Roll No…………………….. **Total No. of Printed Pages: 1**

Total No. of Questions: [09]

B.Sc. (Hons) Math (Semester -1st)
ALGEBRA-1
Subject Code: BMATS1-122
Paper ID: [22131202]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a) Show that a group in which every element is its own inverse is an abelian group.
b) Show that n-nth roots of unity form a cyclic group under multiplication.
c) Define order of a group .Write order of an infinite group.
d) Define subgroup. Write two properties of it.
e) Show that Ha= Hb iff $ab^{-1} \epsilon H$.
f) Every cyclic group is abelian. Jistify
g) Prove that intersection of two normal subgroups is a normal subgroup.
h) Show that product of two odd permutations is an even permutation.
i) Define homomorphism, Isomorphism and automorphism of a group.
j) State Cayley's theorem.

Section – B **(5 marks each)**

Q2. If H & K are two subgroups of a group G, then HK is a subgroup of G iff HK=KH. Justify

Q3. If F is a homomorphism of a group G into group G' with kernel K, Then K is a normal Subgroup of G. Justify

Q4. State and Prove Lagrange's theorem.

Q5. State and Prove Freshman's theorem.

Q6. Show that the set A_n of all even permutations of degree n forms a finite group of order $\frac{n!}{2}$ with respect to permutation multiplication.

Section – C **(10 marks each)**

Q7. Let G be a finite group of order $n=p^kq$ $(k\geq1)$where p is a prime number and q is positive Integer such that (p, q) =1, then for each i $(1\leq i\leq k)$, G has at least one subgroup of order p^i.

Q8. State and rove Fermat's theorem.

Q9. Let a, b and x be any element of group G. Then prove that

i) $O(a^{-1}) = O(a)$

ii) $O(a) = O(x^{-1}ax)$