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3 Levels of Topology

Project Overview

For this video, my main objective was to bring the topics I discussed in my first semester project website to people who might not be familiar with it or initially interested, in a way that's entertaining to watch and learn from. I was also very excited to meet Dr. Stoll, who is the reason I'm interested in topology in the first place.

This writeup is meant to serve as a complement to the video, and to give a fuller and more contextualized appreciation of the discussions, which are presented in the video in a very condensed nature. Thus, the lengthiness and thoroughness of this writeup relative to the video product is so that I do not feel like anything of value was lost or sacrificed to make the video an appropriate length - even with the relatively long video, there were substantial cuts made to approximately 3 hours of raw footage. I want everything that I captured in this project that I find interesting to be accessible in some way.

Level #1 - Arun Reyes

Coincidentally, Arun was also doing his final project for 8th grade geometry on topology, so he was a great level to start at! He came to my house the morning of Sunday, May 19th. We began with the basic notions of what topology is and how you can think of it as analogous to geometry, and Arun actually had a pretty solid understanding of the basics: he knew about homeomorphisms, manifold theory, etc. Since he said he'd focused mainly on knot theory, I thought it would be fun to introduce him to non-orientable manifolds. Since this was level 1, I focused on building it intuitively, starting with the most basic example, the Möbius strip, using it to define non-orientability, then discussing its properties.

We transitioned to talking about Klein bottles by wondering what happens when you glue two Möbius strips together; how would you construct a non-orientable with no edges? I stopped level 1 here since I didn't want to repeat the concepts discussed in the higher levels, but since Arun seemed already pretty familiar with everything we'd discussed and I didn't want him to be bored, I introduced more complex topics like embeddings vs immersions, fundamental polygons, and the real projective plane. I was very impressed that Arun is already exploring such relatively abstract and advanced topics in middle school, since in 8th grade I was just learning trigonometry and things of that nature. However, it does make me grateful that in this class we have the opportunity to do similar projects and sort of branch out beyond what's required to things that interest us.

Thanks to Mrs. Reyes for giving me the opportunity to meet Arun and discuss a fascinating topic that interests both of us!

Level #2 - Paul Lee

I met with Paul outside the Pleasanton Public Library on Saturday, May 18th. Paul was a good person to talk with for the second level because he's heard me talk about Klein bottles before (often not by choice), but doesn't necessarily have a concrete understanding of what they are. To get an idea of where to begin, I asked him what his understanding of topology was, and since he gave a somewhat vague answer I decided to also start from the foundations. I explained my understanding of what topology entails (which may or may not be the best, given that most of my reading is Wikipedia) as rubber-sheet geometry. I learned this term from Dr. Stoll in one of his Numberphile videos, and I really like it because it gives a sort of immediate intuition as to the kind of transformations that we're dealing with. It makes it easy to understand the notion that we're dealing with properties that don't care about the rigid shapes: the side lengths, the areas, the angles, the volumes.

Since this is level 2, I wanted to start introducing more complex topics, so I used the Klein bottle to introduce the distinction between immersions and embeddings: some manifolds, like Klein bottles and the real projective plane, cannot be embedded in 3-space, they require higher dimensions to exist. Thus, the representations that we can view in our universe are necessarily immersions, which are imperfect versions of the manifolds that have self-intersections or pinch points as a result. Paul asked me a really interesting question here: how do you know when something is an immersion and not an embedding? As far as I can tell, you can tell when a manifold is not in its "true" embedded state if it has cusps, self-intersections or cross-caps, since all immersions of the real projective plane and Klein bottle share these features. However, I'm sure this is not a mathematically rigorous way to figure out something is an immersion, so this could be interesting to look into.

I was curious to see if he could come up with any meaningful applications of topology since he'd just gotten an idea of what it entails, and sometimes newcomers to a topic are the most creative with it since they haven't had any particular way of thinking baked into them. He said something that somewhat reflects my thoughts on topology: maybe a topic doesn't necessarily have to be obviously useful in any way to be cool and interesting to learn about. (He compared it to MoMA.)

