Hoboken Public Schools

Mathematics Curriculum
Grade 7

Updated 25-26

Mathematics Grade 7 HOBOKEN PUBLIC SCHOOLS

Course Description

This Grade 7 math curriculum reflects the **2023 New Jersey Student Learning Standards** and is designed to give students the requisite skills for all future mathematics courses through situational problem solving. In this course, students will focus on five domains of mathematics:

- Ratio and Proportional Relationships
- The Number System
- Expressions and Equations
- Geometry
- Statistics and Probability

Within each domain are standards, which are further broken down into clusters. The standard's language within each domain defines what students should be able to do. Below is the language of the standards, topics, and skills for Grade 7 Mathematics.

Grade 7 Standard	Topic	This topic focuses on...
7.NS.A	Rational Number Operations	Applying and extending previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
7.RP.A	Analyze and Use Proportional Relationships	Analyzing proportional relationships and using them to solve real-world and mathematical problems.
	Analyze and Solve Percent Problems	
7.EE.A	Generate Equivalent Expressions	Using properties of operations to generate equivalent expressions.
7.EE.B	Solve Problems Using Equations and Inequalities	Solving real-life and mathematical problems using numerical and algebraic expressions and equations.
7.SP.A	Use Sampling to Draw Inferences About Populations	Using random sampling to draw inferences about a population.
7.SP.B		Drawing informal comparative inferences about two populations.
7.SP.C	Probability	Investigating chance processes and developing, using, and evaluating probability models.

7.G.A	Solve Problems Involving Geometry	Drawing, constructing and describing geometrical figures and describing the relationships between them.
7.G.B		Solving real-life and mathematical problems involving angle measure, area, surface area, and volume.

In addition to the content standards in Grade 7 Mathematics, students will continue to develop their level of expertise with the Standards for Mathematical Practice.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The progression in Grade 7 Mathematics learning includes the Concrete, Representational, Abstract (CRA) Instructional Model. This progression enhances the eventual abstraction of mathematical concepts by allowing students to progress from concrete modeling with hands-on materials to two-dimensional pictures and representations to abstract thinking with symbols, numbers and procedures.

Course Resources

- enVision Math 2024 (available in English & Spanish)
- Desmos Classroom
- LinkIt! Assessments
- NJ Student Learning Standards in Mathematics
- Common Core Mathematics Standards
- Achieve the Core - Focus Areas
- Khan Academy
- NJCTL.org
- Illustrative Mathematics
- Connected Mathematics 3 (CMP3)

Integrated Course Accommodations and Modifications

- **Students with Disabilities** *(specific accommodations, instructional adaptations, and/or modifications are determined by the child's IEP or 504)*
    - Provide graphic organizers for additional support or encourage students to create digital multimedia to showcase knowledge of mathematical concepts.

    - Provide extended time for revisions or opportunity to identify and develop areas of personal interest.
    - Encourage creative expression and thinking by allowing students to choose how to approach a problem or assignment.
    - Allow extended time to complete classwork and homework assignments where necessary
    - Preferential seating mutually determined by student and teacher
    - Use of computer to complete assignments
    - Unique Learning System enables educators in self-contained classes to deliver differentiated, standards-aligned content from one convenient, cloud-based platform enhanced by robust assessments, data tools, and evidence-based instructional support. Students from pre-K through transition have the advantage of consistent, high-quality instruction, a motivating interactive learning environment, engaging symbol support, and a path to independence.
- **English Language Learners**
    - Invite students to explore different points of view on a particular mathematical standard and compare.
    - Pair visual prompts with verbal presentations
    - Model for mastery
    - Extended time for assignment completion as necessary
    - Highlight key vocabulary
    - Use graphic organizers
    - Provided translated materials and audio versions of text when applicable *(Digital Spanish version of enVision textbook available on Savvas platform)*
- **Skills Fragile Students**
    - Encourage students to make transformations - use a common task or item in a different way
    - Provide graphic organizers for additional support or encourage students to create digital multimedia to showcase knowledge of mathematical concepts.
    - Provide extended time for revisions or opportunity to identify and develop areas of personal interest.
    - Preferential seating determined by teacher and student
- **Gifted & Talented**
    - Encourage students to explore concepts in depth and encourage independent studies or investigations.
    - Provide enrichment activities that include more advanced material
    - Use of Higher Level Questioning Techniques
    - Allow team-teaching opportunities

Integration of NJSLS Standard 9 Career Readiness, Life Literacies & Key Skills

- 9.4.8.CI.2: Repurpose an existing resource in an innovative way
- 9.4.8.CI.3: Examine challenges that may exist in the adoption of new ideas
- 9.4.8.CI.4: Explore the role of creativity and innovation in career pathways and industries.

