Another Way to the Circumcenter

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Host Organization: Stanford University **ETP Type: Traditional**
Subject/Grade: Geometry Honors/9th-10th

Abstract (~150 words)

In teams of 3-4 members, Geometry Honors students will receive a problem on finding the circumcenter of a triangle and its algebraic solution. The solution involves a concept and theorem covered prior to this, but for which a graphical solution has not been covered. Students will need to synthesize the information from Algebra 1 and this class to understand why the solution works. From there, students will produce two of the following: an algebraic solution, a physical solution, a solution involving code in Python, a different problem using the same algebraic procedure, or a problem where this solution wouldn't make sense. Each team will produce a Google document with their problem(s) and/or solution(s). After all teams complete work on their document, the class will review another group's document to review the work of others and to reinforce their own knowledge.

Focal Content & Supporting Practices

CCSS.MATH.CONTENT.HSG.MG.A.3
Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*

21st Century Skills and Applications (1 - 2)

Problem Solving: Secures additional relevant information regarding problem and generates multiple strong alternatives for solving it
Collaboration and communication: Varies communication to meet varied purposes: to **inform, instruct**, motivate or persuade

Students are given an opportunity to produce an alternate problem and/or solution to a given problem statement and solution. To do this, students will work in groups to communicate with each other on what the problem statement is about and how the given solution works. From there, students will use prior knowledge to inform the group of any other possible problem and/or solution and instruct others on how this would work within the context.

Measurable Objective(s)

Students will demonstrate their understanding of a problem and a graphical solution by creating another representation of the solution.

Students will be able to analyze a graphical solution to create a solution using equations or physical drawings (constructions or patty paper), or modify code and/or students will be able to create a problem and identify whether finding the circumcenter is the appropriate solution for it.

Formative Assessment(s)

After giving instructions for this lesson and each team's prompt, I will **circulate and monitor** the class and **check for understanding** by asking questions such as:

- Why do we need to find the circumcenter of the triangle?
- Why do we need the midpoint of a side?
- What are different ways to find the midpoint of a segment?
- Why do we need the perpendicular slope of a side?
- How do we find slope?
- What do you know about the slopes of perpendicular lines?

Summative Assessment(s)

A Google document containing a report of the team's analysis

Fellowship Description (300-500 words)

This summer, I'm working in a Wind Engineering lab of the Civil and Environmental Engineering department at Stanford University. In this lab, we have 5 graduate students working with Professor Catherine Gorlé on studying how wind affects the exterior and interior of buildings. Each of the projects involves using multiple sensors to detect and collect data such as wind pressure, temperature, and wind flow. The data is then analyzed to help with calculations of a formula that makes predictions useful for designing various types of buildings and analyzing current structures and systems within the structure.

My mentor's project involves the connection between the high rate of pneumonia in Bangladesh by measuring the ventilation inside the slum buildings. In this project, he proposes a model for which natural ventilation affects air flow in a building. To do this, sensors were placed in slum buildings in Bangladesh to measure temperature and wind flow inside. The data is used to verify his model and assist in making modifications to the model if necessary.

Another research project involves measuring wind pressure on the windows of high rise buildings. To conduct this experiment, sensors are strategically placed on a physical model to simulate wind pressure on a high rise building. The results of this project provide calculations of pressure on different locations of a high rise building. These calculations provide insight for architects when deciding on where to place windows and the strength of the window.

Another one of the projects involves the building where the lab is located. This project measures the wind flow on the different floors of the Y2E2 building. The purpose of this project is to measure the energy efficiency of the night-time flush.

My role in this lab is to assist my mentor in deploying a barometer connected to a Raspberry Pi to retrieve information about wind pressure and temperature. A website stores and displays the data and one of my tasks involves understanding how to modify the code to differently display the data. To do this, I will create a flowchart of how the code works. In order to fulfill this project, I need skills in programming in Python and HTML to not only write code, but also to understand existing code and manipulate it for the purposes of this project. On the hardware side, I need to understand how to connect a pressure sensor to a Raspberry Pi in order to retrieve information. This means, I need to learn how to connect pins from the sensor to the Raspberry Pi. In addition, I need skills in organizing, communicating, and presenting information on a website.

Fellowship Connection to School/Classroom (300-500 words)

One of my tasks in the Wind Engineering lab is to work with existing code for a website that collects and displays real-time data from various sensors, motes, and stations. With this task, I need to understand the structure of the code and modify parts of the code to show other features or calculations on the website. The skills involved include analyzing the existing solution and synthesizing what I learn from it with prior or new knowledge to generate more data.

