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M.Sc. (Mathematics) (Sem.: 3rd)

FOURIER ANALYSIS & APPLICATIONS

SUBJECT CODE: MMAT1356

Paper ID: [19220515]

Time Allowed: 3 hrs. **Maximum Marks: 60**

Instruction to Candidates:

1. **Section-A** is Compulsory.
2. Attempt any **three** questions from **Section-B.**
3. Attempt any **two** questions from **Section-C.**

Section –A **(4 marks each)**

Q1. Obtain half range Fourier Cosine series for $f(x)=2x-1$ in the interval (0, 1).

Q2. Write a short note on: (i) Convergence and Uniform convergence of Fourier series, (ii) Dirichlet Kernal and Fejer kernel.

Q3. Find the Fourier cosine transform of $f(x)=\frac{1}{1+x^2}$.

Q4. Explain Discrete Fourier Transform and Find the DFT of a sequence x(n)= {1,1,0,0}.

Section –B **(8 marks each)**

Q5. Express $f(x)=x(\pi-x),\ 0<x<\pi$ as a Fourier series of periodicity 2π containing (i) sine terms only & (ii) cosine terms only. Hence deduce, $1-\frac{1}{3^3}-\frac{1}{5^3}+\frac{1}{7^3}+\ldots=\frac{\pi^3}{32}$ and $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots=\frac{\pi^2}{12}$.

Q6. Suppose that ***f*** is periodic and integrable then nth partial sum of Fourier series expansion of ***f*** is given by $S_N(f)=\frac{1}{2\pi}\int_{-\pi}^{\pi}D_N(x-y)f(y)\,dy=\frac{1}{2\pi}D_N(y)\ f(x-y)\,dy.$

Q7. The temperature u(x,y) at any point of an infinite bar satisfies the equation $u_t=u_{xx}, -\infty<x<\infty,\ t>0$ and the initial temperature along the length of the bar is given by $u(x,0)=\begin{cases}1 & for\ \ |x|<1\\ 0 & for\ \ |x|>1.\end{cases}$ Determine the expression for u(x, t).

Q8. (i) Compute the DFT of the following sequences

a) x = [1, 0, −1, 0] b) x = [1, 1, 1, 1, 1, 1, 1, 1]

c) x[n] = cos (0.25 π n), n = 0, ..., 7 d) x[n] = 0.9^n, n = 0, ..., 7

(ii) Explain Periodic and Linear properties of Discrete Fourier Transform.

Section –C **(10 marks each)**

Q9. (i) Find the Fourier series to represent x^2 in the interval $(-l, l)$.

(ii) The fourier series of an integrable function on the circle is Abel summable to **f** at every point of continuity. Moreover, if **f** is continuous on the circle, then the Fourier series of **f** is uniformly Abel summable to **f**.

Q10. (i) Obtain half range Fourier Cosine series $f(x) = x\sin x$ for in the interval $(0, \pi)$ and show that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \ldots = \frac{\pi - 2}{4}$.

(ii) Find the Fourier cosine and sine transform of $[6e^{-4x} + 8e^{-2x}]$

Q11. (i) (a) State and prove Convolution theorem of Fourier transforms,

(b) State and prove Parseval's identity of Fourier transforms.

(ii) Find the Fourier transform of $f(x) = \begin{cases} 1 & for \; |x| < 1 \\ 0 & for \; |x| > 1. \end{cases}$ Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx.$