- 9.4.8.CT.1: Evaluate diverse solutions proposed by a variety of individuals, organizations, and/or agencies to a local or global problem, such as climate change, and use critical thinking skills to predict which one(s) are likely to be effective
- 9.4.8.CT.2: Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option
- 9.4.8.CT.3: Compare past problem-solving solutions to local, national, or global issues and analyze the factors that led to a positive or negative outcome.
- 9.4.8.DC.3: Describe tradeoffs between allowing information to be public (e.g., within online games) versus keeping information private and secure.
- 9.4.8.DC.4: Explain how information shared digitally is public and can be searched, copied, and potentially seen by public audiences.
- 9.4.8.DC.5: Manage digital identity and practice positive online behavior to avoid inappropriate forms of self-disclosure.
- 9.4.8.DC.6: Analyze online information to distinguish whether it is helpful or harmful to reputation
- 9.4.8.DC.7: Collaborate within a digital community to create a digital artifact using strategies such as crowdsourcing or digital surveys.
- 9.4.8.DC.8: Explain how communities use data and technology to develop measures to respond to effects of climate change (e.g., smart cities).
- 9.4.8.GCA.1: Model how to navigate cultural differences with sensitivity and respect
- 9.4.8.GCA.2: Demonstrate openness to diverse ideas and perspectives through active discussions to achieve a group goal.
- 9.4.8.IML.3: Create a digital visualization that effectively communicates a data set using formatting techniques such as form, position, size, color, movement, and spatial grouping
- 9.4.8.IML.4: Ask insightful questions to organize different types of data and create meaningful visualizations.
- 9.4.8.IML.5: Analyze and interpret local or public data sets to summarize and effectively communicate the data
- 9.4.8.TL.1: Construct a spreadsheet in order to analyze multiple data sets, identify relationships, and facilitate data-based decision-making.
- 9.4.8.TL.2: Gather data and digitally represent information to communicate a real-world problem
- 9.4.8.TL.3: Select appropriate tools to organize and present information digitally.
- 9.4.8.TL.4: Synthesize and publish information about a local or global issue or event.
- 9.4.8.TL.5: Compare the process and effectiveness of synchronous collaboration and asynchronous collaboration.
- 9.4.8.TL.6: Collaborate to develop and publish work that provides perspectives on a real-world problem

Course Pacing Guide by Unit

Unit #	Title	Standards	Timeframe (Approximately)
1	Rational Number Operations	7.NS.1a, 7.NS.1b, 7.NS.1c, 7.NS.1d, 7.NS.2a, 7.NS.2b, 7.NS.2c, 7.NS.2d, 7.NS.3, 7.EE.3	September-October
2	Analyze and Use Proportional Relationships	7.RP.1, 7.RP.2a, 7.RP.2b, 7.RP.2c, 7.RP.2d, 7.RP.3	October-November
3	Analyze and Solve Percent Problems	7.RP.2a, 7.RP.2b, 7.RP.2c, 7.RP.3	November-December
4	Generate Equivalent Expressions	7.EE.1, 7.EE.2 7.EE.3, 7.EE.4a	December-January
5	Solve Problems Using Equations and Inequalities	7.EE.3, 7.EE.4a, 7.EE.4b	January-February
6	Use Sampling to Draw Inferences About Populations	7.RP.2c, 7.EE.3, 7.SP.1, 7.SP.2, 7.SP.3, 7.SP.4	February-March
7	Probability	7.RP.2c, 7.EE.3, 7.SP.5, 7.SP.6, 7.SP.7a, 7.SP.7b, 7.SP.8a, 7.SP.8b, 7.SP.8c	March-April
8	Solve Problems Involving Geometry	7.NS.3, 7.EE.3, 7.EE.4a, 7.G.1, 7.G.2, 7.G.3, 7.G.4, 7.G.5, 7.G.6	April-May
9	Automation & Robotics (PLTW)	8.2	May-June

Climate Change standards are denoted with this symbol.

