earns $16 per hour working at Foods Co-op. If she works more than 40 hours a week, she earns time-and-a-half (ie. a 50% pay increase) for those overtime hours. Write an equation for her total weekly pay as a function of hours worked.

Equation:

3 points each

A stamp costs $0.58 for any letter that weighs 1 oz or less. It costs an additional $0.20 per ounce to mail letters that weigh more than 1 oz, up to a maximum of 3.5 oz.

Equation:

Cleo works as a recruiter earning commission. They have a base salary of $35,000 and are paid $5,000 per person hired for the first 8 positions they fill. If they fill more than 8 positions, they are paid $6000 for each additional person hired.

Equation:

Under her health insurance, Toni pays 100% of her medical costs up to $1000. After that, she pays 30% of any additional medical costs. However, her total payments are limited to $8000, no matter her medical costs.

Equation:

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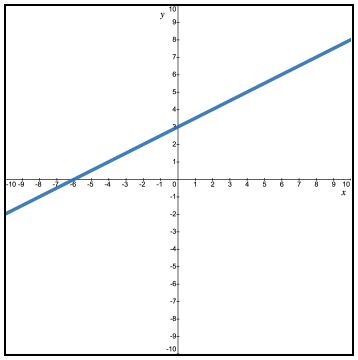
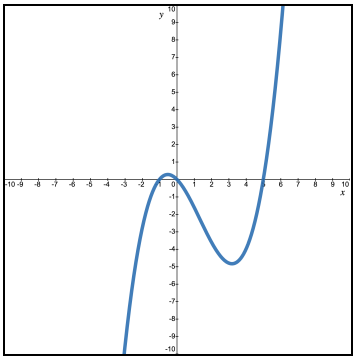
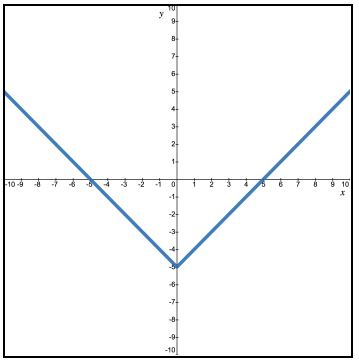
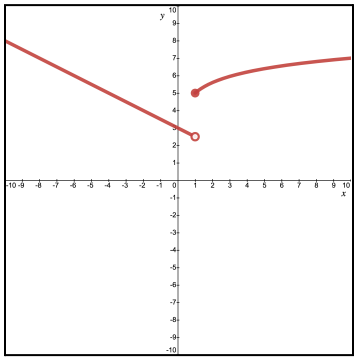
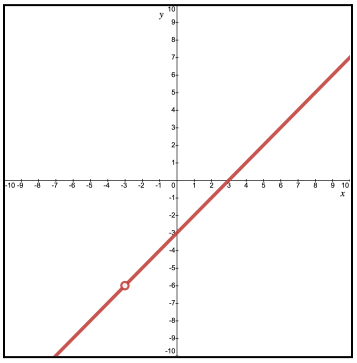
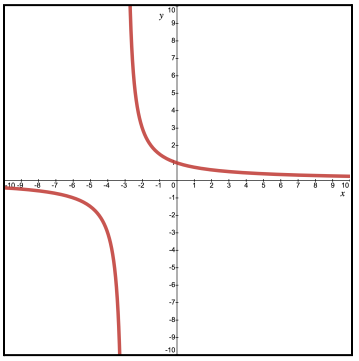
Graphs of Piecewise Functions

Functions can be classified as continuous or discontinuous.

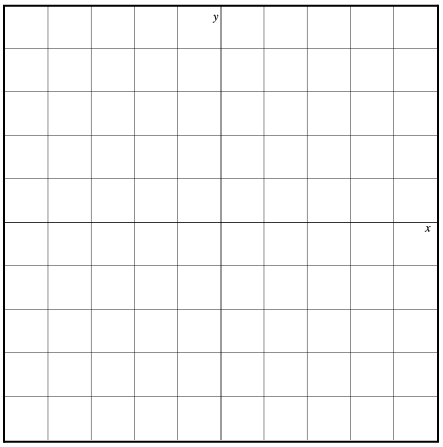
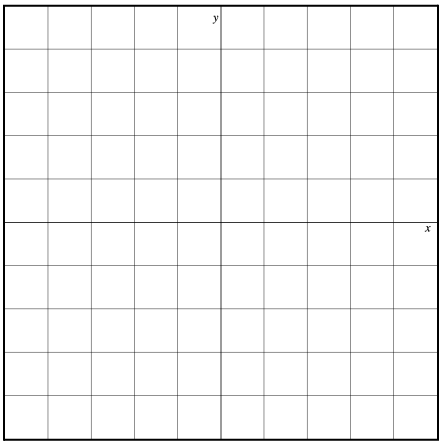
Continuous: A function whose graph you could draw without lifting up your pencil.

Discontinuous: A function with some type of "break", so you could not draw its graph without lifting your pencil. This "break" could be somewhere where the graph has a hole or where it jumps and doesn't connect.

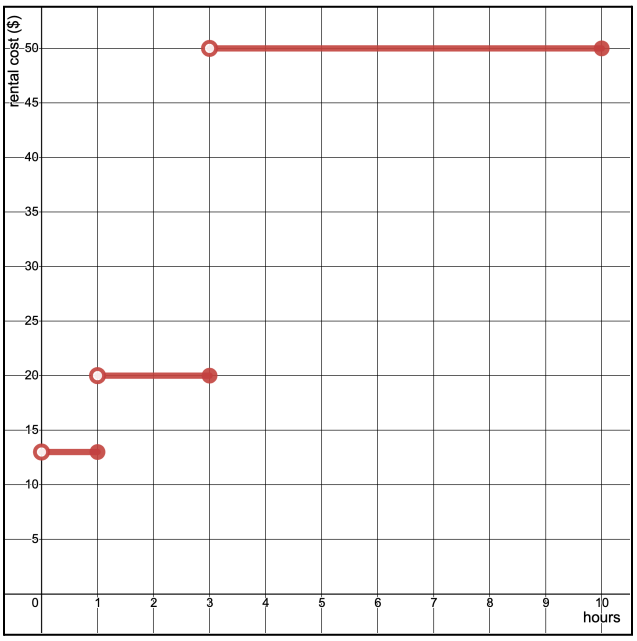




1. **Sketch your own examples of a <u>continuous</u> function and a <u>discontinuous</u> function.**

Continuous	Discontinuous
	

2. Let's look back at the earlier problem about Marshall renting a bike. Here is the equation and graph for that function:

$$f(x) = \begin{cases} 13 & 0 < x \leq 1 \\ 20 & 1 < x \leq 3 \\ 50 & 3 < x \leq 10 \end{cases}$$



a. Is the graph continuous or discontinuous?

b. What is f(2)? What does that mean in this context?

c. Compare the graph with the domain statements in the equation. What does the open circle on the graph tell you? What does the closed circle tell you?

d. What is f(3)?

PRACTICE IT

ACTIVITY: Piecewise Functions Card Sort

1. **Match each word problem to the corresponding function, graph, and point. Write your answers or paste the cards below.**

Match 1	
Match 2	
Match 3	
Match 4	

APPLY IT

Follow your teacher's directions to complete the Application Problems.