ELEC3004 – Systems: Signals & Controls

Study Material: Linear Systems & Signals Chapter Summary

Chapter 1: Signals and Systems

A system receives a set of data (signals as inputs) and turned them into another set of data (signals as output). It's Important to understand how signals and systems interact with each other and how a goal can be achieved by manipulating the input and controlling the output.

WHAT IS THE BIG PICTURE?
To answer this, it is mandatory to identify each component

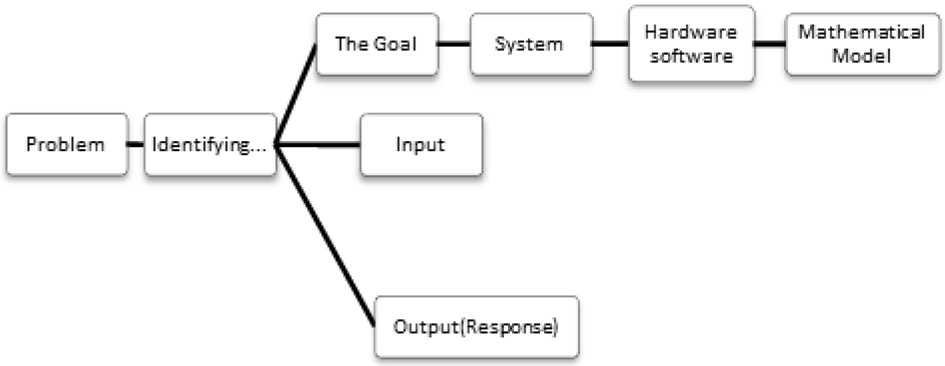


Fig.1szz,mx,r

- **Get to know the System**

Depending on the nature of a goal and the nature of the information being used as input the system may be formed by physical components (Electrical, Mechanical…) or it may be a computing algo euszsrithm.

,iie,f,

Characteristic of input and Ol,f. D (Signal)

In order to be able to process a signal it needs to be measurable. Considering the signal has amplitude and duration, its size can be represented by A_x (Area) under the signal:

$A_{x=} \int_{-\infty}^{\infty} x(t)$

The limitation of this description is that the area above and below the time vector t can cancel each other.,,

To overcome this, we describe another definition E_x which will turn the negative part into a positive value:

Signal Energy: $E_x = \int_{-\infty}^{\infty} |x(t)|^2$

Since these values could be infinite for an infinite signal we define Power as:

Signal Power: $Px = lim_{T\to\infty} 1/T \int_{-T/2}^{T/2} |x(t)|^2$

Please note the the definition of Energy that we used here is not the actual energy because it depends on the load For example if i(t) is the current through a resistor, then the true energy dissipated in the resistor is:

E= $\int_{-\infty}^{\infty} Ri^2(t)dt = lim_{T\to\infty} \int_{-T}^{T} Ri^2(t)dt$

For more information please refer to LATHI. P.70.

Operations on signals

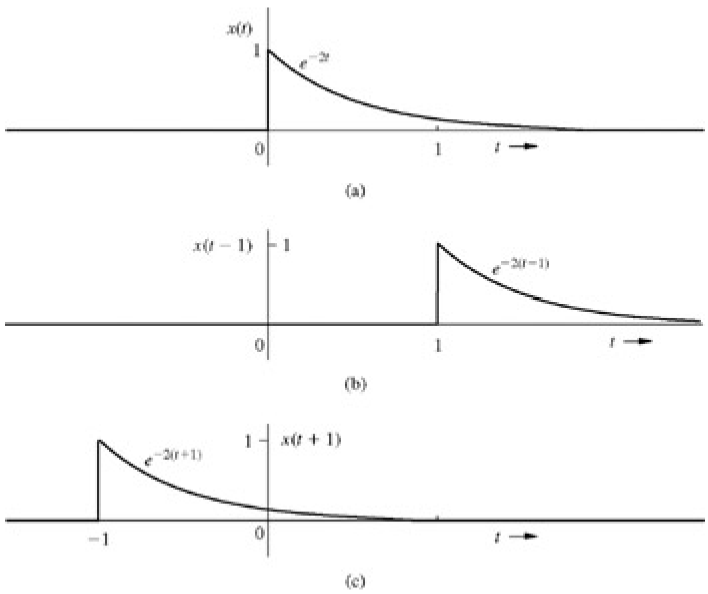
Time Shifting:

For a continuous signal x(t) and a delay/advancement of $\tau > 0$:

Y (t) =x (t+ τ), Advancement

Y (t) =x (t− τ), Delay

Y (t) =x (t), Original (Fig.2)



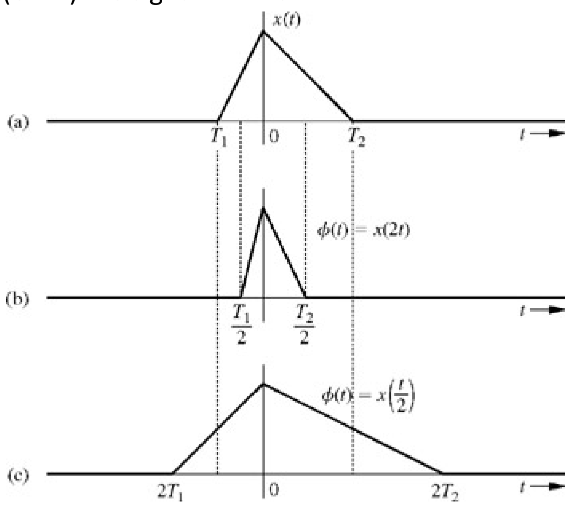
x(t) = e^{-2t} is delayed by 1 second

x(t) = e^{-2t} is advanced by 1 second

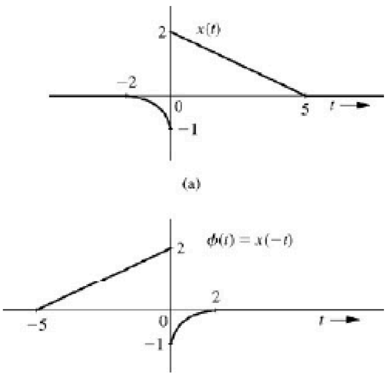
Fig.2

Time scaling:

A signal x (t) is scaled in time by multiplying the time variable by a positive constant b, To produce x (bt). A positive factor of b either expands (0 < b < 1) or compresses (b > 1) the signal in time.



Note: Time reversal is when b=-1.

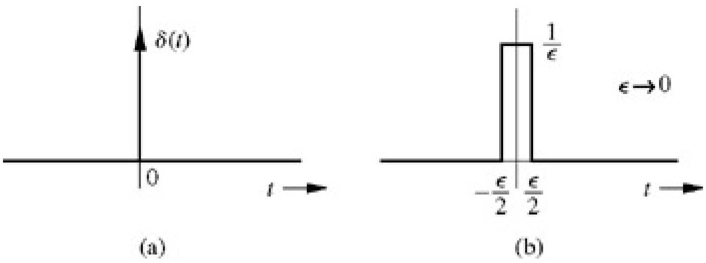


Periodic functions
• A continuous time signal is periodic if there exists a T > 0 such that
x(t + T) = x(t) for all t. T is a period of x.
• The smallest positive T is the fundamental period T0 of the periodic signal, and f0 = 1/T0 is the fundamental frequency of the continuous time periodic signal.
Note that the period of the sum of periodic functions is the least common multiple of the periods of the individual functions summed. If the least common multiple is infinite, the sum function is non-periodic.

Fundamental signals

- *Unit impulse*

AKA Delta Dirac is a tall narrow rectangular pulse with an area of 1.

