PDM: 9.5: Using Derivatives to Analyze Graphs

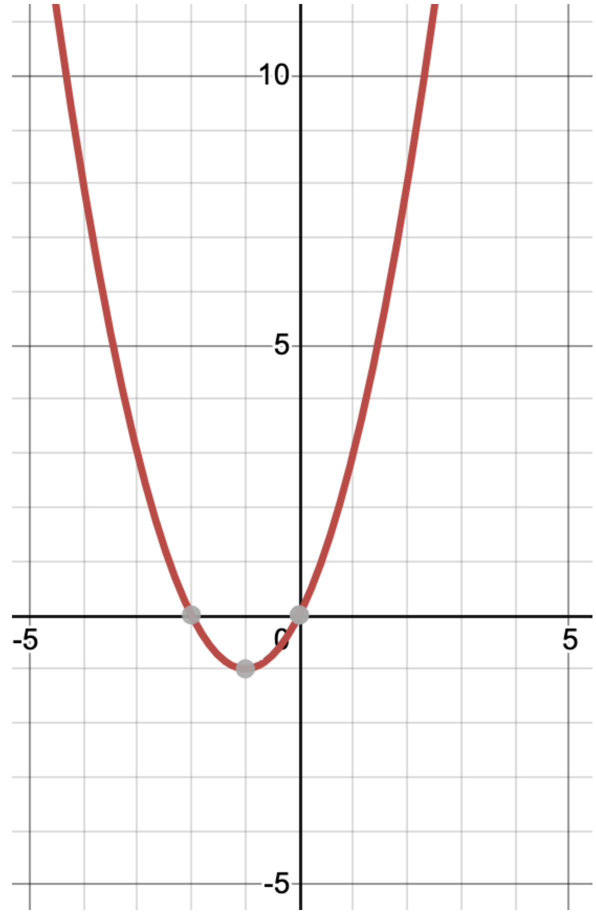
1) Let $f(x) = x^2 + 2x$, which is shown on the graph.
   a) Find $f'(x)$

   b) Graph $f'(x)$ on the same set of axes as $f(x)$

   c) When $f'(x)$ results in positive values, what do you notice about $f(x)$?

   d) When $f'(x)$ results in negative values, what do you notice about $f(x)$?

   e) When $f'(x)$ results in zero, what do you notice about $f(x)$?