Unit 1 - Rational Number Operations
Timeframe: September-October
Overview

Using number lines, students visualize adding and subtracting integers and examine the inverse relationship between integers and their opposites. Students will extend this understanding to apply to multiplying integers and dividing integers. Students will identify rational numbers and write them in equivalent fraction forms and learn about repeating and terminating decimals. Students will also perform operations with rational numbers. Students will use their understanding of rational numbers to solve problems in real world contexts involving operations with rational numbers.

Curricular Sequence

- Looking Back
    - *In 6th grade, students learned to:*
        - Graph integers and their opposites on a number line to understand their distances from zero.
        - Use positive and negative numbers in context relating to their distance from zero.
        - Perform operations with decimals and fractions by following an algorithm or utilizing a number line.
        - Recognize that opposite signs in rational numbers indicate locations on opposite sides of zero on the number line.
- Looking Ahead
    - *In 8th grade, students will:*
        - Use operations with positive and negative integers and rational numbers to solve equations by using inverse relationships and operation rules.
        - Use operations with positive and negative integers and rational numbers to find solutions for systems of linear equations using algebraic models.
        - Examine the use of square and cube roots and discover that square roots of negative numbers cannot be found in the real number system.

Essential Questions

1. How do operations with integers relate to the same operations with rational numbers?
2. How can you determine the correct operation to use to solve problems?

Essential Learning Outcomes

- *At the conclusion of this unit, students will be able to:*
    - Understand how integers and their opposites are related.
    - Identify rational numbers and write them in decimal form.
    - Add positive and negative integers.
    - Model integer addition in real-life applications.
    - Understand subtraction of integers as adding the additive inverse, $p - q = p + (-q)$
    - Use properties of operations to add and subtract rational numbers.

- Multiply positive and negative integers.
- Apply integer multiplication to real-life applications.
- Find the product of rational numbers.
- Understand how to divide integers by applying the rules of multiplying integers.
- Determine equivalencies among integer quotients.
- Understand how the signs of integers in a multiplication sentence relate to the signs in a related division statement.
- Decide which operations to use to solve problems.
- Use precision when solving problems with rational numbers.

Mathematics Standards

- 7.NS.A.1a Describe situations in which opposite quantities combine to make 0. For example, in the first round of a game, Maria scored 20 points. In the second round of the same game, she lost 20 points. What is her score at the end of the second round?
- 7.NS.A.1b Understand as the number located a distance from, in the positive or negative direction depending on whether *q* is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
- 7.NS.A.1c Understand subtraction of rational numbers as adding the additive inverse, . Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
- 7.NS.A.1d Apply properties of operations as strategies to add and subtract rational numbers.
- 7.NS.A.2 Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
- 7.NS.A.2b, Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If and are integers, then . Interpret quotients of rational numbers by describing real world contexts.
- 7.NS.A.2c Apply properties of operations as strategies to multiply and divide rational numbers.
- 7.NS.A.2d Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
- 7.NS.A.3, Solve real-world and mathematical problems involving the four operations with rational numbers. (Clarification: Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)
- 7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar inches long in the center of a door that is inches wide, you will need to place the bar about 9 inches from each edge; this

estimate can be used as a check on the exact computation.

Suggested Supplemental Resources

- Illustrative Mathematics Performance Tasks:
  - Bookstore Account
  - Comparing Freezing Points
  - Differences and Distances
  - Differences of Integers
  - Distances Between Houses
  - Distances on the Number Line 2
  - Operations on the number line
  - Rounding and Subtracting
- CMP3 Investigations:
  - Accentuate the Negative: 1.1 Playing Math Fever: Using Positive and Negative Numbers, 1.2 Extending the Number Line, 1.3 From Sauna to Snowbank: Using a Number Line, 1.4 In the Chips: Using a Chip Model, 2.3 The "+/ –" Connection, 3.4 Playing the Integer Product Game: Applying Multiplication and Division of Integers, 4.1 Order of Operations ,4.2 The Distributive Property, 4.3 What Operations Are Needed?

Integration of Financial Literacy Standards (NJSLS 9.1)

- 9.1.8.CP.2: Analyze how spending habits affect one's ability to save.
- 9.1.8.FP.1: Describe the impact of personal values on various financial scenarios.
- 9.1.8.FP.3: Explain how self-regulation is important to managing money (e.g., delayed gratification, impulse buying, peer pressure, etc.).

