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AIMS STRUCTURED MASTER'S IN MATHEMATICAL SCIENCES
SKILLS AND REVIEW COURSES
January 2024 intake

Week starting	**Course**	**Lecturer**	**Affiliation**
8 - 26 January 2024	Computer Algebra	Gerhard Pfister and Wolfram Decker	Kaiserslautern
8 - 26 January 2024	Mathematics and Art in Africa	Steve Bradlow	University of Illinois at Urbana-Champaign
8 - 26 January 2024	Fluid Dynamics	Richard Katz	University of Oxford
29 January (ongoing)	English language and communication	Lynne Teixeira	AIMS South Africa
29 January - 16 February 2024	Python	Mohau Mateyisi Makhamisa Senekane	CSIR University of Johannesburg
29 January - 16 February 2024	Applying logic	Jeff Sanders	AIMS South Africa
29 January - 16 February 2024	Transformation Groups	Claudiu Remsing	Rhodes University
29 January - 16 February 2024	Machine Learning	Ulrich Paquet	AIMS South Africa
26 February 2024	Mathematical Problem solving	Nancy Neudauer	Pacific University Oregon
26 February 2024	Networks	Phil Knight	Strathclyde University
26 February 2024	Analytical Techniques in Mathematical Biology	Lyndsay Kerr	Strathclyde University
26 February 2024	Information Theory	Stéphane Ouvry	Université Paris Saclay
25 March 2024	Experimental Mathematics with Sage	Eric Andriatiana	Rhodes University
25 March 2024	Model Theory	Dugald MacPherson	University of Leeds
25 March 2024	Geophysics	Grae Worster and Jerome Neufeld	Cambridge University
25 March 2024	Solitons	Patrick Dorey	Durham University

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22 April 2024	Algebraic Biology	Matthew Macauley	Clemson University
22 April 2024	Statistics Skills	Jane Hutton	University of Warwick
13 May 2024	Quantum Information	Joan Simon Soler	University of Edinburgh
13 May 2024	Problem-solving in Physics	Aniekan Ukpong	University of KwaZulu-Natal
29 May 2024	Statistical Inference	Julia Mortera	Warwick Universita Roma Tre.
1 July 2024	Introduction to the Representation of Finite Groups	Nezhla Aghaei	University of Southern Denmark
1 July 2024	Reinforcement Learning Fundamentals	Mehrdad Ghaziasgar	University of the Western Cape
22 July 2024	Ordinary Differential Equations	Fernando de Pestana	Aberta University

Course abstracts

Computational Algebra
Wolfram Decker and Gerhard Pfister (TU Kaiserslautern)
Groebner bases and Buchberger's algorithm for ideals and modules will be studied. Applications to commutative algebra, selected problems in singularity theory and algebraic geometry will be given as well as applications to electronics and engineering. The course includes an introduction to the computer algebra system SINGULAR and its programming language.

Mathematics and Art in Africa
Steven Bradlow (University of Illinois at Urbana-Champaign)
Many motifs in African decorative arts display beautiful distinctive order. Mathematics provides an effective language for describing this order; conversely, the arts and crafts exploit a wide range of mathematical principles. We will explore this two-way relationship in several examples, including Islamic tiling patterns, Ndebele house decorations, basketry from southern Mozambique, and Chokwe sona drawings. The mathematical ideas will include some or all of the following:

1. symmetry and the classification of the 17 wallpaper groups and seven frieze patterns,

2. graph theory and Euler cycles

3. number theory and the Euclidean algorithm

4. knot theory.

Fluid Dynamics
Richard Katz (University of Oxford)
Fluids are all around us, from the air we breathe to the oceans that determine our climate and from oil that powers our industries to metals that are cast into machinery. The study of fluid dynamics requires sophisticated applications of mathematics and the ability to translate physical problems into mathematical language and back again. The course begins by building a fundamental understanding of viscous fluid flows in the context of unidirectional flows. In more general, higher dimensional flows, pressure gradients are generated within a fluid to deflect the flow around

obstacles rather than the fluid being compressed in front of them, and an understanding of the coupling between momentum and mass conservation through the pressure field is key to the understanding and analysis of fluid motions. We will use simple experiments to illustrate and motivate our mathematical understanding of fluid flow. Prerequisite for the course is fluency with differential equations and vector calculus. No previous knowledge of fluid dynamics will be assumed.

