Linear Equation (Maximization)

A farmer raises **goats and cattle**. Each animal gives a certain **profit**, and requires **space** and **feed**. The farmer has limited resources and wants to determine the optimal number of goats and cattle to raise in order to **maximize profit**.

Details:

			it per Unit (Rs) Space
Goat	1,500	1 4	q.m) Feed Required kg/day) 3
Cattl e	3,000	10	8

Available Resources:

- Total space available = 100 sq.m
- Total feed available = 80 kg/day

Let:

- x = number of **goats**
- y = number of cattle

Objective Function:

Maximize Profit:

• Maximize Z=1500x+3000y

Subject to Constraints:

Space constraint (sq.m): 4x+10y≤100
Feed constraint (kg/day): 3x+8y≤80
Non-negativity constraint: x≥0, y≥0x

Step 1: Convert to Standard Form

Add slack variables s1 and s2 to convert inequalities into equalities.

 $4x+10y+s_1=100$ $3x+8y+s_2=80$

Objective function: Z=1500x+3000v+0s1+0 s2

Objective it	Cj	1500	3000	0	0	RHS	Ratio
	B.V.	X	у	S 1	S2		
0	S1	4	10	1	0	100	10
0	S2	3	8	0	1	80	10
	Zj	0	0	0	0	0	
	Zj - Cj	-1500	-3000	0	0		

Identify Entering and Leaving Variables

- Most negative in Z row: -3000, so y enters.
- Ratios (RHS / y-column):
 - o Row 1: 100/10=10
 - o Row 2: 80/8=10

Choose tie-breaker \rightarrow pick Row 1

So, **s1 leaves**, y enters.

Step 4: Pivot Operation (Make pivot = 1 and others = 0)

Key element = 10 in Row 1.

Row operations:

• New Row 1 (pivot row): divide all by 10

 \rightarrow R1=R1/10

 $R1:0.4x+1y+0.1s_1=10$

New Row 2: R2= R2-8×New R1

New R1	New Row 2 : R2= R2-8×New R1
0.4	3-8*0.4 = -0.2
1	8-8*1 = 0
0.1	0 - 8*0.1 = -0.8
0	1 - 8*0 = 1
10	80 - 8*10 = 0

	Cj	1500	3000	0	0	RHS	Ratio
	B.V.	X	у	S1	S2		
3000	у	0.4	1	0.1	0	10	25
0	S2	-0.2	0	-0.8	1	0	
	Zj	1200	3000	300	0	30000	
	Zj - Cj	-300	0	300	0		

Step 5: Next Entering and Leaving Variables

- Most negative in Z row: -300, so x enters.
- Ratios (RHS / x-column):
 - o Row 1: 10/0.4=25
 - o Row 2: Not valid (x coefficient is negative)

So, **Row 1** is pivot row again, y leaves, x enters.

Step 6: Pivot Again (Pivot = 0.4 in Row 1)

• New Row 1: R1=R1/0.4

 $x+2.5y+0.25s_1=25$

New Row 2: $R2 = R2 + 0.2 \times New R1 0x + 0.5y - 0.75s_1 + 1s_2 = 5$

New R1	New Row 2 : R2= R2+0.2×New R1
1	-0.2+0.2*1 = 0
2.5	0+0.2*2.5 = 0.5
0.25	-0.8 + 0.2*0.25 = -0.75
0	1 + 0.2*0 = 1
25	0 + 0.2*25 = 5

	Cj	1500	3000	0	0	RHS	Ratio
	B.V.	X	у	S1	S2		
1500	X	1	2.5	0.25	0	25	25
0	S2	0	0.5	0.75	1	5	
	Zj	1500	3750	375	0	37500	
	Zj - Cj	0	750	375	0		

Final Answer

- x=25 goats
- y=0 cattle
- Maximum Profit = Rs. 37,500