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Tema: Derivada direccional

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Utilizando la definición hallar la derivada direccional del campo escalar F utilizando un punto p, en dirección v

a) $F(x,y) = 2y + 3y$; $P(1,1)$; $V = [1,-1] \Rightarrow \hat{V} = \left[\frac{1}{\sqrt{2}} ; -\frac{1}{\sqrt{2}}\right]$

$$\lim_{t \to 0} \frac{F(1,1) + t\left[\frac{\sqrt{2}}{2} ; \frac{\sqrt{2}}{2}\right] - F(1,1)}{t}$$

$$\lim_{t \to 0} \frac{2\left(1+\frac{\sqrt{2}}{2}t\right) + 3\left(1+\frac{\sqrt{2}}{2}t\right) - 2(1) - 3(1)}{t}$$

$$\lim_{t \to 0} \frac{\cancel{2} + \sqrt{2}t + \cancel{3} + \frac{3}{2}\sqrt{2}\,t - \cancel{2} - \cancel{3}}{t}$$

$$\lim_{t \to 0} \frac{\cancel{t}\left(\sqrt{2} + \frac{3}{2}\sqrt{2}\right)}{\cancel{t}} = \frac{5}{2}\sqrt{2} \; //$$

b) $F(x,y) = x^2 + xy$; $P = (1,0)$; $V = [3,4] \Rightarrow \hat{V} = \left[\frac{3}{5} ; \frac{4}{5}\right]$

$$\lim_{t \to 0} \frac{F(1,0) + t\left(\frac{3}{5} ; \frac{4}{5}\right) - F(1,0)}{t}$$

$$\lim_{t \to 0} \frac{\left(1 + \frac{3}{5}t\right)\left[\left(1+\frac{3}{5}t\right) + \frac{4}{5}t\right] (1)^2 + 1(0)}{t}$$

$$\lim_{t \to 0} \frac{\left(1 + \frac{3}{5}t\right)\left(1 + \frac{7}{5}t\right)}{t} \quad L'H$$

$$\frac{3}{5} + \frac{7}{5} = 2 \; //$$

c) $F(x,y,z) = xy + yz + xz$; $P(0,1,-1)$; $V = [1,2,2] \Rightarrow \hat{V} = [1/3; 2/3; 2/3]$

$$\lim_{t \to 0} \frac{F[0,1,-1] + t\,[1/3; 2/3; 2/3] - F(0,1,-1)}{t}$$

$$\lim_{t \to 0} \frac{F(1/3;\ 1+2/3t;\ -1+2/3t) - F(0,1,-1)}{t}$$

$$\lim_{t \to 0} \frac{t/3(1+2/3t) + (1+2/3t)(-1+2/3t) + (2/3)(-1+2/3t) - (0)(1) - (1)(1) + (0)(-1)}{t}$$

$$\lim_{t \to 0} \frac{(1+2/3t)(-1+(0/3)t)+1}{t}$$

$$\lim_{t \to 0} \frac{\cancel{t} + 10/9\,t^{\cancel{2}}}{\cancel{t}} = 1 + \frac{10}{9}(0) = 1 \ //$$

Hallar la derivada direccional del campo escalar F en el punto P, en direcció V.

a) $F(x,y) = (2x+3y)^3$; $P = (1,1)$; $V = [-1,1]$

$P = (1,1)$
$V = (-1,1)$

$\vec{\nabla} f(x,y) = \left(\frac{\partial f}{\partial x} ; \frac{\partial f}{\partial y}\right)$