Python
Mohau Mateyisi (CSIR) and Makhamisa Senekane (University of Johannesburg)
The overarching goal of the course is to teach students how to program for mathematical problem-solving using the programming language Python. The course will lay the students' foundation for the application of programming for problem-solving by allowing them to spend time developing ideas in design space, starting with abstract designs, and then moving to more mathematical specifications ahead of solution implementations in Python. The anticipated outcome of the course is that students will be able to make use of discrete mathematics in formulating algorithms, analyse their efficiency, and before achieving their own code implementation in Python. The scope of Python programming will cover:

- programming fundamentals,
- writing, and running Python code using Jupyter Notebooks,
- variables and data types (including Pandas and arrays),
- operators, expressions,
- control flow statements,
- functional and object-orientated programming basics and
- Introduction to version control with git/gitlab**.**

Applying logic
Jeff Sanders (AIMS South Africa)
Much of Mathematics supports numerical reasoning. Typically we observe the rate at which observables change in some system of interest and model them by differential equation, use calculus to find a solution, optimisation theory to constrain it in some way, numerical analysis to approximate it, computing to simulate observable values and finally use statistics to calibrate the results or predictions of our model. The AIMS courses have been chosen to support that methodology. Many of you will choose such projects and will go on to related mathematical careers.

There is another avenue of Mathematics, more recent though no less important, to which those traditional numerical techniques do not contribute. Computing and information systems, on which we now rely for communication, work and commerce, are discrete. For them, anything other than exact correct behaviour is wrong, so the methods of approximation used for continuous systems are irrelevant. Such systems need to be specified, constrained to meet acceptable behaviour (like efficiency, security and privacy), designed and validated. Appropriate techniques are provided by Mathematical Logic.

This course contains as much of the theory of logic as is needed to model representative, contemporary case studies. The theory begins with propositions, their syntax and truth-table semantics, their use in formalising properties, reasoning by laws, and finding a proposition to match a given truth table. The fundamental role of implication is emphasised and used in proving completeness: the equivalence of the algebraic and semantic views.

Propositional logic is extended to predicate logic, by the introduction of variables and quantifiers, to facilitate expression of and reasoning about more general systems. Implication is again fundamental but now algebraic reasoning dominates. Use of $\exists$ to abstract or hide variables provides our methodology of ‘exact approximation’ for discrete dynamical systems.

Finally modal logic is studied in the forms of (a) temporal logic and (b) epistemic logic. Those facilitate the specification and reasoning about systems (a) whose evolution with time is important, and (b) in which ‘who knows what’ is important.

As the theory unfolds examples are given to show the power of each technique. The course concludes with two stand-alone case studies: the specification and correctness of (i) Valiant’s algorithm for machine-learning monomials, and (ii) security in the WhatsApp messaging system.

Outcome. Students will pass by acquiring the skill of specifying the behaviour of discrete systems and of applying the laws of logic to reason about the result. Such quantitative reasoning is a mathematician’s primary asset.

Transformation Groups
Claudiu Remsing (Rhodes University)
Transformation groups arguably form the oldest part of group theory; they continue to play an important role in modern mathematics, particularly, in group theory, representation theory, mathematical physics, and geometry. Today, (both finite and infinite) transformation groups are lively topics of research.

This course is intended as an introduction to the theory of transformation groups—that is, the study of symmetries of mathematical structures (e.g., sets, groups, rings, vector spaces, topological spaces, or manifolds); it contains a large number of examples and exercises.

We start with a survey on group theory (groups, subgroups, cyclic groups, permutations groups, isomorphisms, conjugation, cosets, homomorphisms, and automorphisms) and a little Lie theory (linear groups, Lie algebras, the exponential map). The main part is devoted to groups acting on sets. Finally, the formidable relationship between (transformation) groups and geometry is explored in some detail.

Machine Learning
Ulrich Paquet (AIMS South Africa)
ML and AI drive the back-ends and front-ends of many large online companies, and are set to play a transformative role in the “internet of things”. This is a practical module that looks at how ML is applied to internet-scale systems. In this module, students will build their own recommender systems from scratch. Topics covered will include A/B testing, ranking, recommender systems, and the modelling of users and entities that they engage with online (like news stories).

