(16) Constructing Electromagnetic Field tensor F in Clifford Algebra $Cl_{3,0}$(R) using generalized quaternion commutator.

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ABSTRACT

This article describes the application of a generalized quaternion commutator operator GQC(**p**,**q**) = $\mathbf{pq}^{\sim} + e^{2i\alpha}\,\mathbf{qp}^{\sim}$ [eq.I] in $Cl_{3,0}$(R) ("Clifford Algebra" - CA) for constructing a full electromagnetic field 8-vector **F** = -S - **E**/c - *i* **B** - *i*cM (S,M are scalar components).

The main purpose of this article is to show that parameter α of the GQC determines a 'permeability' - a coupling between the charge and current densities related to the scalar fields S,M and the **E,B** field strength.

The second objective of this article is to attempt to represent the 4-vector time-space as the zero **F** field isoclines in the argument space $\{\mathbf{p}_L,\mathbf{q}_L\}$ of the GQC($\mathbf{p}_L,\mathbf{q}_L$), under the condition that $\{\mathbf{p}_L,\mathbf{q}_L\}$ are left-handed spinors.

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INTRODUCTION

Without losing a generality of the formula, the GQC can be rewritten, applying a complex scaling factor $e^{i\alpha}$ to one of its arguments, as

$$GQC(\mathbf{p},\mathbf{q}) = e^{-i\alpha}\,\mathbf{p}\,(e^{i\alpha}\mathbf{q})^{\sim} + e^{i\alpha}\,(e^{i\alpha}\mathbf{q})\,\mathbf{p}^{\sim} = GQC(\mathbf{p},\mathbf{r}) = e^{-i\alpha}\,\mathbf{p}\,\mathbf{r}^{\sim} + e^{i\alpha}\,\mathbf{r}\,\mathbf{p}^{\sim}$$

where $\mathbf{r} := e^{i\alpha}\mathbf{q}$

Note that applying a scaling factor to a left-handed spinor does not change the handedness, thus if the argument domain for {p,q} is left-handed spinor domain, then {**p**,**r**} is also left-handed. We can rename **r** back to **q** and use the new form of the GQC as the alternative commutator formula.

$GQC(\mathbf{p},\mathbf{q}) = e^{-i\alpha}\,\mathbf{p}\,\mathbf{q}^{\sim} + e^{i\alpha}\,\mathbf{q}\,\mathbf{p}^{\sim}$ [eq.II]

Expanding the exponents using Euler's formula will allow us to rewrite GQC as a linear combination of a quaternion commutator QC and quaternion anticommutator QA:

$$GQC(\mathbf{p},\mathbf{q}) = e^{-i\alpha}\,\mathbf{p}\,\mathbf{q}^{\sim} + e^{i\alpha}\,\mathbf{q}\,\mathbf{p}^{\sim} = (\cos\alpha - i\sin\alpha)\ \mathbf{p}\,\mathbf{q}^{\sim} + (\cos\alpha + i\sin\alpha)\ \mathbf{q}\,\mathbf{p}^{\sim} =$$
$$\cos\alpha\,(\mathbf{p}\,\mathbf{q}^{\sim} + \mathbf{q}\,\mathbf{p}^{\sim}) - i\sin\alpha\,(\mathbf{p}\,\mathbf{q}^{\sim} - \mathbf{q}\,\mathbf{p}^{\sim}) =$$
$$\cos\alpha\ QC(\mathbf{p},\mathbf{q}) - i\sin\alpha\ QA(\mathbf{p},\mathbf{q})$$

where

$$QC(\mathbf{p},\mathbf{q}) = \mathbf{p}\,\mathbf{q}^{\sim} + \mathbf{q}\,\mathbf{p}^{\sim}$$
$$QA(\mathbf{p},\mathbf{q}) = \mathbf{p}\,\mathbf{q}^{\sim} - \mathbf{q}\,\mathbf{p}^{\sim}$$