Level #3 - Clifford Stoll

For the final level, I was lucky enough to get to meet Dr. Clifford Stoll, who first got me interested in Klein bottles and topology through his Numberphile videos. After sending multiple emails and finally leaving a voicemail which he responded to, I drove to his house in Oakland, right next to Berkeley, on Sunday, May 12th. He invited me and my dad inside and offered us tea, and showed us around to his cluttered workshop area, where he finished up shipping his last Klein bottle order of the day (his fifth one). We talked a bit about physics, since I told him I'd be majoring in physics at UCLA and his undergraduate degree was also in physics. Interestingly, he expressed a bit of skepticism towards the current direction of the field, explaining that he felt that the theories and models were getting too abstract and inaccessible to the average person to the point that he wasn't sure it was really immediately applicable anymore (as physics ought to be) and as an example he described how the Michelson-Morley experiment uncovered a fundamental truth and disproved luminiferous aether using relatively simple equipment that doesn't compare to CERN and other labs today. However, in the past he's acknowledged being wrong in his skepticisms before, like claiming in 1995 that e-commerce and online news platforms were nonviable, so it's possible that he will change his views as models become more fleshed out and supported by observation.

For our topology discussion, we set up in a little cottage in his garden he'd built himself, where he told me many mathematicians visiting Oakland/Berkeley had stayed over the past decades. In hindsight I'm pretty sure I didn't explain his role in the project clearly enough, and he understood it to mean that he was responsible for covering all the levels, so we started at the very basics of topology and worked our way up through to topics like the real projective plane during a two hour discussion. Dr. Stoll has a way of getting really animated when discussing topics he's passionate about, with mannerisms that remind me of what people tell me I'm like when talking about Klein bottles or other things that are important to me. It felt as though he was so eager to get every word out and explain everything he wanted to show, ripping up paper and drawing on his glass bottles with no hesitation, scrambling to find his glass Boy's surface model. It was such a unique experience to capture the *je ne sais quoi* of his excitement and eccentric personality that I'd seen in videos, and other interviews, in person.

Level 3 in the video is an extremely condensed version of only the part of our discussion that focused on topology, which is why the conversation seems stilted and rushed. Since Dr. Stoll started at the very foundations of topology, much of what he said was also covered in the previous levels, so I cut them.

He showed me a fold-up bed he'd made for visiting mathematicians, and the various quilts he'd made, one of which was a solution to a specific case of an np-hard problem: what's the minimum number of squares needed to perfectly make a 13x13 grid? (Turns out, the answer is 11.)

After finishing with the “formal” interview, I asked him about some things I was curious about, like why he was so interested in topology after a career in astronomy and cybersecurity. He gave a pretty insightful and beautiful explanation: he has decided to focus his time and energy on having fun, not to spend his time publishing papers or trying to make money. I noticed that everything he’s doing nowadays is just stuff he wants to do: building a cottage in his garden, tiling his own bathroom, sewing, woodworking, and running a home business selling Klein bottles. When I asked about applications of topology, it sparked another tangent where he dug out his old ingenious binary calculator that worked using marbles rolling down a track, flipping wood “bits” to perform operations like counting, adding, and multiplying.

Dr. Stoll also surprised me with really good Chinese, which he explained he’d picked up when he visited China to teach astronomy 45 years ago. In conversational Chinese, he told me and my dad how sad it made him to think of the brilliant professors much older than him who he had to teach because they’d been banned from intellectual activity during the Cultural Revolution.

When I asked about his crawlspace storage system from one of his Numberphile videos, he led me back to his cluttered workshop and opened a little hatch near the floor, revealing an entrance to his makeshift warehouse. He demonstrated the remote-controlled homemade forklift by making it pick up a box of “unsellable” imperfect Klein bottles, which he let me take one of, explaining that he wouldn’t sell any bottle with even the slightest scratch because he might meet the mathematicians he sells to at a conference someday, and wants them to think that they got not a good Klein bottle, but a *great* Klein bottle. He let me drive around the robot and tour the warehouse - which was chock full of boxes of Klein bottles - using monitors that streamed camera feeds from the robot. I was pretty amazed by his engineering spirit, having created such a from-scratch warehouse system in his crawlspace to run a home business, without being an engineer by trade. I hope that I have that kind of mindset and ability throughout my career. Finally, around two and a half hours later, I thanked him for his time and left.

