Differential Geometry
MAT432 and MAT733
Professor Sormani
Final Projects

Googledocs: All work will be submitted by sharing your googledoc for this week with the professor using the correct title on that doc stating the course number, the week number and your name:

MAT432F23-Final-YourNameHere

MAT733F22-Final-YourNameHere

Final Projects require a selfie with your work or your id placed next to your work.

Start your final project by creating the above document and then emailing it to me with the following information cut and pasted into the doc with boxes checked.

Name:

Course: MAT733 or MAT432

Career Goals:

Major, Minor, and/or Graduate Program:

Have you taken Analysis?

Favorite Topics from this course: (check all that apply)

- ☐ **Metric Spaces, Open Sets, Continuity**
- ☐ **Curves including Frenet-Serrat**
- ☐ **Diffeomorphisms and Coordinate Charts**
- ☐ **Surfaces and Submanifolds**

Topics you didn't like or skipped: (to avoid)

- ☐ **Week 4**
- ☐ **Week 7 Part II**
- ☐ **Week 8 Part II**
- ☐ **Anything else?**

Geometric shapes that you find interesting:

- ☐ **Conic Sections: hyperbolas, parabolas, ellipses**

- ☐ **Minimal Surfaces: Catenoids and Helicoids**
- ☐ **Black holes, General Relativity**
- ☐ **Add a photo or mention a shape from a course you took if you wish**

Applications you might be interested in:

- ☐ **Physics**
- ☐ **Chemistry**
- ☐ **Biology**
- ☐ **Data Science**
- ☐ **Business**
- ☐ **Advanced Mathematics**
- ☐ **Teaching High School**
- ☐ **Other**

I will use this information to design a final project for you.

One last question:

What deadline would you personally prefer for your final project (depending on your personal holidays):

- ☐ **due Dec 17 fix by Dec 23**
- ☐ **due Dec 22 fix by Dec 26**
- ☐ **I might need an incomplete and finish in January because I am behind schedule**

Each student has been assigned an implicitly defined surface, S, and been given a point, p, on that surface. You will find this in your doc.

$$S = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mid F(x,y,z) = 0 \right\}$$

where $F(x,y,z) =$

$p = \left(\quad \right)$ ← Each student has a different F and p.

All students will complete the following questions for their surface and add photos of their work to their final googledoc.

(1) Verify that your point, p, is in your surface and use MATLAB implicitplot to plot your surface and the point.
(2) Find the matrix DF and use the implicit function theorem to explain why your function is a smooth surface as in HW3 of Week 10.
(3) Check that DF has rank 1 at p and choose a leading variable. Then find a chart around p describing the domain and formula for the chart. Call this chart Y as in HW2 of Week 11. Use MATLAB to plot this chart and check that it lands on your surface.
(4) Find DY and define the vector fields: V is the first column and W is the second column as in HW6 of Week 11.
(5) Find V and W at p and write the tangent plane at p as the span of these two vectors as in HW4 of Week 11. Check that V at p is perpendicular to the gradient of F at p and check that W at p is perpendicular to the gradient of F at p using dot products. So their span can also be written as the collection of vectors perpendicular to this gradient.
(6) Find the Projection map at p as in HW6' of Week 11 Part II.
(7) Use the vector fields V and W to find the first fundamental form by taking the dot products as in HW9 of Week 11 Part II.
(8) Find the covariant derivatives at p of V with respect to W, W with respect to V, and V with respect to V, and W with respect to W as in HW7 of Week 12 Part II.
(9) Find the area of your patch as taught in Week 13 Part I.
(10) Find the Second Fundamental Form at p as taught in Week 13 Part II and find the Mean Curvature at p as taught in Week 14 Part II. Extra Credit: find the Gauss curvature at p.