Integration of Career Awareness Standards (NJSLS 9.2)

- 9.2.8.CAP.3: Explain how career choices, educational choices, skills, economic conditions, and personal behavior affect income.

Integration of Civics (NJSLS - Social Studies)

- 6.3.8.CivicsPR.4: Use evidence and quantitative data to propose or defend a public policy related to climate change.

Integration of Diversity, Equity, and Inclusion (N.J.S.A. 18A:35-4.36a): (See Appendix A)

- Amistad: Students explore the ancient African origins of mathematical reasoning—such as the Ishango Bone (early counting tool from the Congo Basin)—and compare early numeric systems to modern integers and rational numbers.

Interdisciplinary Connections
Social Studies

- **Example:** Trace how African innovations in mathematics influenced trade, engineering, and architecture across ancient civilizations.

Unit 2 - Analyze and Use Proportional Relationships
Timeframe: **October-November**
Overview

Using ratio tables, students generate equivalent ratios for rates with rational terms. Students compute unit rates for ratios with rational terms by interpreting the rate as division and applying the standard algorithm for division of fractions. Students also will interpret unit rates to solve multi-step problems. Students will use verbal descriptions, tables, and diagrams to determine if quantities are in a proportional relationship and select an appropriate solving strategy for real world problems.

Curricular Sequence

- Looking Back
    - *In 6th grade, students learned to:*
        - Reason about ratios by using equivalent ratios, tables of equivalent ratios, bar diagrams, and double-numbered line diagrams.
        - Use special types of ratios called percentages.
        - Use special types of ratios called rates.
- Looking Ahead
    - *In 8th grade, students will:*
        - Understand the connections among proportional relationships, lines, and linear equations.
        - Graph proportional relationships and compare proportional relationships represented in different ways.
        - Use similar triangles to investigate slope, and derive equations such as $y = mx$ and $y = mx + b$.

Essential Questions

1. How can you recognize and represent proportional relationships and use them to solve problems?

Essential Learning Outcomes

- *At the conclusion of this unit, students will be able to:*
    - Use ratios and rates to describe the relationship between two quantities.
    - Find equivalent ratios and use unit rates to solve problems.
    - Find Unit rates with ratios of fractions.
    - Use unit rates to solve multi-step problems.
    - Determine whether quantities are proportional by testing for equivalent ratios.

- Use the constant of proportionality to write equations that represent proportional relationships.
- Use equations to solve problems involving proportional relationships.
- Use a graph to recognize proportionality.
- Identify a constant of proportionality from a graph.
- Interpret a point on a graph of a proportional relationship.
- Explain whether a situation represents a proportional relationship.
- Use representations to find entry points into problems.

Mathematics Standards

- 7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks a mile in each hour, compute the unit rate as the complex fraction miles per hour, equivalently 2 miles per hour.
- 7.RP.A.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- 7.RP.A.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- 7.RP.A.2c Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as .
- 7.RP.A.2d Explain what a point on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and where the unit rate is.
- 7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Suggested Supplemental Resources

- Illustrative Mathematics Performance Tasks:
  - Climbing the steps of El Castillo
  - Drill Rig
  - Dueling Candidates
  - Perfect Purple Paint II
  - Sale!
  - Stock Swaps, Variation 2
  - Stock Swaps, Variation 3
  - Temperature Change
  - Thunder and Lightning
  - Track Practice
- CMP3 Investigations:
  - Comparing & Scaling: 1.1 Surveying Opinions: Analyzing Comparison Statements, 1.2 Mixing Juice: Comparing Ratios, 1.3 Time to Concentrate:

Scaling Ratios, 1.4 Keeping Things in Proportion: Scaling to Solve Proportions, 2.1 Sharing Pizza: Comparison Strategies, 2.2 Comparing Pizza Prices: Scaling Rates, 2.3 Finding Costs: Unit Rate and Constant of Proportionality

Integration of Financial Literacy Standards (NJSLS 9.1)

- 9.1.8.CP.1: Compare prices for the same goods or services.
- 9.1.8.FP.6: Compare and contrast advertising messages to understand what they are trying to accomplish.