Mathematical Problem Solving
Nancy Neudauer (Pacific University Oregon)
In this course we shall consider a variety of elementary, but challenging, problems in different branches of pure mathematics. Investigations, comparisons of different methods of attack, literature searches, solutions and generalizations of the problems will arise in discussions in class. The objective is for students to learn, by example, different approaches to problem solving and research.

Networks
Phil Knight (Strathclyde University)
One cannot ignore the networks we are part of, that surround us in everyday life. There's our network of family and friends; the transport network; the banking network- it doesn't take much effort to come up with dozens of examples. Network theory aims to provide a mathematical framework for analysis of the huge networks that drive the global economy (directly or indirectly) and this course provides an introduction to the key tools and an opportunity to employ them to gain new insight into complex behaviours and structures in real-world data.

The intimate connection between matrix algebra and graph theory will be highlighted and students will use this connection to develop a practical approach to analysing networks. Python provides an ideal computational environment for large-scale simulation and analysis, in particular for identifying the key members of a network and for uncovering local and global structure that can be hidden by the scale of the data.

Analytical Techniques in Mathematical Biology
Lyndsay Kerr (University of Strathclyde)
In this class we will explore how analytical techniques and traditional “pure" mathematics can be applied to real biological problems. Mathematical models are mathematical descriptions of real-life systems. In this class we will examine mathematical models that describe important biological systems. Posing such systems in terms of mathematics often proves to be very useful in allowing us to answer questions of interest, such as: What is the long-term outcome of the system? Are there any factors (known as the model parameters) that could cause the long-term behaviour of the system to change? For example, we will examine models that describe the interaction between two competing species. Here, we will be interested in whether both species can coexist in the long term or whether one of the species will die out. Models describing tumour cell growth will also be analysed, where we are interested in whether or not the tumour cells will die out. In addition, we will study models that describe the spread of an infectious disease through a population. Here we will determine what factors contribute to the outbreak or termination of a disease, we will examine the long-term behaviour of the number of infected individuals and we will investigate how immunisation can prevent or halt a pandemic. We will also learn about how real-to-life data can be used in combination with infection models to estimate important factors such as the infection rate of a disease. The final topic that we will consider relates to the patterning that is found on the coats of animals. We shall examine models describing a potential mechanism that could cause such patterns to arise and we will learn how these models can be used to produce patterns akin to those found on the coats of animals.

Information Theory
Stéphane Ouvry (Université Paris Saclay)
This course is an introduction to various random systems, probability theory, Shannon information theory and some related topics, with a special emphasis on their mathematical aspects. In particular I will present selected lectures on

- Probability calculus and the central limit theorem
- Application to random walks on a line and Brownian curves
- Notions of random numbers and pseudo random numbers
- Application to Monte Carlo sampling
- Shannon statistical entropy and information theory
- LZW compression algorithm
- Diaconis riffle shuffle: how to “randomize” a deck of cards?
- Random permutations and application to the statistical “curse” problem in sailing boat regattas

Experimental Mathematics with Sage
Eric Andriatiana
This course introduces an approach to doing mathematics that is founded on experiment and inquiry. It does so in the medium of Sage, a Python-based tool for computation and experiment. Some of the problems studied come from (but not limited to) Geomery, Probability Theory, Number Theory and Graph Theory. Skill with Sage will be important for many subsequent courses.

Model Theory and Homogeneous Structures
Dugald MacPherson (University of Leeds)
Mathematical Logic is a discipline overlapping with mathematics, philosophy, and computer

science, and tackling questions such as: can we build a rigorous foundation for all of mathematics based on self-evident principles? Can we deal rigorously with infinite objects, possibly of different infinite sizes? What does it mean for something to be (algorithmically) computable? How do we model and justify rigorous reasoning?

Model Theory is the branch of (mathematical) logic which relates most closely to the rest of mathematics. Its objects of study are mathematical objects such as graphs, groups, fields, metric spaces, viewed as structures in formal logical languages. Surprisingly, such abstract frameworks can have important applications in many parts of mathematics.