It's easy to see[1] that for all arguments **p** and **q**, QC(**p**,**q**) is always a complex scalar, while QA(**p**,**q**) is the Clifford 6-vector of only the primary and secondary components with no scalar components. That is, if we take the full Clifford base vectors as {1,**g**,**e**,**f**,**i**,**j**,**k**,*i*} then QC(p,q) maps the Clifford double spinor space into Complex scalar subdomain {1,*i*} while QA into {**g**,**e**,**f**,**i**,**j**,**k**}. We have already explored the fact (see article [15]) that QC reproduces correct rotation and Lorentz transformations for electromagnetic scalar fields S,M while QA reproduces the electric and magnetic field components **E** + *i***B** of the F field vector.

Full field vector **F** can be thus represented as:

$$\mathbf{F} = \cos\alpha\,(-S-icM) - i\sin\alpha\,(-i\,\mathbf{E}/c+\mathbf{B}) = \cos\alpha\,(-S-icM) + \sin\alpha\,(-\mathbf{E}/c-i\,\mathbf{B})$$
$$QC(\mathbf{p},\mathbf{q}) = -S - icM$$
$$QA(\mathbf{p},\mathbf{q}) = -i\mathbf{E}/c + \mathbf{B}$$

Let us look at the full Maxwell equation in Clifford Algebraic form (see [4] and [4A]).
An original formula:

$$\nabla'\mathbf{F} = \partial/\partial t - c(\partial/\partial x\,\mathbf{g} + \partial/\partial y\,\mathbf{e} + \partial/\partial z\,\mathbf{f})\ (-S-\mathbf{E}/c - i{*}\mathbf{B} - icM) = \rho/\varepsilon_0 + \mathbf{J}/(c\varepsilon_0)$$

is rewritten to include the $\cos\alpha$ and $i\sin\alpha$ scaling factors as:

$$\nabla'\mathbf{F} = \partial/\partial t - c(\partial/\partial x\,\mathbf{g} + \partial/\partial y\,\mathbf{e} + \partial/\partial z\,\mathbf{f})\ (\ \cos\alpha\,(-S-icM) + \sin\alpha\,(-\mathbf{E}/c - i\mathbf{B}) = \rho/\varepsilon_0 + \mathbf{J}/(c\varepsilon_0)$$

then it can be expanded into the base vector components (like in [4A]):

$$\nabla'\mathbf{F} = \partial/\partial t - c(\partial/\partial x\,\mathbf{g} + \partial/\partial y\,\mathbf{e} + \partial/\partial z\,\mathbf{f})$$
$$(-S\cos\alpha - ((E_x\mathbf{g} + E_y\mathbf{e} + E_z\mathbf{f})/c)\sin\alpha - i{*}(B_x{*}\mathbf{g} + B_y{*}\mathbf{e} + B_z{*}\mathbf{f})\sin\alpha - icM\cos\alpha) =$$

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[1] I love this expression

$-\partial S/\partial t \cos\alpha + \mathrm{div}\mathbf{E} \sin\alpha\ +$ // ρ/ε_0 Clifford scalar (note[2])

$+i\ \mathrm{div}\mathbf{B} \sin\alpha - ic\partial M/\partial t \cos\alpha$ // $i\rho_M/\varepsilon_0$ Clifford pseudoscalar

$-(1/c)\partial\mathbf{E}/\partial t \sin\alpha + c\ \mathrm{rot}\mathbf{B} \sin\alpha + c\ \mathrm{grad}S \cos\alpha +$ // $\mathbf{J}/(c\varepsilon_0)$ primary Clifford vector

$-i\ \partial\mathbf{B}/\partial t \sin\alpha\ - i\ \mathrm{rot}\mathbf{E} \sin\alpha\ +\ ic^2\ \mathrm{grad}M \cos\alpha$ // $i\mathbf{J}_M/(c\varepsilon_0)$ secondary Clifford vector[3]

$= \rho/\varepsilon_0 + \mathbf{J}/(c\varepsilon_0) + i\rho_M/\varepsilon_0 + i\mathbf{J}_M/(c\varepsilon_0)$ // units $[\mathrm{v/m}^2]$

is sinα equal to, or proportional to the fine structure constant $\boldsymbol{\alpha}$ (=1/137) ?