Integration of Career Awareness Standards (NJSLS 9.2)

- 9.2.8.CAP.15: Present how the demand for certain skills, the job market, and credentials can determine an individual's earning power.

Integration of Diversity, Equity, and Inclusion (N.J.S.A. 18A:35-4.36a): (See Appendix A)

- Amistad: Through proportional reasoning, students analyze data on the transatlantic trade economy to understand the mathematical impact of supply, demand, and human labor systems. Students create ratio tables to model population and economic change.

Interdisciplinary Connections
Social Studies:

- **Example:** Evaluate historical trade networks and the effects on African and American societies, connecting mathematics to historical data analysis.

Unit 3 - Analyze and Solve Percent Problems
Timeframe: November-December
Overview

By analyzing the connection between proportions and percentages, students will use algebraic expressions and equations to solve real-world percent problems. Students will understand that there can be percents greater than 100% and less than 1%, or a non-whole number. Students will apply proportional reasoning to solve a percent equation. Students will determine whether a situation represents a percent increase or decrease, and to calculate the percent change. Students will also apply their prior knowledge of absolute value to solve percent error problems. Students will use the percent equation as it relates to markup, markdown, sales tax, and simple interest.

Curricular Sequence

- Looking Back
    - *In 6th grade, students learned to:*
        - Students learned to reason about ratios by using equivalent ratios and tables of equivalent ratios.
        - Work with special types of ratios called percentages.
        - Work with special types of ratios called rates.

- Looking Ahead
    - *In 8th grade, students will:*
        - Utilize the connections between percentages and proportions to scale drawings.
        - Create corresponding scale drawings.
        - Determine the scale factor as a percent, when given the original drawing and the scale of the drawing.
        - Use scale factor to differentiate between reduction and enlargement.
        - Use similar triangles to investigate slopes, and develop equations for lines.

Essential Questions

1. How can percents show proportional relationships between quantities and be used to solve problems?

Essential Learning Outcomes

- *At the conclusion of this unit, students will be able to:*
    - Understand that equivalent rates can be used to find percentages.
    - Analyze percentages of numbers in a real-world context.
    - Construct a percent proportion.
    - Use a percent proportion to find an unknown part, whole, or percent.
    - Understand the relationship between proportional reasoning and percent.
    - Interpret the results of a percent equation in a real-life scenario.
    - Solve real world-problems involving percent change and percent error.
    - Understand the percent equation and the different ways it can be used.
    - Understand and calculate markups and markdowns.
    - Relate percent change to percent markup and percent markdown.
    - Identify the parts of interest problems and how the values are related.
    - Understand what simple interest is and how it is calculated.

Mathematics Standards

- 7.RP.A.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- 7.RP.A.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- 7.RP.A.2c Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as .
- 7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Suggested Supplemental Resources

- Illustrative Mathematics Performance Tasks:
  - Anna in D.C.
  - Buying Protein Bars and Magazines
  - Chess Club
  - Comparing Years
  - Double Discounts
  - Finding a 10% increase
  - Friends Meeting on Bikes
  - Gotham City Taxis
  - How Fast is Usain Bolt?
  - Lincoln's math problem
  - Music Companies, Variation 2
  - Sand Under the Swing Set
  - Selling Computers
  - Tax and Tip
  - The Price of Bread
  - Two-School Dance
- CMP3 Investigations:
  - Comparing and Scaling: 3.1 Commissions, Markups, and Discounts: Proportions With Percents, 3.2 Measuring to the Unit: Measurement Conversions, 3.3 Mixing It Up: Connecting Ratios, Rates, Percents, and Proportions

Integration of Financial Literacy Standards (NJSLS 9.1)

- 9.1.8.CP.2: Analyze how spending habits affect one's ability to save.
- 9.1.8.FP.3: Explain how self-regulation is important to managing money (e.g., delayed gratification, impulse buying, peer pressure, etc.).

Integration of Career Awareness Standards (NJSLS 9.2)

- 9.2.8.CAP.3: Explain how career choices, educational choices, skills, economic conditions, and personal behavior affect income.
- 9.2.8.CAP.15: Present how the demand for certain skills, the job market, and credentials can determine an individual's earning power.

Integration of Diversity, Equity, and Inclusion (N.J.S.A. 18A:35-4.36a): (See Appendix A)

- Persons with Disabilities: Students analyze how accessibility costs and modifications (ramps, elevators, adaptive technologies) can be represented as percentages of overall budgets. They calculate percent increases in inclusive design initiatives for public buildings.

