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Total No. of Questions: [09]

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B.Sc. (Hons.) Physics (Semester – 2nd)
MATHEMATICS-II
Subject Code: BMATH5201
Paper ID: [19131508]

Time: 03 Hours

Maximum Marks: 60

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A

(2 marks each)

Q1. Attempt the following:

- a. A family has two children. What is the probability that both the children are boys given that at least one of them is a boy?
- b. A random variable X has following probability function:

Value of X,

x	0	1	2	3	4	5	6
$p(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$

- (i) Find k
 - (ii) Evaluate $P(X < 5)$.
- c) Seven coins are thrown simultaneously. Find the probability of getting at least five heads.
 - d) What do you mean by skewness in Statistics?
 - e) If $f = \log(xy + 2y^2 - 2x)$. Find $f_x(2, 3)$ and $f_y(2, 3)$.
 - f) If $f_1(x_1, x_2, x_3) = x_1^3$, $f_2(x_1, x_2, x_3) = e^{x_2}$, $f_3(x_1, x_2, x_3) = x_1 + \sin x_2$. Evaluate $\frac{\partial(f_1, f_2, f_3)}{\partial(x_1, x_2, x_3)}$.
 - g) Show that the alternating series $\sum \frac{(-1)^n}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \dots$ is convergent.
 - h) Define sequence and convergence of a sequence.
 - i) If X is normally distributed and the mean of X is 12 and standard deviation is 4. Find out the probability of the following:
 - i) $X \geq 20$
 - ii) $X \leq 20$.
 - j) Find $\frac{dz}{dt}$ at $t = \frac{\pi}{2}$ when $z = xy$, $x = \cos t$ and $y = \sin t$.

Section – B

(5 marks each)

Q2. Using Cauchy's Integral Test, discuss the convergence or divergence of the

series $\sum_{n=1}^{\infty} \frac{1}{n^p}$, $p > 0$.

Q3. State Euler's theorem on Homogenous functions. If $z = \frac{x-y}{\sqrt{x+y}}$, show that

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = \frac{1}{2} \tan \tan z.$$

Q4. Explain Poisson approximation to the Binomial distribution.

Q5. Calculate the correlation coefficient for the following heights (in inches) of father (X) and their sons (Y):

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

Q6. If $x^y + y^x = (x + y)^{x+y}$ then by using partial derivatives, find $\frac{dy}{dx}$.

Section – C

(10 marks each)

Q7. (a) Discuss the convergence or divergence of the series :

$$\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \dots$$

(b) Let $f_1 = \sqrt{x^2 + y^2}$, $f_2 = \frac{y}{x}$ for $x \neq 0$. Evaluate $\frac{\partial(f_1, f_2)}{\partial(x, y)}$ at $(1, 2)$.

Q8. (a) Find median of the Normal distribution.

(b) Two dice are rolled. Let X denote the random variable which counts the total number of points on the upturned faces. Construct a table giving the non-zero values of the probability mass function and draw the probability chart. Also find the distribution function of X.

Q9. (a) If $x = r \cos \theta$, $y = r \sin \theta$, prove that

$$\frac{\partial^2 \theta}{\partial x \partial y} = \frac{\partial^2(\log r)}{\partial x^2} = -\frac{\partial^2(\log r)}{\partial y^2} = -\frac{1}{r^2} \cos 2\theta.$$

(b) Discuss the convergence or divergence of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots$