

1) Given the vectors $\mathbf{M} = -10\mathbf{a}_x + 4\mathbf{a}_y - 8\mathbf{a}_z$ and $\mathbf{N} = 8\mathbf{a}_x + 7\mathbf{a}_y - 2\mathbf{a}_z$, find:

a) a unit vector in the direction of $-\mathbf{M} + 2\mathbf{N}$.

$$-\mathbf{M} + 2\mathbf{N} = 10\mathbf{a}_x - 4\mathbf{a}_y + 8\mathbf{a}_z + 16\mathbf{a}_x + 14\mathbf{a}_y - 4\mathbf{a}_z = (26, 10, 4)$$

$$\text{Thus } \mathbf{a} = (26, 10, 4) / |(26, 10, 4)| = (0.92, 0.36, 0.14)$$

b) the magnitude of $5\mathbf{a}_x + \mathbf{N} - 3\mathbf{M}$:

$$(5, 0, 0) + (8, 7, -2) - (-30, 12, -24) = (43, -5, 22), \text{ and } |(43, -5, 22)| = 48.6.$$

c) $|\mathbf{M}||2\mathbf{N}|(\mathbf{M} + \mathbf{N})$:

$$|(-10, 4, -8)||16, 14, -4||(-2, 11, -10)| = (13.4)(21.6)(-2, 11, -10)$$

$$= (-580.5, 3193, -2902)$$

2) The vector from the origin to the point A is given as $(6, -2, -4)$, and the unit vector directed from the origin toward point B is $(2, -2, 1)/3$. If points A and B are ten units apart, find the coordinates of point B .

With $\mathbf{A} = (6, -2, -4)$ and $\mathbf{B} = 0.33 B(2, -2, 1)$, we use the fact that $|\mathbf{B} - \mathbf{A}| = 10$, or $|(6 - 0.666 B)\mathbf{a}_x - (2 - 0.666 B)\mathbf{a}_y - (4 + 0.333 B)\mathbf{a}_z| = 10$

Expanding, obtain

$$36 - 8B + \frac{4}{9}B^2 + 4 - \frac{8}{3}B + \frac{4}{9}B^2 + 16 + \frac{8}{3}B + \frac{1}{9}B^2 = 100$$

$$\text{or } B^2 - 8B - 44 = 0. \text{ Thus } B = \frac{8 \pm \sqrt{64 - 176}}{2} = 11.75 \text{ (taking positive option) and so}$$

$$\mathbf{B} = \frac{2}{3}(11.75)\mathbf{a}_x - \frac{2}{3}(11.75)\mathbf{a}_y + \frac{1}{3}(11.75)\mathbf{a}_z = \underline{7.83\mathbf{a}_x - 7.83\mathbf{a}_y + 3.92\mathbf{a}_z}$$