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Week 11
Centrality 2

Agenda

Last time we focused on algebraic measures. Today we add graph-theoretic measures, among other measures

inverse -weighted degree
Closeness
Betweenness
Induced centralities
Group centralities
key player

INverse-weighted degree

Negative ties

- Receiving negative ties is bad, but not from people that hate everybody
- Weight ties inversely by outdegree of sender

Positive ties too

- Receiving positive ties from people who give them indiscriminately isn't as rewarding a receiving ties from those that give few

->coldb = colstoch(dbconet) //columps of coldb all add to one
->dsp rowsums(coldb)

Closeness

Freeman closeness is sum of geodesic distances to all other nodes

->draw borg4cent

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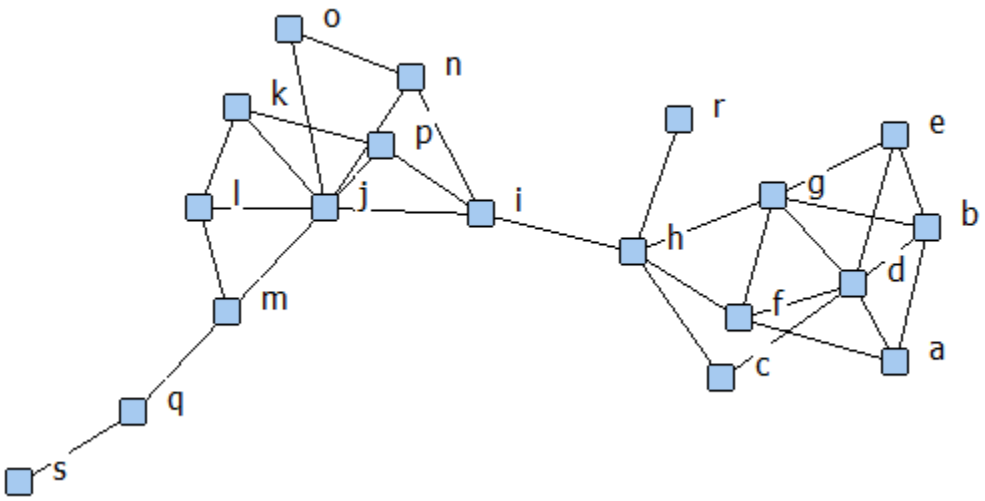

->g = geodesic(borg4cent)
->dsp rowsums(g)
Note that closeness is inverse measure :bigger numbers mean less centrality

Disconnected networks

->women = xxt(davis) //multiply davis by its transpose
->women3 = dichot(women gt 3)
->draw women3

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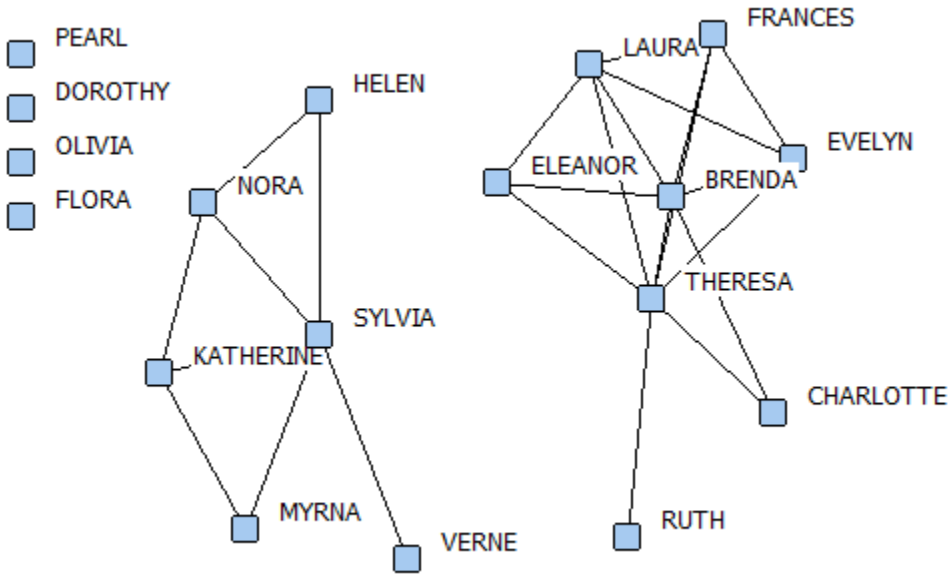

