1.

a. Show that $\frac{5}{\sqrt{75}-\sqrt{50}}$ can be written in the form $\sqrt{a}+\sqrt{b}$, where *a* and *b* are integers.

(5)

b. Express 27^{2x+1} in the form 3^{y}, stating *y* in terms of *x*.

(2)

2.

a. For this question, $\mathrm{f}(x)=4kx^2+(4k+2)x+1$, where *k* is a real constant.

i. Find the discriminant of $\mathrm{f}(x)$ in terms of *k*.

(3)

ii. By simplifying your answer to part (a) or otherwise, prove that $\mathrm{f}(x)$ has two distinct real roots for all non-zero values of *k*.

(2)

iii. Explain why $\mathrm{f}(x)$ cannot have two distinct real roots when $k=0$

(1)

b. Lynn is selling cushions as part of an enterprise project. On her first attempt, she sold 80 cushions at the cost of £15 each. She hopes to sell more cushions next time. Her adviser suggests that she can expect to sell 10 more cushions for every £1 that she lowers the price.

i. The number of cushion sold *c* can be modelled by the equation $c=230-Hp$, where £*p* is the price of each cushion and *H* is a constant. Determine the value of *H*.

(1)

To model her total revenue, £*r*, Lynn multiplies the number of cushions sold by the price of each cushion. She writes this as $r=p(230-Hp)$

ii. Rearrange *r* into the form $A-B(p-C)^2$, where *A, B* and *C* are constants to be found.

(3)

iii. Using your answer to part (b) or otherwise, show that Lynn can increase her revenue by £122.50 through lowering her prices, and state the optimum selling price of a cushion.

(2)

3.

The points $R(-4,3),\ S(7,4)\text{ and }T(8,-7)$ lie on the circumference of a circle.

a. Show that *RT* is the diameter of the circle.

(4)

b. Find the equation of the circle.

(4)

4.

a. Shade the region that satisfies the inequalities
$y > x^2 + 4x - 12$ and $y < 4 - x^2$

(5)

b.

i. Given that $3^x = 9^{y-1}$, show that $x = 2y - 2$

(1)

ii. Solve the simultaneous equations:
$$x = 2y - 2$$
$$x^2 = y^2 + 7$$

(6)

5.

Given that $\mathrm{f}(x) = \frac{1}{x}$, $x \neq 0$,

a. Sketch the graph of $y = \mathrm{f}(x) - 2$ and state the equations of the asymptotes.

(3)

b. Find the coordinates of the point where the curve $y = \mathrm{f}(x) - 2$ cuts a coordinate axis.

(2)

c. Sketch the graph of $y = \mathrm{f}(x + 3)$

(2)

d. State the equations of the asymptotes and the coordinates of the point where the curve cuts a coordinates axis.

(2)

6.

A is the point $(-1, 5)$. Let (x, y) be any point on the line $y = 3x$.

a. Write an equation in terms of *x* for the distance between (x, y) and $A(-1, 5)$.

(3)

b. Find the coordinates of the two points, *B* and *C*, on the line $y = 3x$ which are a distance of $\sqrt{74}$ from (-1 , 5)

(3)

c. Find the equation of the line l_1 that is perpendicular to $y = 3x$ and goes through the point $(-1, 5)$.

(2)

d. Find the coordinates of the point of intersection between l_1 and $y = 3x$

(2)

e. Find the area of triangle *ABC*.

(2)
