Linear Equation (Maximization)

A farmer raises **goats and cattle**. Each animal gives a certain **profit**, and requires **space** and **feed**. The farmer has limited resources and wants to determine the optimal number of goats and cattle to raise in order to **maximize profit**.

Details:

			it per Unit (Rs) Space
Goat	1,500	4	**q.m) Feed Required kg/day)** 3
Cattle	3,000	10	8

Available Resources:

- Total space available = 100 sq.m
- Total feed available = 80 kg/day

Let:

- x = number of **goats**
- y = number of **cattle**

Objective Function:

Maximize Profit:

- Maximize $Z=1500x+3000y$

Subject to Constraints:

1. **Space constraint** (sq.m): $4x+10y \leq 100$
2. **Feed constraint** (kg/day): $3x+8y \leq 80$
3. **Non-negativity constraint:** $x \geq 0, y \geq 0x$

Step 1: Convert to Standard Form

Add **slack variables** s_1 and s_2 to convert inequalities into equalities.

$$4x+10y+ s_1=100$$
$$3x+8y+ s_2=80$$

Objective function: $Z=1500x+3000y+0s_1+0\ s_2$

	C_j	1500	3000	0	0	RHS	Ratio
	B.V.	x	y	s_1	s_2		
0	s_1	4	10	1	0	100	10
0	s_2	3	8	0	1	80	10
	Z_j	0	0	0	0	0	
	$Z_j - C_j$	-1500	-3000	0	0		

Identify Entering and Leaving Variables

- Most negative in Z row: **-3000**, so **y** enters.
- Ratios (RHS / y-column):
  - Row 1: 100/10=10
  - Row 2: 80/8=10

Choose **tie-breaker → pick Row 1**
So, **s1 leaves**, y enters.

Step 4: Pivot Operation (Make pivot = 1 and others = 0)

Key element = 10 in Row 1.
Row operations:

- **New Row 1** (pivot row): divide all by 10
  → R1=R1/10
  R1:0.4x+1y+0.1s_1=10

New Row 2: R2= R2−8×New R1

New R1	**New Row 2**: R2= R2−8×New R1
0.4	3-8*0.4 = -0.2
1	8-8*1 = 0
0.1	0 - 8*0.1 = -0.8
0	1 – 8*0 = 1
10	80 – 8*10 = 0

	C_j	1500	3000	0	0	RHS	Ratio
	B.V.	x	y	s_1	s_2		
3000	y	0.4	1	0.1	0	10	25
0	s_2	-0.2	0	-0.8	1	0	
	Z_j	1200	3000	300	0	30000	
	$Z_j - C_j$	-300	0	300	0		

Step 5: Next Entering and Leaving Variables

- Most negative in Z row: **-300**, so x enters.
- Ratios (RHS / x-column):
  - Row 1: 10/0.4=25
  - Row 2: Not valid (x coefficient is negative)

So, **Row 1** is pivot row again, **y leaves**, x enters.

Step 6: Pivot Again (Pivot = 0.4 in Row 1)

- **New Row 1**: R1=R1/0.4

$x+2.5y+0.25s_1=25$

New Row 2: R2= R2+0.2×New R1 $\mathbf{0x + 0.5y -0.75s_1 + 1s_2 = 5}$

New R1	**New Row 2**: R2= R2+0.2×New R1
1	-0.2+0.2*1 = 0
2.5	0+0.2*2.5 = 0.5
0.25	-0.8 + 0.2*0.25 = -0.75
0	1 + 0.2*0 = 1
25	0 + 0.2*25 = 5

	C_j	1500	3000	0	0	RHS	Ratio
	B.V.	x	y	s_1	s_2		
1500	x	1	2.5	0.25	0	25	25
0	s_2	0	0.5	0.75	1	5	
	Z_j	1500	3750	375	0	37500	
	$Z_j - C_j$	0	750	375	0		

Final Answer

- x=25 goats
- y=0 cattle
- **Maximum Profit = Rs. 37,500**