->g = geo(women3)
->rg = reciprocal(g)
->clo = rowsum(rg)
->dsp clo

Note that bigger numbers mean more centrality

<u>Directed data</u>

Directed networks are usually disconnected, so we use the reciprocal method

->rg = recip(geo(campnet))
-> dsp marginals(rg) // display row and column sums

```
                1     2
             Rows  Cols
           ----- -----
1   HOLLY     6 8.733
2  BRAZEY 5.588 2.917 //brazey can reach others easily. Reverse not true
3   CAROL 5.333 6.819
4     PAM 4.983 9.150
5     PAT 5.667 8.650
6  JENNIE 5.333 7.152
```

```
 7 PAULINE 5.333 7.819
 8     ANN 4.983 6.652
 9 MICHAEL 5.333 8.150
10    BILL 5.167     0
11     LEE 5.588 4.333
12     DON 5.333 8.067
13    JOHN 8.250     0
14   HARRY 5.333 7.050
15    GERY 7.500 3.667
16   STEVE 6.150 5.500
17    BERT 6.150     5
18    RUSS 6.633     5
```

Issues

- Low variance in large datasets
- Disconnected pairs introduce a great deal of noise / hard to interpret

Betweenness

Betweenness of node k is Share of all geodesic paths from all i to all j that go through k.

$$b_k = \sum_{i,j} \frac{g_{ikj}}{g_{ij}}$$

g(i,j) = number of geodesic paths from i to j
g(i,k,j) = no. of geodesic paths from i to j that pass through k

If there are two equally short paths from i to j, and k is along one of them, k gets ½ point for that.

->bet = betweenness(campnet)
->draw campnet bet

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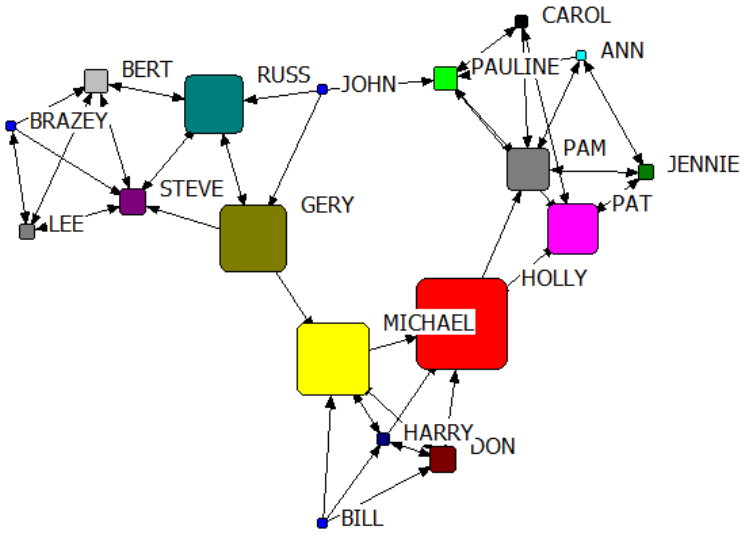

Pitts use case

Pitts, F. R. (1978). The medieval river trade network of Russia revisited. *Social networks*, *1*(3), 285-292.