This course gives an introduction to model theory. It begins with a review of the notions of set (the mathematical idea of a collection of objects), functions (injections, surjections, bijections), and sets of different infinite sizes (countable and uncountable). Equivalence relations, partial and total orders, and graphs are introduced and discussed, partly as a source of examples of 'models'.

The course then introduces the notion of a first order language, first order structure, interpretations, and formulas and sentences. I will discuss definable sets, the back-and-forth method, and ways of proving certain sets are definable or are not definable. The compactness theorem, the result which makes first order logic so powerful, will be stated and not proved, and some applications given. The course will conclude with a brief discussion of the notion of homogeneous structure (infinite structures such as graphs which have a very high level of symmetry), and I will finish by introducing a beautiful example, `the random graph'.

Solitons
Patrick Dorey (Durham University)
Solitons were first identified in 1834 - they are exceptionally long-lasting solutions to certain non-linear partial differential equations, with many intriguing properties and widespread applications in mathematics and physics. This course will explore some aspects of this still-developing story.
Topics covered will include:

- Illustrations of solitons via computer animations and practical demonstrations (this will introduce the KdV and sine-Gordon equations, two key examples for soliton theory)
- Waves, dispersion and dissipation
- Travelling waves
- Numerical simulation of soliton equations
- A quick sketch of Lagrangian mechanics (first for particles and then for field theory)
- Topological lumps and the Bogomolnyi argument; sine-Gordon and phi4 examples (phi4 is a further equation with soliton-like properties)
- Conservation laws; extra conservation laws for the KdV equation
- Backlund transformations as a tool to generate exact solutions of soliton equations

Algebraic Biology
Matthew Macauley (Clemson University)
Mathematical biology has been transformed over the past few decades by researchers using novel tools from discrete math and computational algebra to tackle old and new problems. For example, many systems such as gene regulatory networks have been traditionally modelled using differential equations. However, a new popular trend is to use discrete or Boolean models. In this setting, the local functions and the resulting finite dynamical system map can be expressed as multivariable polynomials. This opens the door to using the powerful toolbox of computational algebra to attack classic problems in systems biology. In this class, students will be introduced to this new and exciting field known as "algebraic systems biology". It will be broken up into three parts of about equal length: (1) biochemical reaction networks and ODE models, (2) Boolean models of molecular networks, (3) algebraic models and methods.

Geophysics
Grae Worster and Jerome Neufeld (Cambridge University)
Fluids are all around us in the natural world, from the air we breath and that determines our weather to the oceans that affect our climate and the 'solid' earth beneath our feet, which is in constant motion, giving rise to continental drift and volcanism. This course will give an introduction to the sorts of mathematical fluid dynamics that allows us to make predictions of such geophysical flows. As time allows: we will explore the role of buoyancy-driven convection and the effect of Earth's rotation in determining weather patterns and extreme events such as hurricanes and tornadoes; we will study flows through porous media relating to ground-water aquifers and storage of CO2 to mitigate climate change; and we will model the change of phase from liquid to solid that determines the freezing of the polar oceans, the formation of solid rocks from molten lava and the growth of the Earth's inner core, which drives our magnetic field.
It is strongly advised that students taking this course have first taken the course in Fluid Dynamics earlier in the term.

Statistical Inference
Jane Hutton (Warwick University)
This course address the process of statistical problem solving, including formulating questions, obtaining data, investigating, summarising and reporting on analyses. Statistical thinking includes mathematical modelling, but also requires an appreciation of context.
We start with real datasets from health care, agriculture and legal systems. Initial data analysis and visualisation are essential to ensuring reliable conclusions, and the skills involved require practice. Common descriptive statistics are defined and their use for different data and aims discussed. A clear description of a dataset is used to decide whether it is adequate. The criteria for good design of data collection for administrative, survey and experimental datasets are considered.
After an introduction to probability as a foundation for statistics, we explain the main concepts of statistics with regard to data collection, estimation of parameters and statistical modelling. Random variables and models for data are informed by summary statistics. The likelihood principle for estimation of the parameters of the models will be introduced, focusing on statistical inference relevant to the initial questions. The R software will be used to apply ideas to the core data sets for e.g. Binomial, Poisson, Normal, and standard linear regression.

