Mathematics, Grade 9 , Fractions

The same number in different forms

Use your calculator and determine the following

- ⅕ of R200
- 2/10 of 200
- 4/20 of 200

All of the answers are the same

- These numbers are equivalent fractions.
- They are different ways of writing the same number.

Converting between mixed numbers and fractions

- Numbers that have both whole number and fraction parts are called **mixed numbers**. Examples of mixed numbers: 3 4/5 , 2 7/8 and 8 3/10
- Mixed numbers can be written in expanded notation, for example: 3 4/5 means 3 + 4/5 and 2 7/8 means 2 + ⅞.

To add and subtract mixed numbers, you can work with the whole number parts and the fraction parts separately, for example:

3 ⅘ + 13 ⅗

=16 7/5

=17 2/5

13 ⅗ - 3 ⅘

12 8/5 - 3 ⅘

= 9 ⅘

- we need to "borrow" a unit from 13, because we cannot subtract 4/5 from ⅗.

3.2 Adding and subtracting fractions

To add or subtract two fractions, they have to be expressed with the *same* denominators first. To achieve that, one or more of the given fractions may have to be replaced with equivalent fractions.

$\frac{3}{20} + \frac{2}{5}$

We will refer to this as the LCM method.

$= \frac{3}{20} + \frac{2 \times 4}{5 \times 4}$

$= \frac{3}{20} + \frac{8}{20}$

$= \frac{11}{20}$

$\frac{5}{12} + \frac{7}{20}$

We will later refer to this method of adding or subtracting fractions as Method A.

$= \frac{5 \times 20}{12 \times 20} + \frac{7 \times 12}{20 \times 12}$

$= \frac{100}{240} + \frac{84}{240}$

$= \frac{184}{240}$

$= \frac{23}{30}$

In the case of $\frac{5}{12} + \frac{7}{20}$, multiplying by 20 and by 12 was a sure way of making equivalent fractions of the same kind, in this case two hundred-and-fortieths. However, the numbers became quite big. Just imagine how big the numbers will become if you use the same method to calculate $\frac{17}{75} + \frac{13}{85}$!

Fortunately, there is a method of keeping the numbers smaller (in many cases) when making equivalent fractions, so that fractions can be added or subtracted. In this method you first calculate the **lowest common multiple** or LCM of the denominators. In the case of $\frac{5}{12} + \frac{7}{20}$, the smaller multiples of the denominators are:

| 12: | 12 | 24 | 36 | 48 | 60 | 72 | 84 |
| 20: | 20 | 40 | 60 | 80 | 100 | 120 | 140 |

The smallest number that is a multiple of both 12 and 20 is 60.

Both $\frac{5}{12}$ and $\frac{7}{20}$ can be expressed in terms of sixtieths:

$\frac{5}{12} = \frac{5 \times 5}{12 \times 5} = \frac{25}{60}$ because to make twelfths into sixtieths you have to divide each twelfth into five equal parts, to get 12 × 5 = 60 equal parts, i.e. sixtieths.

Similarly, $\frac{7}{20} = \frac{7 \times 3}{20 \times 3} = \frac{21}{60}$.

Hence $\frac{5}{12} + \frac{7}{20} = \frac{25}{60} + \frac{21}{60} = \frac{46}{60} = \frac{23}{30}$

We may call this method the LCM method of adding or subtracting fractions.

Multiply a fraction by a whole number

Example:

$8 \times \frac{3}{5} = 8 \times 3 \text{ fifths} = 24 \text{ fifths} = \frac{24}{5} = 4\frac{4}{5}$

Divide a fraction by a whole number

You can divide a fraction by converting it to an equivalent fraction with a numerator that is a multiple of the divisor.

Example:

$\frac{2}{3} \div 5 = \frac{10}{15} \div 5 = 10 \text{ fifteenths} \div 5 = 2 \text{ fifteenths} = \frac{2}{15}$

A fraction of a whole number, and a fraction of a fraction

Examples:

A $\frac{7}{12}$ of R36.

   $\frac{1}{12}$ of R36 is the same as R36 ÷ 12 = R3, so $\frac{7}{12}$ of R36 is 7 × R3 = R21.

B $\frac{7}{12}$ of 36 fiftieths.

   $\frac{1}{12}$ of 36 fiftieths is the same as 36 fiftieths ÷ 12 = 3 fiftieths,

   so $\frac{7}{12}$ of 36 fiftieths is 7 × 3 fiftieths = 21 fiftieths.

   $\frac{7}{12} \times \frac{36}{50}$ means $\frac{7}{12}$ of $\frac{36}{50}$, it is the same.

   $\frac{1}{12}$ of $\frac{36}{50}$ is the same as $\frac{36}{50} \div 12 = \frac{3}{50}$, so $\frac{7}{12}$ of $\frac{36}{50}$ is $7 \times \frac{3}{50} = \frac{21}{50}$.

