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B.Sc. (Hons. Math) (Semester – 3rd)
DIFFERENTIAL EQUATIONS - I
Subject Code: BMAT1-307
Paper ID: [18131211]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a. Define Clairaut's differential equation with the help of an example.

b. Solve the differential equation: $xyy' = 1 + x + y + xy$, where $y' = \frac{dy}{dx}$.

c. Form the differential equation of $y = ae^{x} + be^{2x} + ce^{-3x}$, where a , b, c are constants.

d. Find the integrating factor of

$$x^2 y\,dx - (x^3 + y^3)\,dy = 0.$$

e. Solve the differential equation: $\frac{d^3y}{dx^3} - 5\frac{d^2y}{dx^2} + 7\frac{dy}{dx} - 3y = 0$.

f. Find the Particular integral of $4\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = e^{\frac{x}{2}}$.

g. Define Euler's differential equation. Give an example.

h. Prove that: $J_{-n}(x) = (-1)^n J_n(x)$.

i. Prove that: $P_n'(1) = \frac{n(n+1)}{2}$.

j. State Legendre's differential equation.

Section – B **(5 marks each)**

Q2. Solve the differential equation: $y' + 4xy + yx^3 = 0$, where $y' = \frac{dy}{dx}$.

Q3. For what value of k, the differential equation: $\left(1 + e^{\frac{kx}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$ is exact.

Q4. Solve $\tan y \frac{dy}{dx} + \tan x = \cos y \cos^2 x$.

Q5. Solve the differential equation: $\frac{d^2 y}{dx^2} + 7\frac{dy}{dx} + 12y = x\sin x$.

Q6. State and Prove Rodrigue's formula

Section – C **(10 marks each)**

Q7. Solve:

$$(x\sin x + \cos x)\frac{d^2 y}{dx^2} - x\cos x\frac{dy}{dx} + y\cos x = \sin x(x\sin x + \cos x)^2.$$

Q8. Apply the method of variation of parameter to solve:

$$x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} - y = x^2 e^x.$$

Q9. Obtain the series solution of Bessel's differential equation of order two:

$$x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} + (x^2 - 4)y = 0 \text{ near } x = 0.$$