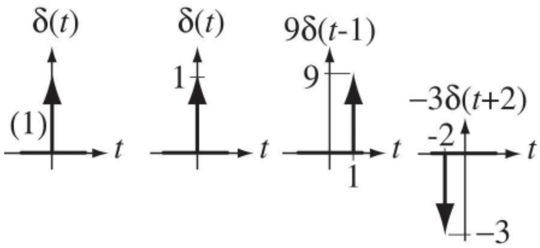


a. Ideal impulse b.the real impulse

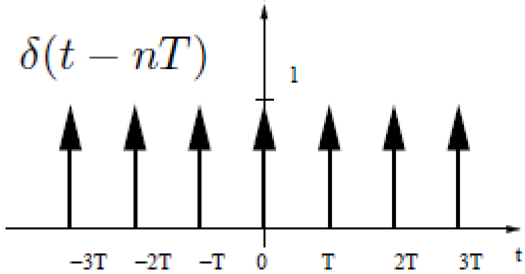
Area of a rectangular is width times by its length so when epsilon goes to infinity the graph(b) will become more closer to what we call an ideal impulse with

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

Multiplication of a signal by Impulse
an impulse function can be used to pick a value of the of signal at a preferred time interval. Based on time shifting :



So as we can see an impulse signal is a good tool to pick the value of other signals at a certain time interval just by a simple multiplication. Translation: the given signal amplitude will be set to zero along its time vector except the specified time by impulse signal.

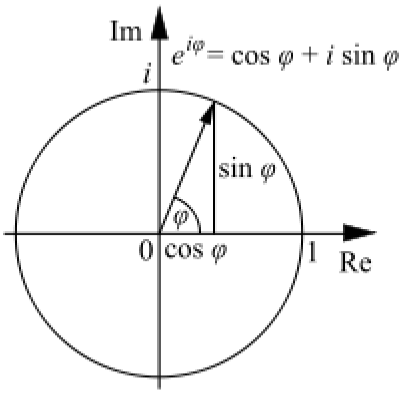


- Exponential Function e^{st}

Reminder for complex numbers:

y=a+ib
Φ=arcTan(b/a)
y=r.(cos(Φ)+isin(Φ))=r.$e^{i\Phi}$
a=rcos(Φ)
b=rsin(Φ)

the same principals apply to complex frequency



$$e^{st} = e^{(\sigma + j\omega)t} = e^{\sigma t} e^{j\omega t} = e^{\sigma t}(\cos\omega t + j\sin\omega t)$$

$e^{st}=e^{\sigma t}\cos\omega t + je^{\sigma t}\sin\omega t$

$e^{\sigma t}\cos\omega t$ is a simple decaying/growing sinusoid signal (c,d) (on real axis)

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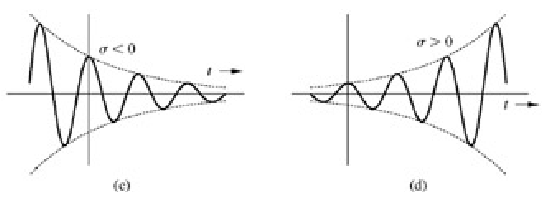
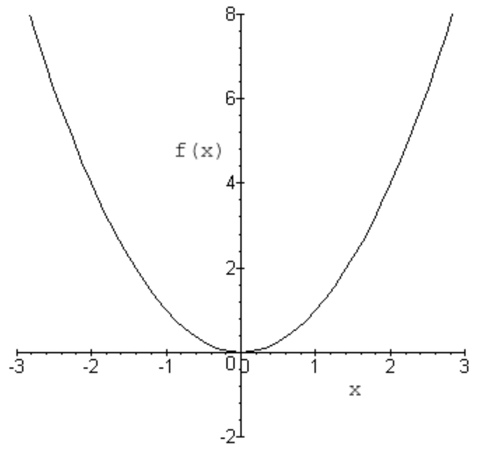
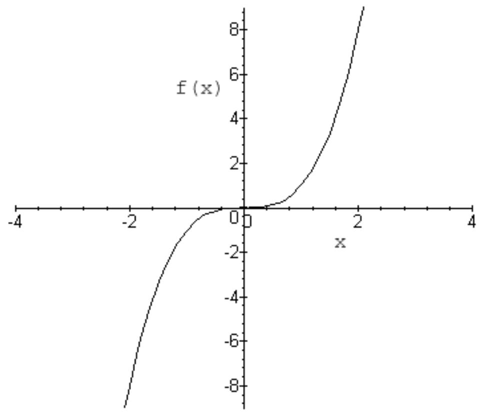


the same applies to $je^{\sigma t}$ sin ωt hence the plot would be the same but the vertical axis would be imaginary.

Even and Odd Function:





Even Odd

(under construction...)