Determining the asymptotic complexity of matrix multiplication is a central problem in algebraic complexity theory. The fact that algorithms for matrix multiplication of a certain complexity exist can be phrased as an orbit problem. The best upper bounds on the so-called exponent of matrix multiplication are obtained by starting with an "efficient" tensor, taking a high power of it and degenerating a matrix multiplication out of it, the so-called "laser" method. In the first part of the talk, we give a gentle introduction to the design of fast matrix multiplication algorithms.

In recent years, several so-called barrier results have been established. A barrier result shows a lower bound on the best upper bound for the exponent of matrix multiplication that can be obtained by a certain restriction starting with a certain tensor. We prove the following barrier over the complex numbers: Starting with a tensor of minimal border rank satisfying a certain genericity condition, except for the diagonal tensor, it is impossible to prove $\omega = 2$ using arbitrary restrictions. This is astonishing since the tensors of minimal border rank look like the most natural candidates for designing fast matrix multiplication algorithms. We prove this by showing that all these tensors are irreversible, using a structural characterization of these tensors.

Joint works with Vladimir Lysikov.