

AB Calculus Practice 5.3
Continuity and differentiability

1. If you can draw a function without picking up your pencil, it's probably _____.
2. For $f(x)$ to be continuous at $x = a$, the left- and right-hand _____ must match, and they must be the same value as $f(a)$.
3. If a function isn't a "smooth curve," it's probably not _____.
4. If you zoom in enough on any _____ function, it'll eventually look like a _____.
5. All _____ functions are _____, but not all _____ functions are _____.
6. For a function to be differentiable, it can't have any _____, _____, _____, nor any _____.
7. For $f(x)$ to be differentiable at $x = a$, the left- and right-hand _____ must match, and the left- and right-hand _____ must also match.

bold numbers - calculator permitted

$$f(x) = \begin{cases} 2x + 5 & \text{for } x < -1 \\ -x^2 + 6 & \text{for } x \geq -1 \end{cases}$$

8. If f is the function defined above, then $f'(-1)$ is
- (A) -2 (B) 2 (C) 3
(D) 5 (E) nonexistent

9. If $\lim_{x \rightarrow a} f(x) = f(a)$, then which of the following statements about f must be true?

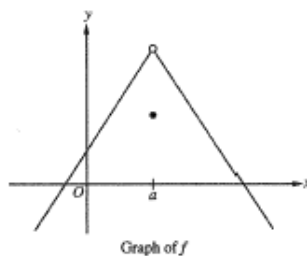
- (A) f is continuous at $x = a$.
- (B) f is differentiable at $x = a$.
- (C) For all values of x , $f(x) = f(a)$.
- (D) The line $y = f(a)$ is tangent to the graph of f at $x = a$.
- (E) The line $x = a$ is a vertical asymptote of the graph of f .

10. If the function f is continuous at $x = -2$, which of the following must be true?

- (A) $f(-2) < \lim_{x \rightarrow -2} f(x)$
- (B) $\lim_{x \rightarrow -2^+} f(x) \neq \lim_{x \rightarrow -2^-} f(x)$
- (C) $f(-2) = \lim_{x \rightarrow -2^+} f(x) = \lim_{x \rightarrow -2^-} f(x)$
- (D) The derivative of f at $x = -2$ exists
- (E) The derivative of f is positive for $x < -2$ and negative for $x > -2$

11. Let f be the function defined by $f(x) = \sqrt{|x + 3|}$ for all x . Which of the following statements is false?

- (A) f is continuous but not differentiable at $x = -3$.
- (B) f is continuous at $x = -3$.
- (C) f is differentiable at $x = -3$.
- (D) $\lim_{x \rightarrow -3} f(x) = 0$
- (E) $x = -3$ is a vertical tangent of the graph of f .



12. The graph of the function f is shown above. Which of the following statements must be true?

- (A) f is continuous at $x = a$.
- (B) $\lim_{x \rightarrow a} f(x) = f(a)$.
- (C) $\lim_{x \rightarrow a} f'(x)$ exists.
- (D) $\lim_{x \rightarrow a} f(x)$ exists.
- (E) $f(x)$ is continuous for all values of x .

$$f(x) = \begin{cases} x+2 & \text{if } x \leq 3 \\ 4x-7 & \text{if } x > 3 \end{cases}$$

13. Let f be the function given above. Which of the following statements are false about f ?

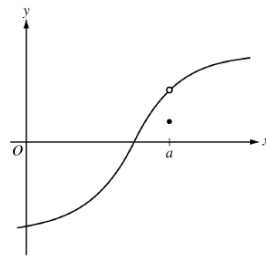
Circle all that apply

- I. $\lim_{x \rightarrow 3} f(x)$ exists
- II. f is continuous at $x = 3$
- III. f is differentiable at $x = 3$

a	$\lim_{x \rightarrow a^-} f(x)$	$\lim_{x \rightarrow a^+} f(x)$	$f(a)$
-1	4	6	4
0	-3	-3	5
1	2	2	2

14. The function f has the properties indicated in the table above. Which of the following must be true?

- (A) f is continuous at $x = -1$
- (B) f is continuous at $x = 0$
- (C) f is continuous at $x = 1$
- (D) f is differentiable at $x = 0$
- (E) f is differentiable at $x = 1$



Graph of f

15. The graph of $y = f(x)$ is shown above. Which of the following is false?

- (A) $\lim_{x \rightarrow a^+} f(x) \neq \lim_{h \rightarrow a^-} f(x)$
- (B) $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ does not exist.
- (C) $\lim_{x \rightarrow a} f(x) \neq f(a)$
- (D) $\lim_{x \rightarrow a} f(x)$ exists.

$$f(x) = \begin{cases} 3x - 2 & \text{if } x < 1 \\ \ln(3x - 2) & \text{if } x \geq 1 \end{cases}$$

16. Let f be the function defined above. Which of the following statements about f are false?

Circle all that apply

- I. $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x)$
- II. $\lim_{x \rightarrow 1^-} f'(x) = \lim_{x \rightarrow 1^+} f'(x)$
- III. f is differentiable at $x = 1$