LESSON 1.2 | RATES OF CHANGE

Name: _____

Date: _____ **Class:** _____

TODAY I AM learning to calculate and interpret rates of change.

SO I CAN explain how quickly one quantity changes compared with another.

I'LL KNOW I HAVE IT WHEN I can calculate a rate from an equation, table, or graph, estimate a rate at one point, and explain its sign and units.

ESSENTIAL QUESTION | How can a graph or a formula tell us how fast something is changing?

Warm-up: What do you already know?

1. The slope between (1, 2) and (4, 8) is _____.
2. A rising line has a _____ slope.
3. Change 30 minutes into hours: _____.

1 | Average Rate of Change (AROC)

The average rate of change compares how much the output changes with how much the input changes. On a graph, it is the slope of the secant line joining two points.

$$\text{AROC on } [a, b] = \frac{(f(b) - f(a))}{(b - a)} = \frac{\Delta y}{\Delta x}$$

Fill in: output change = _____; input change = _____.

Interpretation: AROC > 0 means the output is _____ overall.

EX #1 | Equation — I DO → WE DO

Find the average rate of change of $f(x) = x^3 - 4x$ on [1, 3].

Step 1. $f(3)$ = _____; $f(1)$ = _____.

Step 2. AROC = $\frac{(_____ - _____)}{(3 - 1)}$ = _____.

WE DO | Let $q(x) = x^2 + 2x$. Find its AROC on [0, 2].

$q(0)$ = _____; $q(2)$ = _____; AROC = _____.

WORKSPACE

2 | Tables and Real-Life Units

An input could be a year, minute, or second. A rate must tell both the output unit and the input unit, such as million tons per year.

EX #2 | U.S. municipal solid waste (MSW)

Year	1960	1970	1980	1985	1990	2000	2005	2010	2015	2018
Million tons	88.1	121.1	151.6	166.3	208.3	243.5	253.7	251.1	262.1	292.4

Source table also reports 1965: 104.4; 1975: 127.8; 1995: 217.3; 2016: 268.1; 2017: 268.7 (million tons).

A. Find the AROC from 1985 to 1990: $\frac{(_____ - _____)}{(_____ - _____)}$ = _____.

B. Find the interval when total trash production went down and its AROC:

_____.

EX #3 | Medicine concentration — WE DO

Let C(t) be a medicine concentration t minutes after treatment. Write (do not evaluate) the average rate of change between 15 and 90 minutes.

AROC = $\frac{(C(____) - C(____))}{(____ - ____)}$ = _____.

CHECK FOR UNDERSTANDING

CFU 1. If AROC = −4 °C/hour, is temperature rising or falling? _____

CFU 2. Compute the rate for (2, 7) and (6, 19). _____

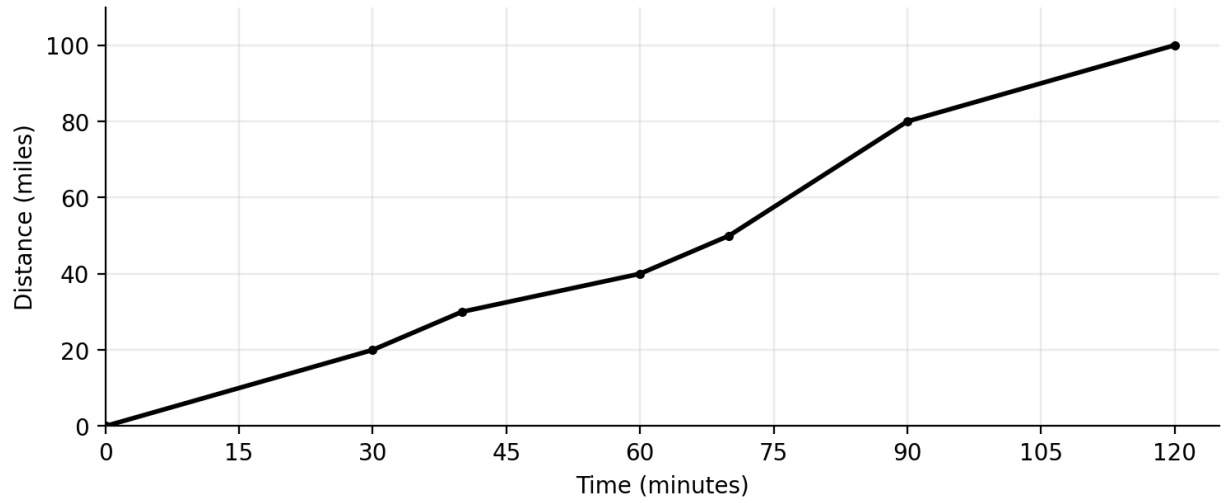
CFU 3. A student finds $\frac{(1990 - 1985)}{(208.3 - 166.3)}$. Explain the mistake.