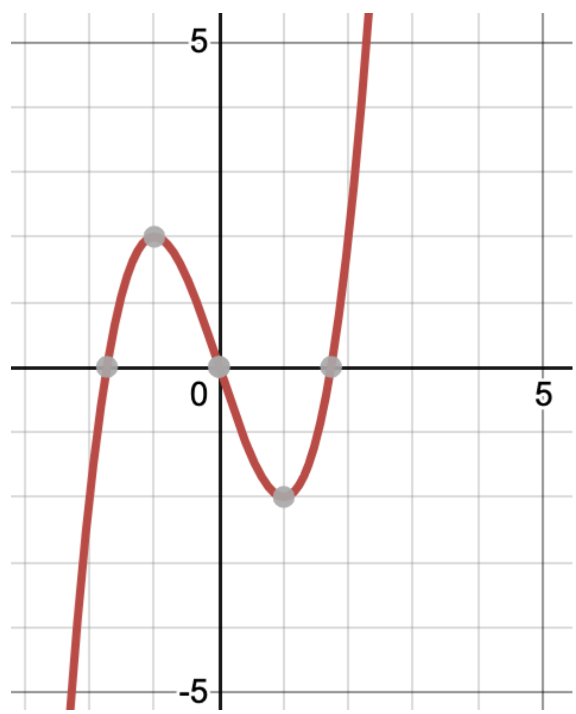


2) Let $f(x) = x^3 - 3x$, which is shown on the graph
   a) Find $f'(x)$

   b) Graph $f'(x)$ on the same set of axes as $f(x)$

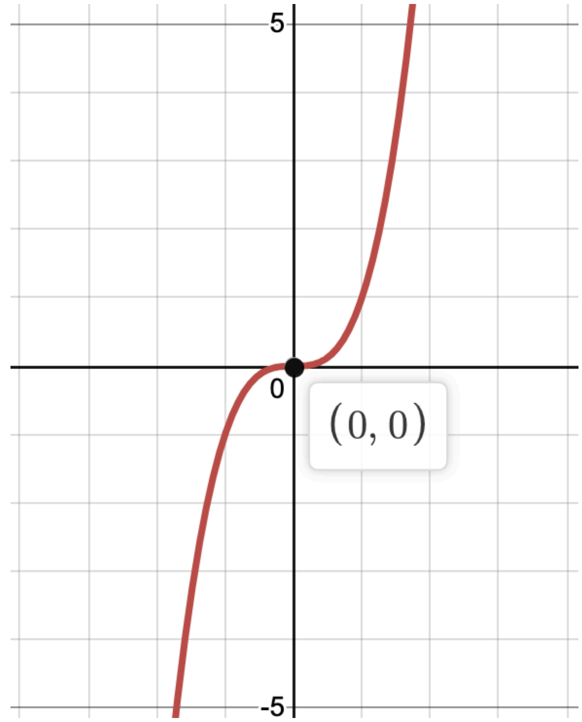
   c) When $f'(x)$ results in positive values, what do you notice about $f(x)$?

   d) When $f'(x)$ results in negative values, what do you notice about $f(x)$?



   e) When $f'(x)$ results in zero, what do you notice about $f(x)$?

3) Let $f(x) = x^3$, which is shown on the graph



a) Find $f'(x)$

b) Graph $f'(x)$ on the same set of axes as $f(x)$

c) When $f'(x)$ results in positive values, what do you notice about $f(x)$?

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e) When $f'(x)$ results in zero, what do you notice about $f(x)$?

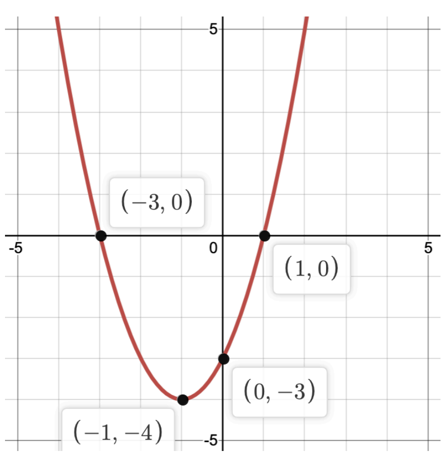
The values of the derivative function are the slopes of lines tangent to the graph of the function.
When the slopes of tangents to the graphs of a function are **positive**, the function is **increasing**.
When the slopes of tangents to the graph of a function are **negative**, the function is **decreasing**.
When the slopes of tangents to the graph of a function are **zero**, the function has a **critical point** (most of the time this means maximum or minimum value, but as you can see in problem #3, that isn't always the case).

Theorem: Suppose *f* is a function whose derivative function $f'(x)$ exists for all *x* in the interval $a < x < b$

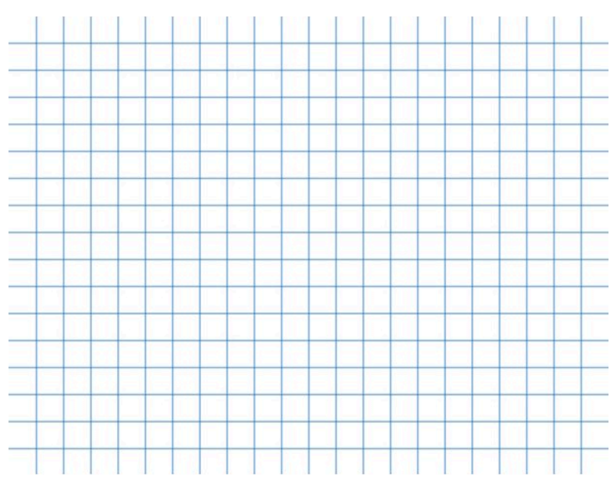
(1) If $f'(x) > 0$ for all $a < x < b$, then *f* is increasing on the interval.
(2) If $f'(x) > 0$for all $a < x < b$, then *f* is decreasing on the interval.