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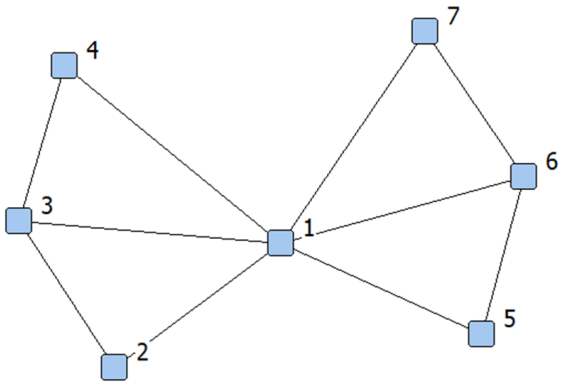
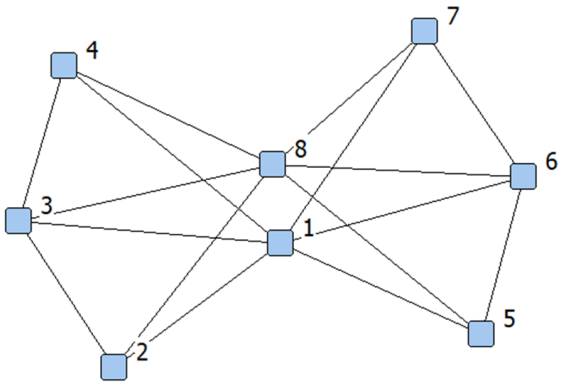
Comparison with structural holes

- SH limited to direct contacts; betweenness is global

Restricting or weighting length of paths





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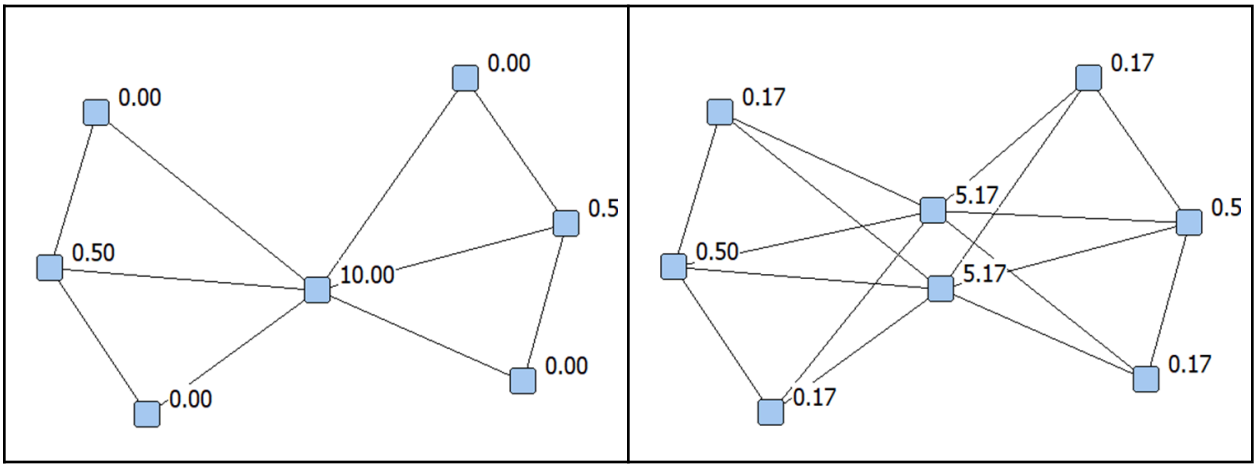
Node 1 has same structural holes score in both networks above. But betweenness does not.



Issues

- Std measure is restricted to dichotomous networks
- Works well with disconnected and directed graphs

Of course, can run multiple measures at the same time using centrality function

->dsp centrality(campnet)

Induced centralities

Normally think of centrality as something the network gives to the node. Induced centrality views things the other way around: how much does the node contribute to the network? I.e., to what extent are the network's characteristics due to each node?

Steps:

1. Calculate graph invariant X(G) (any structural statistic that describes the whole graph)
2. Remove node k
3. Recalculate graph invariant X(G-k)
4. Centrality = X(G) - X(G-k)

Measures the contribution of each node to some overall network property such as average reciprocal distance.

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In the case of “inverse invariants” (where numbers are expected to get larger when you remove a node), we calculate step 4 as X(G-k) - X(G) instead.

Group centrality

KeyPlayer