_____

WORKSPACE

3 | Distance–Time Graphs

Distance–time slope = speed. The steeper the graph, the greater the average speed. A flat section means the distance is not changing.



EX #4 | A car travels for two hours

A. Find average speed over the full 120 minutes. _____

B. Find average speed from 30 to 60 minutes. _____

C. During which interval is the car fastest? How fast? _____

D. Interpret the slope on 90–120 minutes. _____

E. Estimate speed exactly at 45 minutes. _____

WORKSPACE

KEY IDEA + CHECK

A horizontal distance–time segment represents _____.

Why must miles/minute be multiplied by 60 to obtain miles/hour?

_____.

4 | Instantaneous Rate of Change (IROC)

AROC averages change over an interval. IROC describes change at one instant. In precalculus, estimate IROC using AROC on smaller and smaller intervals around the point.

IROC at $t = a \approx \frac{[\,h(a+\varepsilon) - h(a)\,]}{\varepsilon}$, for very small ε

EX #5 | Height of a launched object

A height (in feet) after t seconds is $h(t) = -16t^2 + 48t + 120$. Estimate its rate at $t = 1$ second.

Interval	Your AROC (ft/sec)
A [1, 1.1]	_____
B [1, 1.01]	_____
C [1, 1.001]	_____
D [0.999, 1.001]	_____
WORKSPACE	

E. Estimate IROC at $t = 1$: _____ ft/sec.

I DO / WE DO / YOU DO

I DO: For $y = x^2$ near $x = 2$, the rate on [2, 2.1] is $\frac{(2.1^2 - 2^2)}{0.1} = 4.1$.

WE DO: On [2, 2.01], the rate is _____.

YOU DO: Predict the IROC of $y = x^2$ at $x = 2$: _____.

WORKSPACE

CHECK

CFU 1. AROC uses an interval; IROC uses a _____.

CFU 2. Why try smaller intervals successively? _____.

5 | Positive and Negative Rates

Increasing function → positive AROC | Decreasing function → negative AROC

Look at the function from left to right. If outputs rise, the rate is positive; if they fall, it is negative. Function values themselves can be positive or negative—this is a different question.

EX #6 | Determine only the SIGN of AROC

A. $y = x^2$ on $[2, 3]$ _____.

B. $y = \log x$ on $[0.01, 1]$ _____.

C. $f(x) = (\frac{1}{2})^x$ on $[-1, 0]$ _____.

D. $g(x) = 5 - 2x^2$ on $[1, 4]$ _____.

COMMON ERROR

The sign of a function's output (above/below the x-axis) is not the same as the sign of its rate of change (rising/falling).

WE DO — YOU DO

WE DO: $y = -3x + 5$ on $[1, 4]$ has rate _____ because _____.

YOU DO: $y = 4 - x^2$ on $[-3, -1]$ has rate _____ because _____.

WORKSPACE

CHECK FOR UNDERSTANDING

CFU 1. A function is below the x-axis and rising. Its rate is _____.

CFU 2. How can a decreasing function have positive y-values?