4) Given the graph of the derivative $f'(x)$, use this to sketch a possible graph of the original function $f(x)$

Derivative $f'(x)$

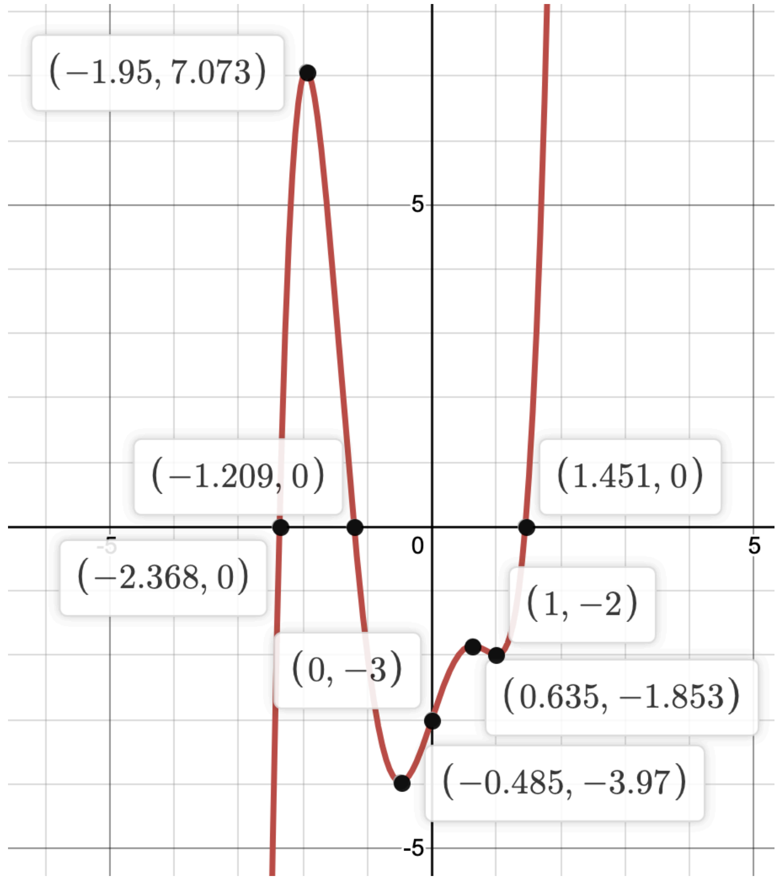


Original $f(x)$

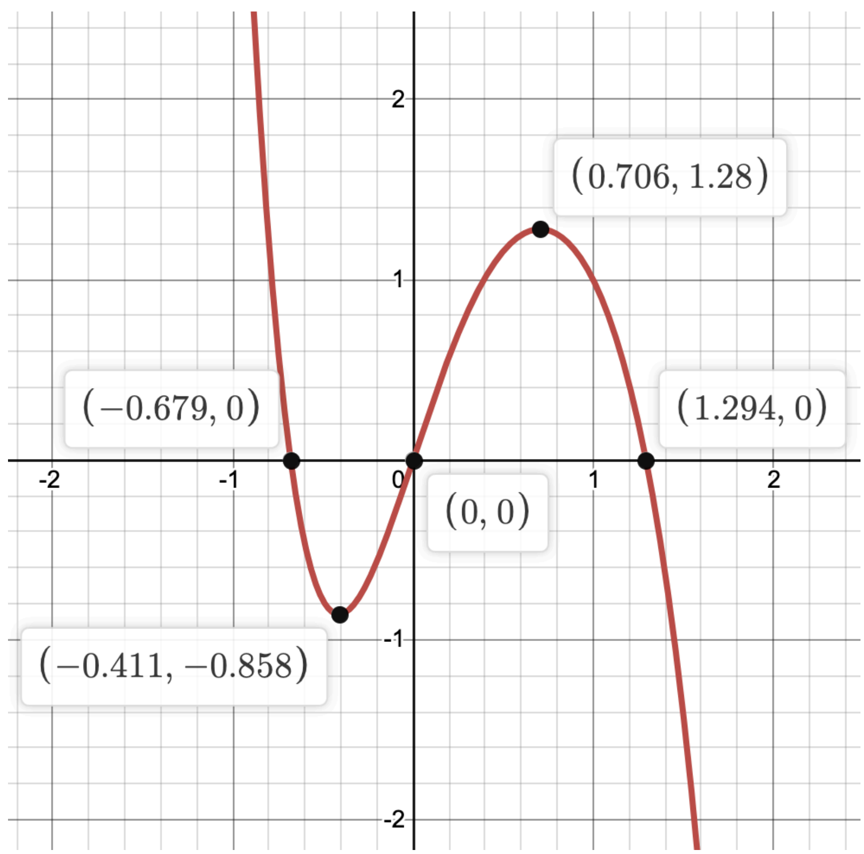


Homework

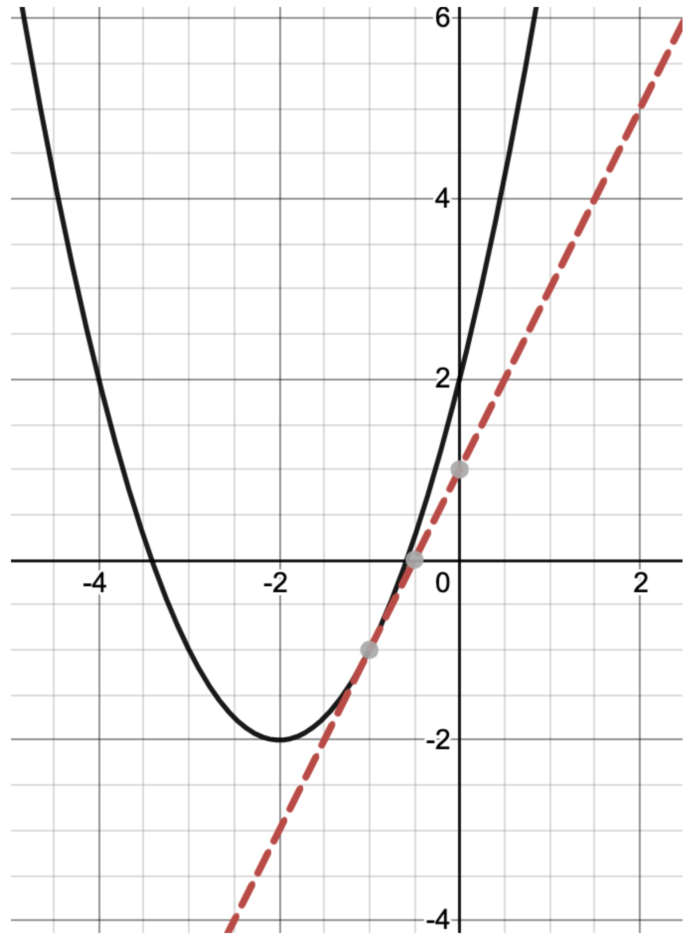
1. The derivative of $g'(x)$ of a function $g(x)$ is graphed.
   a. On what interval(s) is $g(x)$ increasing?
   b. On what interval(s) is $g(x)$ decreasing?
   c. What are the critical values of $g(x)$



2. Refer to the graph of $f(x)$ shown.
   a. On what interval(s) is the derivative of $f(x)$ positive?
   b. On what interval(s) is the derivative of $f(x)$ negative?
   c. Where is the derivative equal to zero?