$-\frac{\partial f}{\partial x} = 6\,(2x+3y)^2$

$-\frac{\partial f}{\partial y} = 9\,(2x+3y)^2$

$\vec{\nabla} f(x,y) = (6\,(2x+3y)^2 ; 9\,(2x+3y)^2)$

$\vec{\nabla}(1,1) = 150 ; 225 \ //$

$\vec{u} \longrightarrow v$

$\vec{u} = (-1,1) - (1,1)$

$\hat{u} = [-1,0]$

$D\vec{v} = \vec{\nabla} f(x,y) . \vec{u}$

$D\vec{v} = (150, 225)\,\frac{1}{2}\,(-2,0)$

$D\vec{v} = -150 \ //$

b) $F(x,y) = \text{sen}\,(x,y)$; $P = (\pi/4 ; \pi/4)$; $\vec{v} = [3,4]$

$\vec{\nabla} f\,(x,y) = \left(\frac{\partial f}{\partial x} ; \frac{\partial f}{\partial y}\right)$

$-\frac{\partial f}{\partial x} = y \cdot \cos(x,y)$

$-\frac{\partial f}{\partial y} = x \cdot \cos(x,y)$

$\vec{\nabla} f(x,y) = (y \cos(x,y) ; x \cos(x,y))$

$\vec{\nabla} f(\pi/4 ; \pi/4) = (0{,}79 ; 0{,}08)$

$\vec{u} = (4,3) - (\pi/4, -\pi/4)$

$\vec{u} = (3{,}92 ; 2{,}71)$

$\hat{u} = \dfrac{< 3{,}92 ; 2{,}71 >}{\sqrt{3{,}92^2 + 2{,}71^2}}$

$D\vec{v} = \vec{\nabla} f(x,y)\ \vec{u}$

$D\vec{v} = (0{,}79)(0{,}08)\,\frac{1}{4{,}5}\,(3{,}92 ; 3{,}2$

$D\vec{v} = (0{,}69 ; 0{,}04)\ //$

c) $F(x,y) = \sqrt{xyz}$; $P = (1,0,1)$; $V = [2,-2,1]$

$\vec{u} = (2,-2,1) - (1,0,1)$

$\hat{u} = \langle \frac{1}{\sqrt{5}} ; -\frac{2}{\sqrt{5}}, 0 \rangle$

$\nabla f = \left(\frac{\partial f}{\partial x} ; \frac{\partial f}{\partial y} ; \frac{\partial f}{\partial z} \right)$

$\frac{\partial f}{\partial x} = \frac{1}{2} (xyz)^{-1/2} (yz)$

$\frac{\partial f}{\partial y} = \frac{1}{2} (xyz)^{-1/2} (xz)$

$\frac{\partial f}{\partial z} = \frac{1}{2} (xyz)^{-1/2} (x,y)$

$\vec{\nabla} F(1,0,1) = (0,0,0)$

$D\vec{v} = \vec{\nabla} F(0,0,0)\,\vec{u}$

$D\vec{v} = \vec{0}$

no se mueve en esa dirección

Encuentre la derivada direccional del campo escalar F en el punto F en el punto P, en dirección de el punto q

a) $F(x,y) = 2x - 7y + 1$; $P = (0,0)$; $q\,(1,1)$

$\vec{u} = (1,1) - (0,0)$

$\vec{u} = (1,1)$

$\hat{u} = (1,1)\ u$

$\vec{\nabla} f(x,y) = \left(\frac{\partial f}{\partial x} ; \frac{\partial f}{\partial y} \right)$