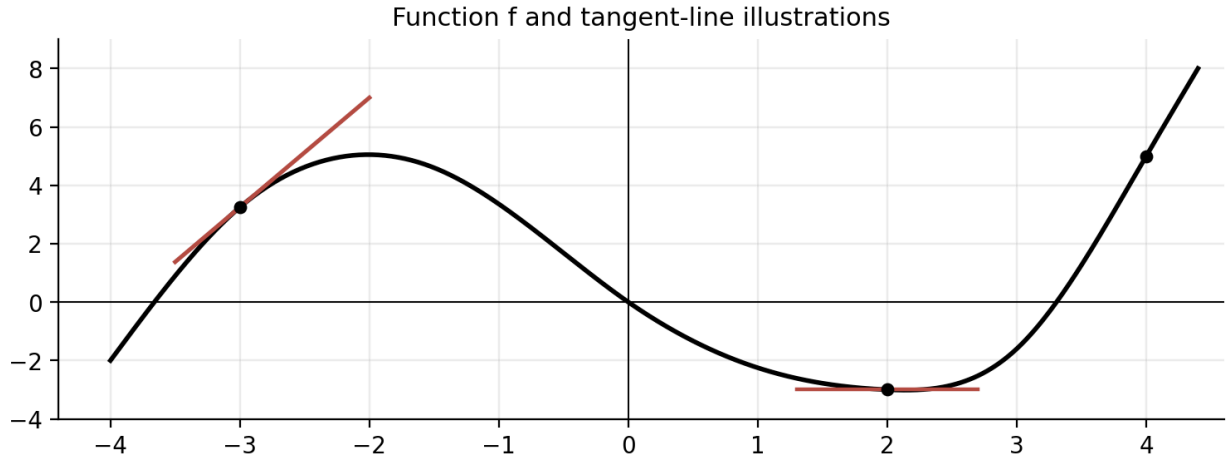
_____.

CFU 3. Sketch any function whose output is positive, but rate is negative. [drawing workspace]

WORKSPACE

6 | Tangent Lines and Rate at a Point

A secant joins two graph points. A tangent line touches the graph locally and gives its rate at one point. A positive tangent slope means increasing; a horizontal tangent has slope 0.



EX #6A–C | Read tangent slopes

6A. The shown tangent near x = −3 passes through (−3, 3.25) and (−2, 7). Estimate its slope: _____.

6B. At the left rising point, x = 2, and right rising point, tangent slopes are respectively _____, _____, _____.

6C. At the low point x = 2, f changes from _____ to _____; IROC = _____.

CHECK FOR UNDERSTANDING

CFU 1. A horizontal tangent means instantaneous rate _____.

CFU 2. Why is a tangent slope more appropriate than a long secant interval for IROC?

_____.

CFU 3. At a smooth local maximum, the rate changes from _____ to _____.

WORKSPACE

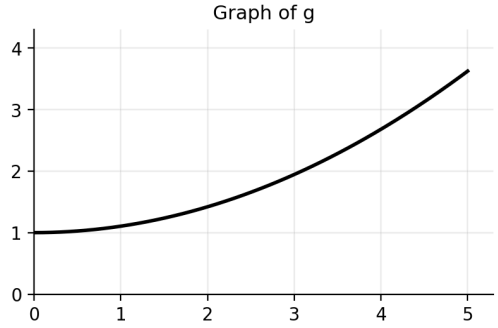
7 | Graph Behavior and Concavity

f increasing ⇔ its rate is positive	f decreasing ⇔ its rate is negative

Rates increasing → concave up	Rates decreasing → concave down

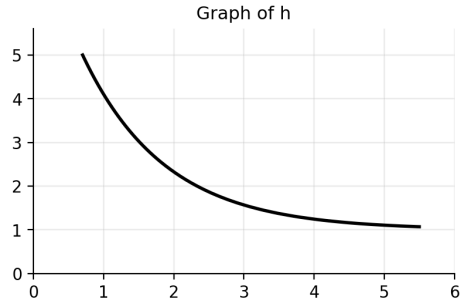
Concavity tells how the slopes are changing. A graph can be decreasing yet concave up (its negative slopes become less negative).

