Aim: Section properties of structural elements

Objectives: At the end of the lesson, the students should be able to:

- Define various cross-section properties: centroid, the neutral axis, and the moment of inertia.
- Determine the location of centroid, the neutral axis, and the magnitude of moment of inertia.

CROSS SECTION PROPERTIES

Centroid:

- The centroid of two or more symmetry axis is located at the interception between both axis:

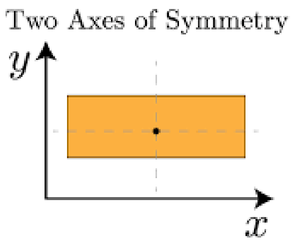


Figure 1. Centroid of a rectangle.

QUIZ

Can you think about another regular shape with two or more axis of symmetry?

The Neutral Axis:

- The neutral axis is an axis or line in the cross section of a beam or shaft along which there are no longitudinal stresses or strains. In other words, there is no tension or compression, resulting in zero stress.
- If the section is symmetric, isotropic (materials have the same mechanical properties, such as elasticity, strength, and conductivity, in all directions), and is not curved before a bend occurs, then the neutral axis is at the geometric centroid.

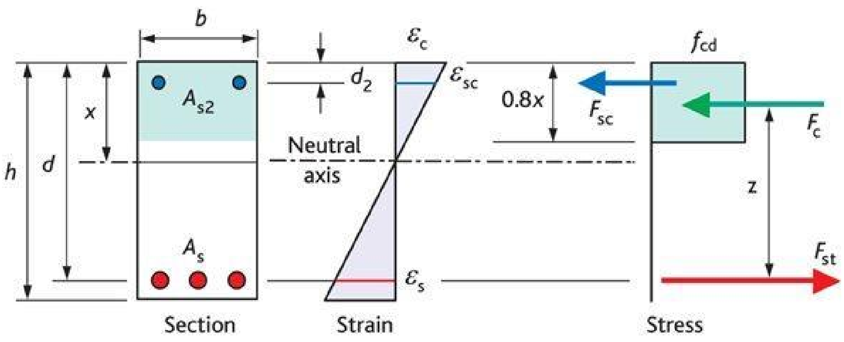
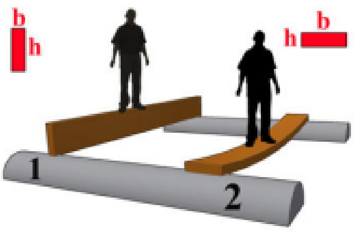


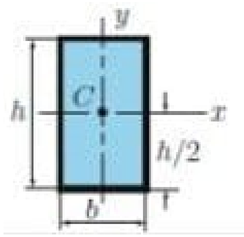
Figure 2. Simplified rectangular stress block for a reinforced concrete beam (Bond et al., 2006)

Area Moment of Inertia / Second Moment of Area

- The second moment of area is a measure of the "efficiency" of a cross-sectional shape to resist bending caused by loading.
- Moments of inertia of areas are used in calculating the stresses and deflections of beams, the torsion of shafts, and the buckling of columns.
- The moment of inertia is always computed with respect to an axis; its value is greatly affected by the distribution of the area relative to the axis.

*Both beams, 1 & 2, have the same area and even the same shape.
*Beam 1 is stronger than Beam 2 because it has a larger second moment of inertia (I).
*Under the same loading conditions, beam 2 will bend before Beam 1.





See Appendix A for moment of inertia for some common shapes

For a rectangle, the moment of inertia

Taking

The Parallel Axis Theorem

- The parallel axis theorem is a fundamental concept in mechanics that relates the moments of inertia of a body about two parallel axes.
- The parallel axis theorem is often used in engineering and physics to calculate the moment of inertia of complex shapes or composite bodies by summing the individual moments of inertia of their component parts.

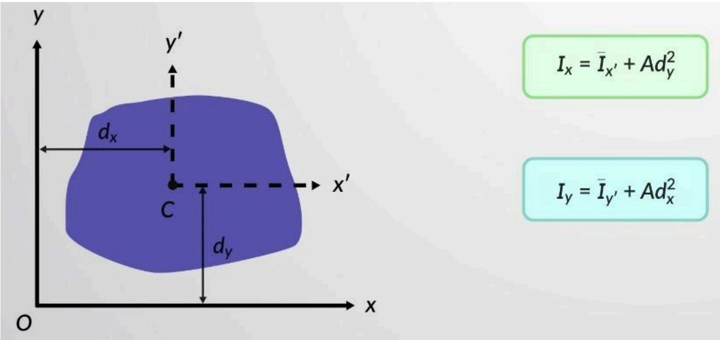


Figure 3. Parallel-axis theorem (MyJoVE Corporation, 2023)

- Mathematically, the parallel axis theorem for the second moment of area (also known as the moment of inertia) can be expressed as:

$$I = I_{cm} + Ad^2$$

Where:

I is the moment of inertia about the parallel axis;

I_{cm} is the moment of inertia about the centre of mass axis;

A is the area of the body;

d is the perpendicular distance between the two axes.

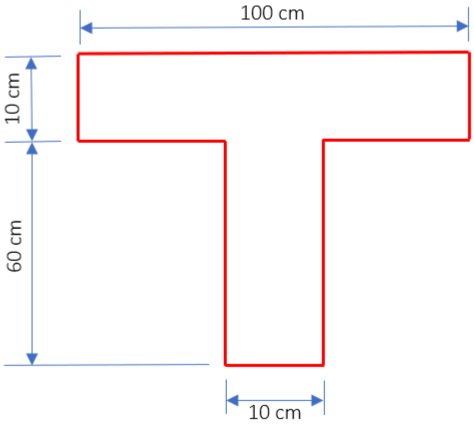
Worked Example 1

You have been provided below with a sectional detail of beam design used in the building structures of a proposed development. Determine, using appropriate mathematical techniques, the following properties:

(i) location of centroidal axes x-x and y-y

(ii) the neutral axis, y in cm

(iii) moment of inertia, I_{XX} in cm^4



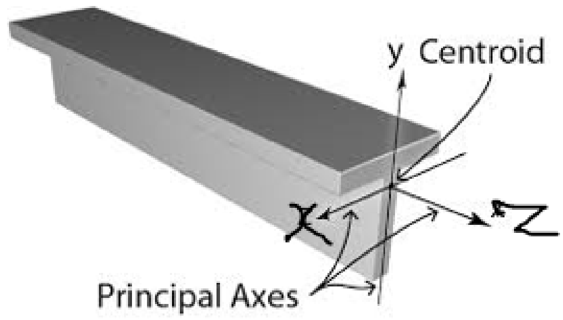


Figure 4. T-shape cross-section.

Solution

Part 1 - Determine the centroid

Centroid of the cross-section is given as, $\overline{y} = \dfrac{\Sigma A_i y_i}{\Sigma A}$

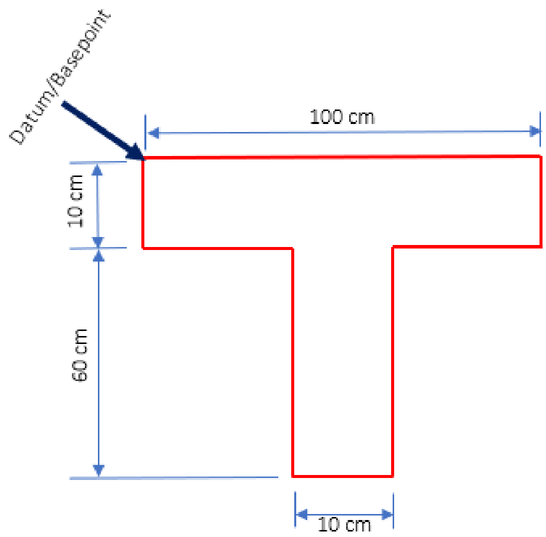
Where:

A_i is the area of each section i;

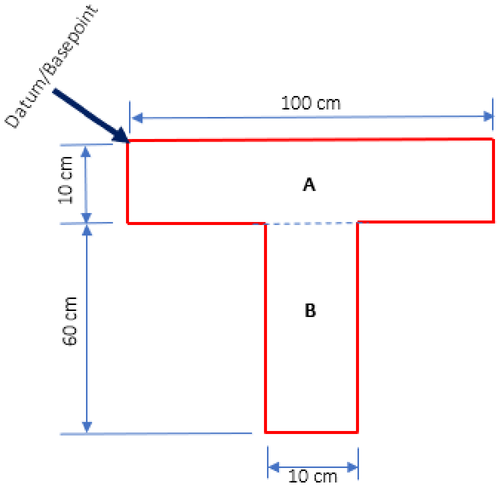
y_i is the centroid of each section from the reference point or datum;

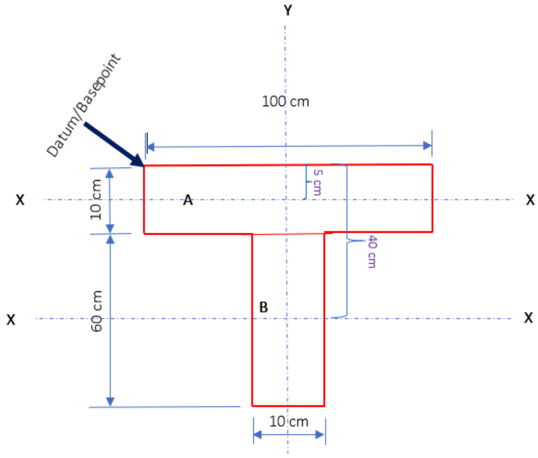
A is the total area of the cross-section.

Step 1.1: Choose a datum or baseline.



Step 1.2: Divide the cross-section into manageable areas of different shapes and locate the centroid y_i.





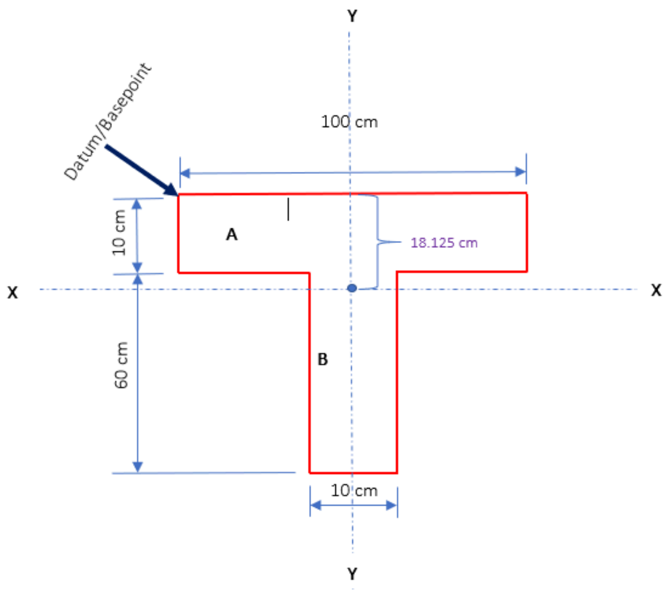
Step 1.3: Create a table to determine the required section properties.

Centroid, $\bar{y} = \dfrac{\sum A_i y_i}{\sum A}$

Section, i	Area, A_i (cm^2)	y_i (cm)	$A_i y_i$ (cm^3)
A	100 X 10 = 1000	5	5,000
B	60 X 10 = 600	40	24,000
Total, $\sum$	1600		29,000

Centroid, $\bar{y} = \dfrac{\sum A_i y_i}{\sum A} = \dfrac{29{,}000}{1{,}600} = 18.125\ cm$

Its x-axis $\bar{x}$ will be at 50 cm from the datum.



Part 2 - Determine the neutral axis

Since the shape is symmetric, the neutral axis will pass through its cross-sectional centroid. Therefore, neutral axis, NA = Centroid, $\bar{y} = 18.125\ cm$

Part 3 - Determine the moment of inertia

Moment of inertia about the x-axis, $I_{xx} = \sum \bar{I}_{xxi} + \sum A_i {dy_i}^2$

Where,

$\bar{I}_{xxi}$ is the moment of inertia of each section of the cross-section i;

A_i is the area of each section i;

dy_i is the difference between the centroid of each section and the centroid of the whole cross-section, $(y_i - \bar{y})$.

Step 3.1: Determine the moment of inertia for each section.

Moment of inertia for a rectangular shape, $I_{xx} = \frac{bh^3}{12}$

$$I_{xx-A} = \frac{bh^3}{12} = \frac{100\ X\ 10^3}{12} = 8,333.33\ cm^4$$

$$I_{xx-B} = \frac{bh^3}{12} = \frac{10\ X\ 60^3}{12} = 180,000\ cm^4$$

Step 3.2: Create a table to determine the other required section properties.

Section, i	Area, A_i (cm^2)	y_i (cm)	$A_i y_i$ (cm^3)	$\bar{I}_{xxi}$ (cm^4)	dy_i $(y_i - \bar{y})$ (cm)	$dy_i^{\ 2}$ (cm^2)	$A_i dy_i^{\ 2}$ (cm^4)
A	100 X 10 = 1000	5	5,000	$8,333.33$	5 – 18.125 = -13.125	172.27	172,270
B	60 X 10 = 600	40	24,000	$180,000$	40 – 18.125 = 21.875	478.52	287,112
Total, ∑	1600		29,000	188,333.33			459,382

Moment of inertia,

$$I_{xx} = \sum \bar{I}_{xxi} + \sum A_i dy_i^{\ 2} = 188,333.33 + 459,382 = 647,715.33\ cm^4$$

Self-assessment Task

You have been provided below with sectional details of various beam designs used in the building structures of a proposed development. Determine, using appropriate mathematical techniques, the following properties:

(i) location of centroidal axes x-x and y-y

(ii) the neutral axis, y in cm

(iii) moment of inertia, I_{XX} in cm^4

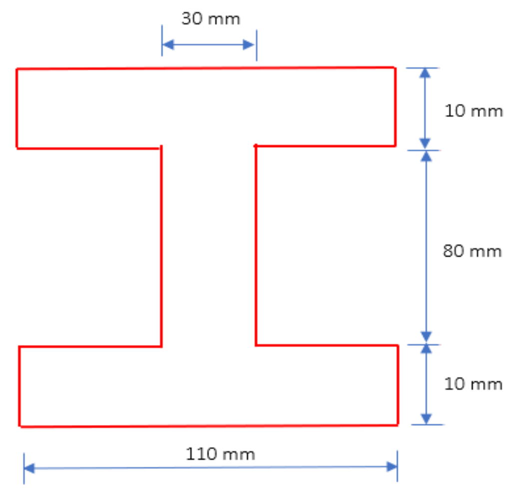


Figure 5. I-shape cross-section.

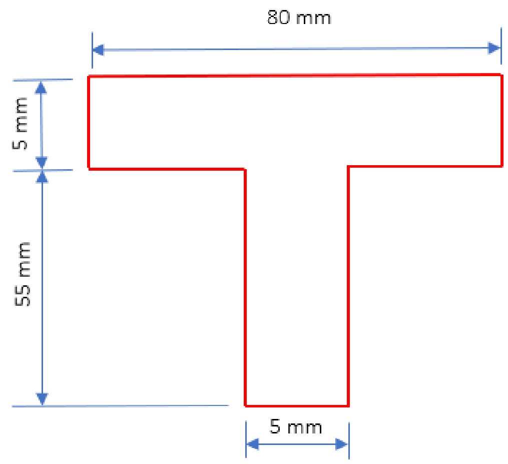
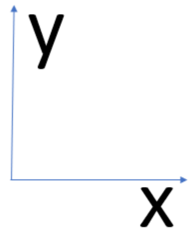
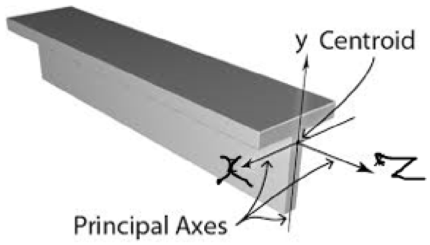


Figure 6. T-shape cross-section.





References

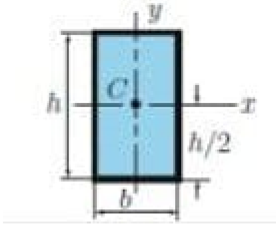
Bond, A. J., Brooker, O., Harris, A. J., Harrison, T., Moss, R. M., Narayanan, R. S. and Webster, R. (2006) *How to design concrete structures using eurocode 2*. Surrey: The Concrete Centre.

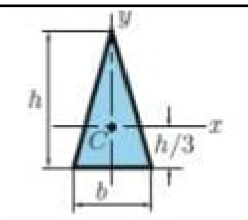
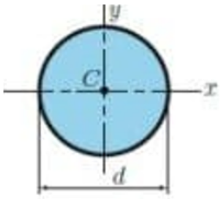
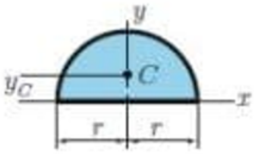
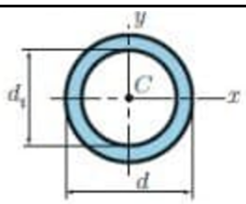
Learneasy (2007). *2nd moment of area*. Available at: http://mem50212.com/MDME/MEMmods/MEM30006A/Area_Moment/Area_Moment4.html (Accessed: 08 October 2023).

MyJoVE Corporation (2023) 10.2: *Parallel-axis theorem for an area* Available at: https://www.jove.com/science-education/14344/parallel-axis-theorem-for-an-area (Accessed: 20 September 2023).

Appendix A

Moment of Inertia for some commom shapes

	$Area,\ A = bh$ $I_{xx} = \frac{bh^3}{12}$ $I_{yy} = \frac{b^3h}{12}$

	$Area,\ A = \frac{bh}{2}$ $I_{xx} = \frac{bh^3}{36}$ $I_{yy} = \frac{b^3h}{36}$
	$Area,\ A = \frac{\pi d^2}{4}$ $I_{xx} = I_{yy} = \frac{\pi d^4}{64}$
	$Area,\ A = \frac{\pi r^2}{2}$ $I_{xx} = I_{yy} = \frac{\pi r^4}{8}$
	$Area,\ A = \frac{\pi}{4}\left(d^2 - d_1^{\ 2}\right)$ $I_{xx} = I_{yy} = \frac{\pi}{64}\left(d^4 - d_1^{\ 4}\right)$