$\frac{\partial f}{\partial x} = 2$; $\frac{\partial f}{\partial y} = -7$

$\vec{\nabla} f(x,y) = (2,-7)$

$\vec{\nabla} f(0,0) = (2,-7)$

$D\hat{q} = \nabla f(x,y) . \hat{u}$

$= (2,-7)(1,1)$

$D\hat{q} = (2,-7)$ //

b) $F(x,y) = x^2 - 2xy + 5y^2$; $P = (-1,2)$; $q = (3,5)$

$\vec{u} = (3,5) - (-1,2)$

$\vec{u}\ (4,3)$

$\hat{u} = \langle \frac{4}{5} ; \frac{3}{5} \rangle$ //

$\frac{\partial f}{\partial x} = 2x - 2y$

$\frac{\partial f}{\partial y} = -2x + 10y$

$\vec{\nabla} f(x,y) = (2x - 2y ; -2x + 10y)$

$\vec{\nabla}(-1,2) = (-6, 22)$ //

$D\hat{q} = \nabla f(x,y) . \hat{u}$

$D\hat{q} = (-4{,}8 ; 13{,}2)$ //

c) $F(x,y,z) = xy + xz + yz$; $P(0,1,2)$; $q = (3,1,-4)$

$\vec{\mu} = (3,-1,4) - (0,1,2)$

$\vec{u} = (3,-2,2)$

$\hat{u} = \left(\frac{3}{\sqrt{17}} ; -\frac{2}{\sqrt{17}} ; \frac{2}{\sqrt{17}} \right)$ //

$\vec{\nabla} F(x,y,z) = \left(\frac{\partial f}{\partial x} ; \frac{\partial f}{\partial y} ; \frac{\partial f}{\partial z} \right)$

$\frac{\partial f}{\partial x} = y+z$; $\frac{\partial f}{\partial y} = x+z$; $\frac{\partial f}{\partial z} = x+y$

$\vec{\nabla} F(x,y,z) = (y+z ; x+z ; x+y)$

$\vec{\nabla} f(0,1,2) = (3,2,1)$ //

$D\hat{q} = \nabla f(x,y,z) \vec{\mu}$

$= (3,2,2) \frac{1}{\sqrt{17}} (3,-2$

$D\hat{q} = \left(\frac{9}{\sqrt{17}} ; -\frac{4}{\sqrt{17}} ; \frac{2}{\sqrt{1}}\right.$

Halle la derivada direccional del campo escalar F, en el punto P, en la dirección indicada θ

a) $F(x,y) = x^2y^3 + 2x^4y$; $P(1,2)$ y $\theta = \pi/3$.

$D_{\hat{u}} f(x,y) = \frac{\partial f}{\partial x} \cos(\pi/3) + \frac{\partial f}{\partial y} \operatorname{sen}(\pi/3)$

$= (2y^3x + 8x^3y) \cos \pi/3 + \frac{\partial f}{\partial y} (3x^2y^2 + 2x^4) \operatorname{sen} \pi/3$

$= \frac{2y^3x + 8x^3y}{2} + \frac{\sqrt{3}\,(3x^2y^2 + 2x^4)}{2}$

$D\hat{u} f(1,2) = 16 + 7\sqrt{3}$ //

b) $F(x,y) = (x^2 - y)^3$; $P = (3,1)$; $\theta = 3\pi/4$

$D\hat{u} f(x,y) = \frac{\partial f}{\partial x} \cos 3\pi/4 + \frac{\partial f}{\partial y} \operatorname{sen} 3\pi/4$

$= 6(x^2 y) x \cos 3\pi/3 - 3(x^2 - y) \operatorname{sen} 3\pi/4$

$= -3\sqrt{2}x\,(x^2 - y) - \frac{3\sqrt{2}\,(x^2 - y)}{2}$

$D\hat{u} f(3,1) = -72\sqrt{2} - 12\sqrt{2} -$

$= -84\sqrt{2}$ //

c) $F(x,y) = \frac{xy^2}{x^2+y^2}$; $P = (0,0)$; $\theta = \pi/4$

$D\hat{u} f(x,y) = \frac{\partial f}{\partial x} \cos(\pi/4) + \frac{\partial f}{\partial y} \operatorname{sen}(\pi/4)$

$= \frac{y^2(x^2+y^2) - 2x^2y^2}{(x^2+y^2)^2} \cos \pi/4 + \frac{2xy\,(x^2+y^2) \cdot 2xy^4}{(x^2+y^2)^2} \operatorname{sen} \pi/4$

$D\hat{u}(0,0)$ = indeterminado ; $(x^2+y^2)^2 \neq 0$.