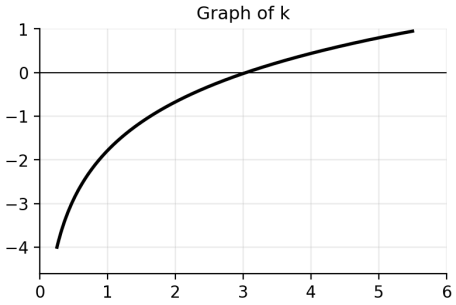
EX #7 | Increasing and concave up



Graph g is _____ and concave _____ because its rate is _____ and _____.

EX #8 | Describe the graph AND its rate





A. Graph h is _____ and _____.

Rate of change of h is _____ and _____.

B. Graph k is _____ and _____.

Rate of change of k is _____ and _____.

CHECK

CFU 1. Negative but increasing slopes give concavity _____.

CFU 2. Positive but decreasing slopes give concavity _____.

8 | Trigonometric Traffic-Flow Application

EX #9 | Model and interpretation

The traffic-flow model is $F(t) = 96 + 5 \sin\left(\frac{t}{2}\right)$, where $0 \le t \le 30$ minutes. Use radians on your calculator. Treat F as the modeled number of vehicles, so changes per minute are vehicles/minute.

A. Find AROC on [3, 8]: $\frac{[F(___) - F(___)]}{(___ - ___)} \approx$ _____.

B. Can precalculus methods estimate the rate at precisely t = 8? Explain.

_____.

C. Calculate three shrinking-interval AROCs

Interval	AROC estimate
[8, 8.1]	_____
[8, 8.01]	_____
[8, 8.001]	_____

WORKSPACE

D. At $t = 8$ minutes, the traffic count is approximately _____ at a rate of _____.

CHECK

CFU 1. What does the negative sign mean? _____.

CFU 2. Why must the calculator be in radians? _____.

9 | Practice: Basic and Application

LEVEL 1 — BASIC UNDERSTANDING

P1. Compute AROC of $f(x) = 3x^2 - 2$ on $[1, 3]$.

WORKSPACE

P2. A table gives (time, distance): (0, 5), (2, 17), (5, 35). Find AROC on $[2, 5]$.

WORKSPACE

P3. On $[1, 5]$, $y = -2x + 9$ is increasing or decreasing? What is its AROC?

WORKSPACE

P4. An output falls by 18 gallons over 6 minutes. State its average rate with units.

WORKSPACE

LEVEL 2 — APPLICATION

P5. Water in a tank: $W(t) = 80 - 2t^2$ (liters). Find AROC on $[2, 4]$, include units.

WORKSPACE

P6. An object moves from 15 miles at 10 min to 45 miles at 40 min. Find the average speed in mph.

WORKSPACE

P7. $y = 2^x$ increases for x from 0 to 1. What is the sign of its AROC? Explain.

WORKSPACE

P8. A graph falls, but its tangent slopes move from −6 to −2 to −1. State direction and concavity.

WORKSPACE

10 | AP-Level Challenge + Exit Ticket

LEVEL 3 — AP-LEVEL CHALLENGE

P9. A ball's height is $s(t) = -16t^2 + 64t + 5$ feet. Calculate AROC on [1, 1.1] and [1, 1.01]. Estimate the IROC at t = 1 and interpret its sign.

WORKSPACE

P10. A function has positive but decreasing instantaneous rates over an interval. Explain whether its graph rises or falls and state its concavity. Give an example.

WORKSPACE

P11. AP-style multiple choice: If a function is decreasing and concave up on an interval, which is true? A. Rate positive and increasing B. Rate negative and increasing C. Rate negative and decreasing D. Rate positive and decreasing

WORKSPACE

WHAT I LEARNED TODAY

1. AROC = output change divided by _____.
2. AROC is the slope of a _____ line.
3. IROC is the slope of a _____ line.
4. Rates becoming larger mean concave _____.

EXIT TICKET

E1. Define average rate of change in one sentence. _____

E2. For $f(x) = x^2 - 1$, find AROC on $[2, 4]$. _____

E3. A curve is below the x-axis and rising with smaller and smaller slopes. Describe its function values, rate sign, and concavity. _____

WORKSPACE