3. Shown is the graph of a function $h(x)$ and it's tangent line at $x = -1$.

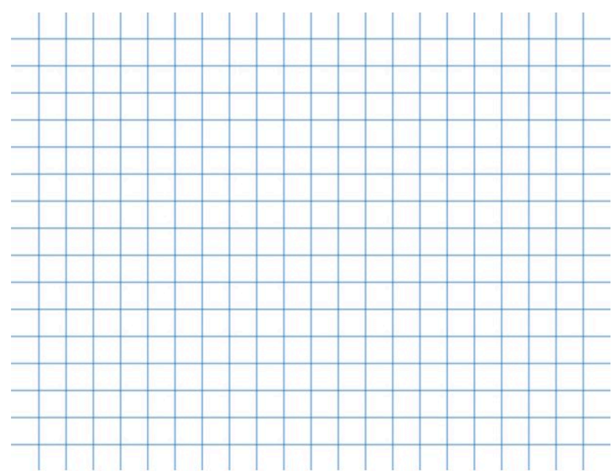


   a. What is $h'(-1)$?

   b. Write an equation for the tangent line in point-slope form $y - y_1 = m(x - x_1)$

   c. Does the equation that you found in part b represent the derivative of the function? Explain your reasoning.

   d. Sketch a reasonable graph of $h'(x)$on the axes below. Be sure to pay attention to increasing & decreasing intervals as well as any critical points of $h(x)$.



4. *True or False.*
   a. The derivative of a cubic function is always a quadratic function

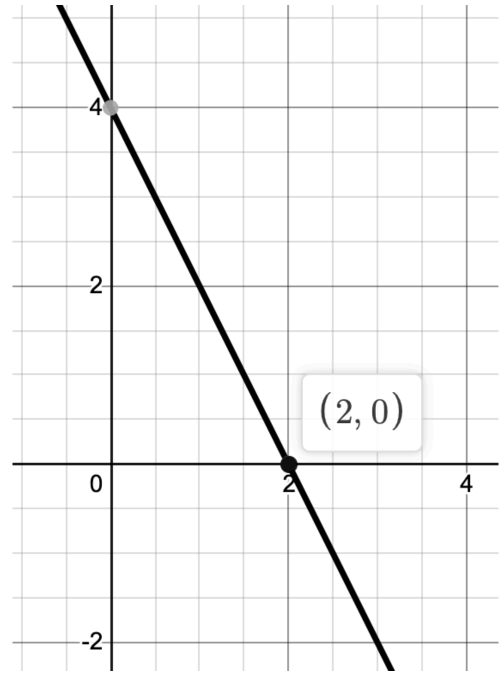
   b. If $g'(x)$is a linear function, then $g(x)$ is a quadratic function.

   c. If $f'(x)$ is below the x-axis from $0 < x < 3$, that means that $f(x)$ is increasing on that interval.

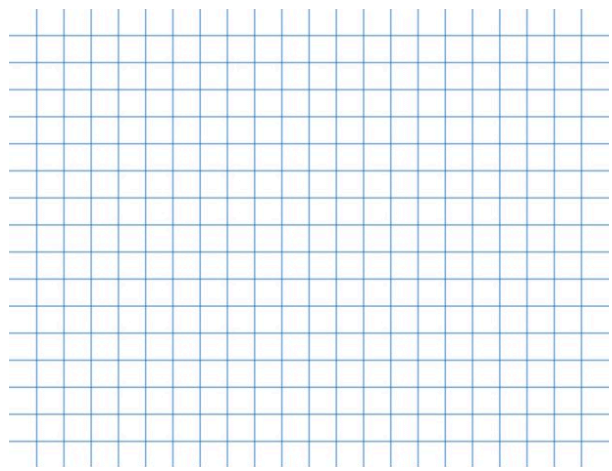
5. Given the graph of $f'(x)$, the derivative of $f(x)$, shown, answer the following:
   a. On what interval(s) would the graph of $f(x)$ be increasing?

   b. On what interval(s) would the graph of $f(x)$ be decreasing?

   c. What are the critical point(s) of $f(x)$?



   d. Use this information to sketch a reasonable graph of $f(x)$ on the axes below.



6. **(Advanced)** Let $g'(x) = 5x^2 - 25x$. Show your work algebraically without using graphing software.
   a. What are the critical points of $g(x)$?

   b. Where is $g(x)$ increasing?

   c. Where is $g(x)$ decreasing?