Interdisciplinary Connections
STEM / Civics:

- **Example:** Examine architectural blueprints and ADA design standards, connecting percent applications to community accessibility improvements.

Unit 4 - Generate Equivalent Expressions
Timeframe: **December-January**
Overview

Using the context of real-life situations, students write and evaluate algebraic expressions. Students will use the Associative and Commutative Properties to reorder and group like terms. Students will build on their knowledge of common factors to factor algebraic expressions and use area models and common factors to factor expressions. Students will add and subtract expressions with rational coefficients.

Curricular Sequence

- Looking Back
    - *In 6th grade, students learned to:*
        - Read and interpret parts of an expression by using mathematical terms and viewing expressions as single entities.
        - Write and evaluate expressions.
        - Identify equivalent expressions by applying the properties of operations.
- Looking Ahead
    - *In 8th grade, students will:*
        - Analyze and solve linear equations in the form of $y = mx$ and $y = mx + b$.
        - Combine like terms and solve equations with variables on both sides.
        - Solve systems of linear equations by graphing, substituting, and eliminating.
        - Use properties of radicals and integer exponents to generate equivalent expressions.

Essential Questions

1. How can properties of operations help to generate equivalent expressions that can be used in solving problems?

Essential Learning Outcomes

- *At the conclusion of this unit, students will be able to:*
    - Understand how variables are used to represent unknown values in problems.
    - Recognize when two expressions are equivalent.
    - Use properties of operations to write equivalent expressions.
    - Combine like integer and rational terms.
    - Use the Distributive Property to expand expressions.

- Understanding expanding expressions is the reverse of factoring.
- Identify the GCF of algebraic terms in expressions.
- Use properties of operations to add expressions.
- Model addition of expressions in real-life applications.
- Use properties of operations to subtract expressions.
- Model subtraction of expressions in real-life applications.
- Write equivalent expressions to show how quantities are related in real-life applications.

Mathematics Standards

- 7.EE.A.1, Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
- 7.EE.A.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, means that “increase by 5%” is the same as “multiply by 1.05.”
- 7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar inches long in the center of a door that is inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.
- 7.EE.B.4a, Solve word problems leading to equations of the form and , where, , and are specific rational numbers. Solve equations of these forms with accuracy and efficiency. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

Suggested Supplemental Resources

- Illustrative Mathematics Performance Tasks:
  - Equivalent Expressions?
  - Miles to Kilometers
  - Writing Expressions
  - Ticket to Ride
- CMP3 Investigations
  - Moving Straight Ahead: 3.1 Solving Equations Using Tables and Graphs, 3.2 Mystery Pouches in the Kingdom of Montarek: Exploring Equality, 3.3 From Pouches to Variables: Writing Equations, 3.4 Solving Linear Equations

Integration of Financial Literacy Standards (NJSLS 9.1)

- 9.1.8.FP.1: Describe the impact of personal values on various financial scenarios.

Integration of Career Awareness Standards (NJSLS 9.2)

- 9.2.8.CAP.12: Assess personal strengths, talents, values, and interests to appropriate jobs and careers to maximize career potential.

Integration of Civics (NJSLS - Social Studies)

- 6.3.8.CivicsPR.4: Use evidence and quantitative data to propose or defend a public policy related to climate change.

Unit 5 - Solve Problems Using Equations and Inequalities
Timeframe: **January-February**
Overview

Using variables and the relationship between the coefficient and the constant, students will move from a visual representation using bar diagrams to the abstract representation using variables. Students will make connections between arithmetic and algebraic solutions and manipulate equations to solve for an unknown variable. Students will extend their understanding of solving equations to solve inequalities and explore inverse relationships. Students will write and solve two-step and multi-step inequalities that represent real-world applications.

Curricular Sequence

- Looking Back
    - *In 6th grade, students learned to:*
        - Evaluate expressions.
        - Write and solve one-step equations and inequalities.
        - Represent and analyze the quantitative relationship between dependent and independent variables.
        - Apply the Distributive Property.
        - Identify equivalent expressions using the Distributive Property and the properties of operations.
        - Use graphs, tables, and number lines to model expressions, equations, and inequalities.
- Looking Ahead
    - *In 8th grade, students will:*
        - Extend their understanding of expressions, equations, and inequalities to equations that contain two variables.
        - Make connections between models and equations including proportional relationships, lines, and linear equations.