Quantum Information
Joan Simon Soler (University of Edinburgh)
This is an introductory course on information and quantum theory emphasising the axioms and mathematical structures behind quantum mechanics. It plans to cover an introduction to classical information theory (Shannon entropy), followed by an axiomatic presentation of quantum mechanics, including the properties of density matrices and the meaning of entanglement, and it will end by discussing von Neumann entropy and some of its applications. We shall work in finite number of dimensions and students are not required to have any prior experience with quantum mechanics.

Problem-Solving in Physics
Aniekan Ukpong (University of KwaZulu-Natal)
In this course, we will discuss fundamental concepts in physics. Key principles that are essential for effective problem-solving will be presented. Hands-on problem-solving will be demonstrated through in-class discussions, which will be integrated with laboratory-based computational investigations. These learning activities will showcase the essential skills that are needed to address problems in classical and quantum physics. The aim is for students to learn, through concrete examples, the various approaches to problem-solving in physics.

Statistical Inference
Julia Mortera (Universita Roma Tre.)
A first part of the course will include a thorough grounding in classical and Bayesian methods of statistical inference. It will also focus in particular on exploratory data analysis, linear models and generalised linear models (GLMs). It will also show how Bayesian networks are used to evaluate forensic evidence, and the use of GLMs in the case of a nurse accused of murdering several patients. Making extensive use of real data examples from health and law and their analysis with R through RStudio and with Hugin software. The student will learn to distinguish between parameter estimation problems, hypothesis testing and prediction. Therefore, the student will be taught not only to apply statistical techniques but also to choose the most appropriate technique and to comment on the output for decision-making purposes.

Introduction to the Representation of Finite Groups
Nezhla Aghaei -
In this course, we review the definition of groups and several properties of groups, such as abelian groups, Coset, Normal subgroups, etc. We consider several examples like, Symmetric groups, Permutations groups, Cyclic groups, Alternating groups, etc, and relations between them. We introduce the definition of representation theory in abstract language and use it for the given examples and explore some properties like irreducibility and adjoint representations. In the second part, we review the basic definitions of topology, dimension, 2d surfaces and their properties, and Knot theory and we focus on the classification of Knots and groups Knots. We end the lecture by explaining how the matrix representation of the group knots can be a nice application of representation theory in another topic of mathematics.

Reinforcement Learning Fundamentals
Mehrdad Ghaziasgar (University of the Western Cape)
This course provides a comprehensive introduction to reinforcement learning (RL). The course starts out with the foundational concepts of Markov Decision Processes (MDPs). Students explore the theoretical underpinnings of MDPs and learn to solve them using key algorithms such as value iteration, policy iteration, policy evaluation, and policy extraction. The course then transitions to reinforcement learning techniques, initially focusing on model-based RL methods. This includes studying how transition and reward models of the environment can be constructed using experiences, and utilized to make informed decisions. Finally, the course delves into model-free RL approaches, emphasizing their practical applications and concluding with an in-depth examination of epsilon-greedy Q-learning. By the end of the course, students will have a solid understanding of the core principles and methodologies of reinforcement learning, equipping them to apply these techniques to real-world problems.

Differential Equations
Fernando Pestana da Costa (Aberta University)
The goal of this course is to be an introduction to the theory and applications of Ordinary Differential Equations, with emphasis in methods of qualitative theory. It starts by revisiting basic techniques to solve ODEs: separable equations, integrating factors, and changes of variables. After these preliminaries the general theorem of Picard-Lindelof for existence and uniqueness of solutions to initial value problems, as well as results on dependence of solutions on the initial data and parameters are studied. These preliminary general results are followed by the main part of the course, viz. a study of important tools from qualitative theory. These consist in the introduction of the main concepts (phase space, flows, critical points, orbits, conservative systems, first integrals, phase portraits, etc.), notions and results about stability of solutions, Lyapunov functions, limit sets, and a study of planar autonomous systems. The final part of the course deals with linear systems (including the computation of the matrix exponential) and the study of the linearization method for nonlinear systems (including the notion of conjugation, and the statement ---without proofs--- and use of the Hartman-Grobman and Hadamard-Perron theorems.) All topics are motivated and

exemplified by adequate examples. The discussion and resolution of exercises and problems illustrating the notions, results, and methods studied is an integral part of the course.