Department of Physics
University of Rajasthan

Third Semester Mid-Term Test 2019

Paper PHY 901: Advanced Quantum Mechanics (unit 2 & 3)

Time allowed: 2 hrs (Shared paper with Dr Chhagan ji) M.M.: 40

Attempt all questions. Answer all questions in separate answer-sheets for corresponding units.
Note: *All notations are used following Bjorken-Drell. If you are following any other notation, please first explain your way of notation.*

1) Short type questions, all carries same marks: (each carries 4 marks)
   (a) Show that Schrödinger equation is not form invariant in relativistic limit.
   (b) Using properties of gamma matrices, prove the following:
      (i) $\gamma^{\mu}\gamma_5 + \gamma_5\gamma^{\mu} = 0$
      (ii) $\gamma_5 = \gamma^5$
   (c) Write down adjoint spinor and their Lorentz transformation.
   (d) Prove that matrix $P = \frac{1+\sigma_z}{2}$ is a projection, what do you understand by this projection?
   (e) Write down the coefficient requirements of Dirac equation.

2) Write down all 16 linearly independent 4×4 matrices says $\Gamma_{\mu\nu}{}^{n}$ and prove that for each Γ^{n}, except Γ^{S} there exists a Γ^{m} such $\Gamma^{n}\,\Gamma^{m} = -\,\Gamma^{m}\,\Gamma^{n}$. [5 + 15]

OR

Write down the plane-wave solutions of Dirac equation and using Lorentz transformation (for velocity in any random direction) to calculate the transformation matrix. [5 + 15]

End of Examination

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Solution of midterm question paper

These answers are not unique. You may have your own way of writing and you may have different ways of solving the same problem. These are few of correct answers. You can find similar solutions in many books of quantum field theory and advanced quantum mechanics. Since, I followed Bjorken-Drell notation. If you find any errors or typos, please inform me as soon as possible.

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1)

(a) $H\psi = E\psi \Rightarrow -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = -\iota\hbar\frac{\partial\psi}{\partial t}$

If we consider one dimensional (say x-axis) and for free particle (V = 0) then,

$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = -\iota\hbar\frac{\partial\psi}{\partial t}$

In Lorentz transformation, we know:

$t' = (t - \beta x/c)/\sqrt{1-\beta^2}$

$x' = (x - \beta ct)/\sqrt{1-\beta^2}$

$y' = y, z' = z$

Similarly, if we write differential form, we will find linear relation for time and space. Since, Schrödinger equation is not linear in time and space derivation. So, it can't remain it's form in time and space coordinates. So, it can't be form invariant for Lorentz transformation.

(b) (i) Easy to do, just use the relation $\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2g_{\mu\nu}$

(ii) Just a convention of Bjorken-Drell. I shall update the proof later.

(c) $\bar{\psi} = \psi^\dagger\gamma^0$ is an adjoint spinor.

$\bar{\psi}'(x') = \bar{\psi}(x)S^{-1}$

(d) For projection matrix, we have to prove $P^2 = P$ and $P^T = P$

Since $P = \frac{1+\sigma_z}{2} \Rightarrow P^2 = \frac{1}{4}(1 + 2\sigma_z + \sigma_z^2) = \frac{1}{2}(1+\sigma_z)$ | Since, $\sigma_z^2 = 1$

Similarly $P^T = \frac{1}{2}(1+\sigma_z)^T = \frac{1}{2}(1+\sigma_z^T) = \frac{1}{2}(1+\sigma_z) = P$ | Since, $\sigma_z^T = 1$

So, P is a projection, which represents the projection in x-axis direction. So, if we operate P on a vector in x-y plane, we will get the projection in x-direction.

(e) For $[\frac{\hbar}{\iota}\alpha_j\frac{\partial}{\partial x^j} + \beta mc^2]\psi = \iota\hbar\frac{\partial\psi}{\partial t}$, where

$\alpha_i\alpha_j + \alpha_j\alpha_i = 2\delta_{ij}$

$\alpha_i\beta + \beta\alpha_i = 0$ and $\alpha_i^2 = \beta^2 = 1$

2) Please have a look at the notes of date 27 September 2019.

OR

Please have a look at the notes of date 28 September 2019.