Essential Questions

1. How can you solve real-world problems and mathematical problems with numerical and algebraic equations and inequalities?

Essential Learning Outcomes

- *At the conclusion of this unit, students will be able to:*
    - Analyze word problems to write two-step equations.
    - Understand the relationship between the terms of the equation and the values they represent.
    - Use models to solve two-step equations.
    - Compare algebraic and arithmetic solutions.
    - Solve equations using the Distributive Property.
    - Graph the solution of inequalities on a number line.
    - Solve inequalities using the Addition and Subtraction Properties of Inequality.
    - Write inequalities and solve them using Multiplication and Division Properties of Inequality.
    - Graph the solutions of an inequality on a number line.
    - Write and solve two-step inequalities to solve a problem.
    - Solve an inequality by multiplying or dividing by a negative rational number.
    - Explore the relationship between two-step inequalities and multi-step inequalities.
    - Apply the Distributive Property to simply and solve multi-step inequalities.

Mathematics Standards

- 7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar inches long in the center of a door that is inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.
- 7.EE.B.4a, Solve word problems leading to equations of the form and , where, , and are specific rational numbers. Solve equations of these forms with accuracy and efficiency. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?
- 7.EE.B.4b Solve word problems leading to inequalities of the form or , where , , and are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.

Suggested Supplemental Resources

- Illustrative Mathematics Performance Tasks:
    - Guess My Number
    - Bookstore Account
    - Drill Rig

    - Fishing Adventures 2
    - Gotham City Taxis
    - Sports Equipment Set
- CMP3 Investigations:
    - Moving Straight Ahead:3.5 Finding the Point of Intersection: Equations and Inequalities

Integration of Financial Literacy Standards (NJSLS 9.1)

- 9.1.8.FP.2: Evaluate the role of emotions, attitudes, and behavior (rational and irrational) in making financial decisions.
- 9.1.8.FP.7: Identify the techniques and effects of deceptive advertising.

Integration of Career Awareness Standards (NJSLS 9.2)

- 9.2.8.CAP.3: Explain how career choices, educational choices, skills, economic conditions, and personal behavior affect income.

Integration of Civics (NJSLS - Social Studies)

- 6.3.8.CivicsPR.4: Use evidence and quantitative data to propose or defend a public policy related to climate change.

Integration of Diversity, Equity, and Inclusion (N.J.S.A. 18A:35-4.36a): (See Appendix A)

- Amistad / African- American History: Students learn about **Katherine Johnson,** the African American mathematician whose calculations helped NASA send astronauts into orbit and safely return. Using equations and inequalities, students solve rate-time-distance problems to simulate aspects of space travel—such as determining when two spacecraft meet in orbit or comparing ascent paths.

Interdisciplinary Connections

STEM / Science:

- **Example:** Connects mathematics to space exploration and the physics of motion. Students discuss how algebraic modeling and precision are essential in scientific careers, linking African American historical contributions to modern STEM learning.

Unit 6 - Using Sampling to Draw Inferences About Populations
Timeframe: **February-March**
Overview

Using the concept of a population, students focus on how a sample, or subset, of a population can yield information about the entire population. Students will learn the best way to characterize a population and understand generating a random sample. Students will extend previous concepts of data displays and descriptive statistical measures to draw informal conclusions. Students combine their previous knowledge of ratios and proportional reasoning with their knowledge of drawing inferences to make predictions about populations. Using different data displays and their appropriate measures of center and spread, students will compare populations and make inferences.

Curricular Sequence

- Looking Back
    - *In 6th grade, students learned to:*
        - Use dot plots and extend their understanding of data displays by using histograms and box plots.
        - Describe data sets in terms of symmetry, skew, and outliers.
        - Identify clusters, peaks, and gas in data sets.
        - Apply measures of center (mean, median, mode) and spread (range, IQR, MAD) to summarize data displays.
- Looking Ahead
    - *In 8th grade, students will:*
        - Extend their understanding of sample data sets and data displays to include bivariate data sets and plot them on a scatter plot instead of a histogram, box plot, or dot plot.
        - Apply the concepts of clusters, gaps, and outliers to bivariate data in a scatter plot.
        - Extend their understanding of measures of center and spread by using a trend line to determine if there is an association between bivariate data on a scatter plot.
        - Use the equation of a linear model as a measure of center, and the distance of each data point from the line as a measure of variation.