Reflection

It was interesting that in all three levels of knowledge, there were still fundamental topics that were necessary to discuss. For example, all three conversations went through the physical example of tracing out a path on a Möbius strip of showing that you end up on the opposite side, and I think this is indicative of how universal our intuitive way of understanding non-orientability is as well as the satisfaction of having a physical model to visualize abstract concepts and surfaces, which are parts of my Why for doing this project.

However, the levels also progressed in difficulty in a very natural way that I didn't have to plan. When talking about properties of non-orientable surfaces, Dr. Stoll explained that the inability to assign a vector field on a Möbius loop implied that we can't use non-orientable surfaces for surface integrals, e.g. Maxwell's equations. Each of the higher levels briefly recaps what was discussed in the previous ones, then adds on a new and more complex concept. In level 2, we introduce immersions vs embeddings. In level 3, we introduce fundamental polygons and the real projective plane.

I really enjoyed this project because at the lower levels, it gave me the opportunity to lead others towards the fascination I have with topology and why I think it's so cool, as well as to meet Dr. Stoll, who I've wanted to talk with for a while. Sometimes I feel like I'm not qualified to be the "interviewer" on this project, since I by no means have a concrete and rigorous understanding of the topics that I simply find cool and interesting to think about, but I think of something Dr. Stoll said to me before we began the interview:

> "When astronomers talk about me, they say, 'Oh, Cliff's no good at astronomy. But he's great at topology!' When topologists talk about me, they say, 'Oh, Cliff's no good at topology. But he's great at glassblowing!' When glassblowers talk about me, they say, 'Oh, Cliff's no good at glassblowing. But he's great at astronomy!'"

It makes me realize that he has no reservations about talking about things he's interested in out of impostor syndrome or fear that he's not qualified, and he's just focused on having fun and learning along the way, which is what I have aimed to do with this project as well.

On the Video Length

I'm acutely aware that the video exceeds the suggested runtime of 9 minutes, having gone through many late hours trying to shave as much extraneous content as possible. (Such is the curse of the video editor.) However, I am unwavering in my belief that the final product is as concise as I could encapsulate all the discussions I had without losing the direction or substance of each one. Please keep in mind I had a total of more than 3 hours of raw footage to cut down, not including extraneous non-topology discussions with Dr. Stoll. I cut entire sections of interviews to avoid redundancy. (I discussed fundamental polygons with Paul, but I cut that because Dr. Stoll covered it better. I discussed the coffee cup/donut transformation with Paul, but I cut that because it was more appropriate for Arun's discussion. I discussed the 0-volume property and embeddings vs immersions with Arun, but I cut that because it fit better in Paul's discussion.) Including necessary video introductions and transitions, a 9 minute video necessitates around 2.5 minutes per interview, which is a somewhat unreasonable expectation to cover an in-depth substantial discussion without reducing it to fluff.

I would note that many of the actual WIRED 5 Levels videos also fall around the 20-30 minute range, albeit placing more emphasis on the number of levels than depth of discussion. While it's unfortunate that the relatively lengthy video might discourage some viewers from watching it, I would rather face this than deny a full, gratifying experience of the interesting discussions I had and people I met during this project to those who are willing to watch the video in full.

Conclusion

I want to express my deep gratitude to Mrs. Reyes for giving the unique opportunity and encouragement to do this kind of project in class, which I would not have otherwise tried to pursue, and for humoring my unreasonable fascination with Klein bottles through the whole year. Meeting Dr. Stoll and discussing topology with him is an opportunity I've hoped for for a while, so this project was exactly the exigence I needed.

And finally, thank you for reading and watching!