3. (a) You calculated $\frac{7}{12} \times \frac{36}{50}$ in the example above. What was the answer?

   (b) Calculate $\frac{7 \times 36}{12 \times 50}$, and simplify your answer.

Example:

$\frac{2}{3} \times \frac{5}{8} = \frac{2}{3} \text{ of } \frac{15}{24} = \frac{1}{3} \text{ of } \frac{30}{24} = \frac{10}{24} = \frac{5}{12}$

The same answer is obtained by calculating $\frac{2 \times 5}{3 \times 8}$.

To multiply two fractions, you may simply multiply the numerators and the denominators.

$\frac{2}{3} \times \frac{9}{20} = \frac{2 \times 9}{3 \times 20} = \frac{18}{60} = \frac{3}{10}$

Division by a fraction

When we divide by a fraction, we have a very different situation. Think about this:

If you have 40 pizzas, how many learners can have $\frac{3}{5}$ a pizza each?

To find the number of fifths in 40 pizzas: 40 × 5 = 200 fifths of a pizza.

To find the number of three fifths: 200 ÷ 3 = 66 portions of $\frac{3}{5}$ pizza and two fifths of a pizza is left over.

Since the portion for each learner is three fifths, the two fifths of a pizza that remains is two thirds of a portion.

So, to calculate 40 ÷ $\frac{3}{5}$, we multiplied by **5** and divided by **3**, and that gave us 66 and two thirds of a portion.

In fact, we calculated 40 × $\frac{5}{3}$.

> Division is the inverse of multiplication.
>
> So, to divide by a fraction, you multiply by its inverse.

Example:

$\frac{18}{60} \div \frac{2}{3} = \frac{18}{60} \times \frac{3}{2} = \frac{54}{120} = \frac{9}{20}$

SQUARES, CUBES, SQUARE ROOTS AND CUBE ROOTS

1. Calculate each of the following:

(a) $\dfrac{3}{4} \times \dfrac{3}{4}$

(b) $\dfrac{7}{10} \times \dfrac{7}{10}$

(c) $2\dfrac{5}{8} \times 2\dfrac{5}{8}$

(d) $1\dfrac{5}{12} \times 1\dfrac{5}{12}$

(e) $3\dfrac{5}{7} \times 3\dfrac{5}{7}$

(f) $10\dfrac{3}{4} \times 10\dfrac{3}{4}$

$\dfrac{9}{16}$ is the square of $\dfrac{3}{4}$, because $\dfrac{3}{4} \times \dfrac{3}{4} = \dfrac{9}{16}$. $\dfrac{3}{4}$ is the square root of $\dfrac{9}{16}$.

FRACTIONS, DECIMALS AND PERCENTAGE FORMS

1. The rectangle on the right is divided into small parts.

   (a) How many of these small parts are there in the rectangle?
   (b) How many of these small parts are there in one tenth of the rectangle?
   (c) What fraction of the rectangle is blue?
   (d) What fraction of the rectangle is pink?

   Instead of "six hundredths" we may say "6 per cent" or, in short, "6%". It means the same thing.

15 per cent of the rectangle on the right is blue.

2. (a) What percentage of the rectangle is green?
   (b) What percentage of the rectangle is pink?

   0,37 and 37% and $\dfrac{37}{100}$ are different ways of writing the same value (**37 hundredths**).

3. Express each of the following in three ways, namely as a decimal, a percentage and a fraction (in simplest form):

   (a) three tenths
   (b) seven hundredths
   (c) 37 hundredths
   (d) seven tenths
   (e) two fifths
   (f) seven twentieths
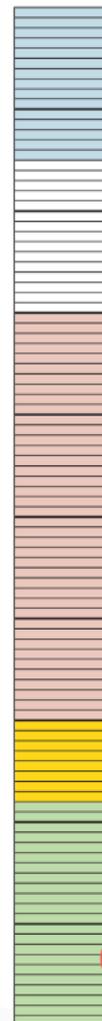
4.1 Equivalent forms

Decimal fractions and common fractions are simply different ways of expressing the same number. They are different **notations** showing the same value.

To write a decimal fraction as a common fraction: Write the decimal with a denominator that is a power of ten (10, 100, 1 000, etc.) and then simplify it if possible.

For example: $0{,}35 = \dfrac{35}{100} = \dfrac{7}{20} \times \dfrac{5}{5} = \dfrac{7}{20}$

To write a common fraction as a decimal fraction: Change the common fraction to an equivalent fraction with a power of ten as a denominator.

For example: $\dfrac{3}{4} = \dfrac{3}{4} \times \dfrac{25}{25} = \dfrac{75}{100} = 0{,}75$

If you are permitted to use your calculator, simply type in $3 \div 4$ and the answer will be given as 0,75. On some calculators you will need to press an additional button to convert the exact fraction to a decimal.

Notation means a set of symbols that are used to show a special thing.