$\varepsilon_0 = e^2/(2\boldsymbol{\alpha}hc)$

α = 0.4182 deg

[UNFINISHED]

Generalised Quaternion Commutator and De Sitter metrics

See the article:

[Observers are All You Need: How Observer-Synchronization Creates All of Physics](//muellerberndt.medium.com/observers-are-all-you-need-how-observer-synchronization-creates-all-of-physics-8ebb7e9783e7)

Bernhard Mueller

(//muellerberndt.medium.com/observers-are-all-you-need-how-observer-synchronization-creates-all-of-physics-8ebb7e9783e7)

de Sitter metrics $ds^2 = -\,dt^2 + e^{2Ht}\,\mathbf{dx}^2$ where $\mathbf{dx} = (dx,dy,dz)$

Notice similarity to Eq.I

In Clifford notation it could be rewritten as

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[2] S and ρ/ε_0 are real numbers

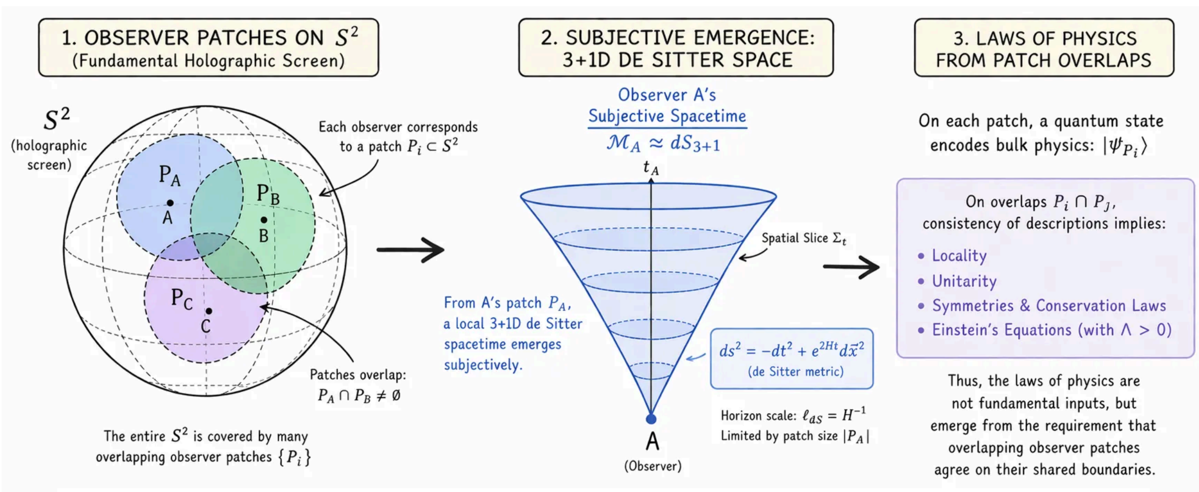
[3] More common nomenclature is to call primary Clifford vectors "polar" vectors and secondary Clifford vectors, bivectors or "axial" vectors.

this Clifford bilinear norm

$GQC(\mathbf{p},\mathbf{q}) = \ \mathbf{p}\tilde{\mathbf{q}} + e^{2Ht}\,\mathbf{q}\tilde{\mathbf{p}}$

produces the following square modulus (metric) function

$|\mathbf{p}|^2 = \ \mathbf{p}\tilde{\mathbf{p}} + e^{2Ht}\,\mathbf{p}\tilde{\mathbf{p}} = \mathbf{p}\tilde{\mathbf{p}}\,(1 + e^{2Ht})$



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