To connect my work this summer with the classroom, I plan to give teams of 3-4 students a problem they have never seen before along with a possible solution for it. The problem consists of finding a point that is equidistant to three noncollinear points and in this case, it would require finding the circumcenter of the triangle. While the solution may be found using equations or constructions, I will provide a graphical solution. To prepare students for this, we will go over the definition of perpendicular bisectors and identify the concurrency of perpendicular bisectors in a triangle as the circumcenter. We will do a patty paper activity where students have the opportunity to discover the circumcenter is equidistant to the vertices of the triangle.

Their task is to do (at least) two of the following:

1. Create a different problem that uses a similar solution and write the algebraic solution for it
2. Create an algebraic solution (optional: scaffolding available)
3. Create a physical solution using constructions or using patty paper along with step-by-step instructions
4. Given a framework of code written in Python, students modify the code appropriately to solve the problem to reach the same solution
5. Write a problem that is similar to the provided problem but which could not use the same solution of finding the circumcenter to solve it (i.e. a situation where finding the circumcenter is not the solution)

The core focal content is a math standard using modeling to solve design problems. This standard is related to my fellowship project because I'm asking students to modify an existing solution to apply to other problems and/or to modify the existing solution by presenting it in a different representation (through equations or coding).

Instructional Plan (This is the bulk of your ETP and may take several pages.)

Timing: Two 45-minute lessons (for block schedule, it is recommended to do across two days)

Content standard: Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*
**Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*).*

Previous Content Knowledge/Vocabulary:
- The definition of equidistant
- The definition of slopes and how to find slopes of perpendicular lines
- Writing linear equations given a slope and a point
- Solving system of linear equations
- The definition of a perpendicular bisector of a segment
- The definition of a circumcenter of a triangle

Day One:
Objective: Students will be able to analyze a graphical solution to create a solution using equations or physical drawings (constructions or patty paper), or modify code.

- 3-5 minutes: Team formation - Students choose teams or teacher chooses teams of 3-4 students
- 5-8 minutes: Teacher introduces activity and gives an overview of expectations
- 10-15 minutes: Students read problem using page 1 of the 3 reads protocol
    - Teacher circulates to listen in on conversations and asks students what would need to be computed in order to solve the problem
- 10-15 minutes: Teacher passes out prompt and students read the solution using page 2 of the 3 reads protocol
    - Teacher circulates to listen in on conversations and asks students the following questions:
        - Why do we need to find the circumcenter of the triangle?
        - Why do we need the midpoint of a side?
        - What are different ways to find the midpoint of a segment?
        - Why do we need the perpendicular slope of a side?
        - How do we find slope?
        - What do you know about the slopes of perpendicular lines?
- 5 minutes: Students choose two options as a group and fill out page 2 of prompt
- [Flexible activity] Start in class and complete for homework: Student handout with scaffolding questions / alternate solution

Assessment: formative assessment questions embedded in Day One

Day Two:
Main Objective: Students will demonstrate their understanding of a problem and a graphical solution by creating another representation of the solution.

Secondary Objective: Students will be able to create a problem and identify whether finding the circumcenter is the appropriate solution for it.

- 5 minutes: Students review what they have with their teams
- 15-20 minutes: Students continue work as needed as agreed upon on the previous lesson
    - Students may need to use graph paper, compasses, straightedges, patty paper, or Chromebooks
- 20 minutes: Students write up their analysis on Google Docs (students may upload photos / videos of their physical project in addition to submitting physical papers)
- Teacher may choose a time outside of class for students to submit their Google Doc
- Teacher uses rubric to assess understanding shown in the Google Doc

Assessment: rubric for summative assessment
Possible Student Solution:
- Algebraic Solution
- Code / Solution

Supply List

1. Prompt with problem and solution
2. Electronic device (Chromebook, tablet, etc.) for students to access Google Docs, Desmos, and/or code
3. Patty paper
4. Compass
5. Straightedge
6. Graph Paper

References

1. High School: Geometry » Modeling with Geometry. (n.d.). Retrieved July 2, 2019, from http://www.corestandards.org/Math/Content/HSG/MG/
2. High School: Modeling. (n.d.). Retrieved July 2, 2019, from http://www.corestandards.org/Math/Content/HSM/
3. Kelemanik, G., & Lucenta, A. (2008). Retrieved July 2, 2019, from http://www.fosteringmathpractices.com/wp-content/uploads/2019/05/Three-Reads-Instructional-Routine-Planner-2019.pdf

Keywords

Geometry
Circumcenter
Triangles

Links to Files in this ETP

Google Slides to introduce lesson
Prompt
Student handout (deconstructing problem and solution)
Student handout (scaffolding for understanding graphical solution) / Slides
Student handout (scaffolding for algebraic solution) / Key
Student handout: Code / Solution
Rubric for assessment (assess objectives and standard)