Essential Questions

1. How can sampling be used to draw inferences about one or more populations?

Essential Learning Outcomes

- *At the conclusion of this unit, students will be able to:*
    - Distinguish between a population and a sample.
    - Establish whether a sample is representative of a population.
    - Generate random samples.
    - Make qualitative inferences from a sample data set.
    - Make quantitative inferences from a sample data set.
    - Make estimates about a population based on a sample data set, and assess whether the inferences are valid.
    - Use box plots to compare and make inferences about populations.
    - Use the median and IQR of data sets to informally compare and make inferences about two populations.
    - Use the mode, range, mean, and mean absolute deviation (MAD) to compare populations.

Mathematics Standards

- 7.RP.A.2c Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as .
- 7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar inches long in the center of a door that is inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.
- 7.SP.A.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
- 7.SP.A.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.
- 7.SP.B.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the

soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.
- 7.SP.B.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.

Suggested Supplemental Resources
- Illustrative Mathematics Performance Tasks:
  - Election Poll, Variation 1
  - Election Poll, Variation 2
  - Election Poll, Variation 3
  - Estimating the Mean State Area
  - College Athletes
  - Offensive Linemen
- CMP3 Investigations:
  - Samples & Populations: 2.1 Asking About Honesty: Using a Sample to Draw Conclusions, 2.2 Selecting a Sample: Different Kinds of Samples, 2.3 Choosing Random Samples: Comparing Samples Using Center and Spread, 2.4 Growing Samples: What Size Sample to Use?, 2.1 Asking About Honesty: Using a Sample to Draw Conclusions, 2.2 Selecting a Sample: Different Kinds of Samples, 2.3 Choosing Random Samples: Comparing Samples Using Center and Spread, 2.4 Growing Samples: What Size Sample to Use?

Integration of Financial Literacy Standards (NJSLS 9.1)
- 9.1.8.CR.2: Compare various ways to give back through strengths, passions, goals, and other personal factors

Integration of Career Awareness Standards (NJSLS 9.2)
- 9.2.8.CAP.10: Evaluate how careers have evolved regionally, nationally, and globally.

Integration of Diversity, Equity, and Inclusion (N.J.S.A. 18A:35-4.36a): (See Appendix A)
- Holocaust: Students investigate how **bias in sampling** can lead to inaccurate or unfair conclusions—mirroring how prejudice and stereotypes distort understanding of groups in society. I. Students are given several small “data samples” that describe fictional populations (e.g., students’ interests, participation, or talents). II. Some samples are **representative** (diverse, balanced), while others are **biased** (missing or overrepresenting certain groups). III. Using measures of center (mean, median, mode) and variability, students analyze how the biased samples give misleading results compared to the full population. IV. Students then discuss: *How can collecting or reporting biased data reinforce stereotypes or unfair judgments?*

Interdisciplinary Connections
Social-Emotional Learning / Civics

- **Example:** Students connect their statistical findings to the idea that bias in data and bias in thinking both distort truth. They reflect on how fair sampling—listening to all voices—leads to more accurate understanding in science, history, and society.

Unit 7 - Probability
Timeframe: **March-April**
Overview

Using probability to examine fairness, students analyze how likely an event is to occur. Students will compare and contrast theoretical and experimental probability and construct probability models through predictions, estimations, and evaluations of situations. Students will explore all possible outcomes of a compound event and learn the concept of sample space. Using a spinner, a coin, or a random number generator, students will learn to simulate compound events and assign probabilities to the different outcomes in order to determine the probability of compound events.

Curricular Sequence

- Looking Back
    - *In 6th grade, students learned to:*
        - Examine data sets to determine statistical measures.
        - Make predictions by analyzing statistical measures.
        - Write fractions as decimals and decimals as fractions.
- Looking Ahead
    - *In 8th grade, students will:*
        - Find probabilities of simple and compound events.
        - Extend this knowledge to finding probabilities and making inferences and predictions using a two-way frequency table.

Essential Questions

1. How can you investigate chance processes and develop, use, and evaluate probability models?

Essential Learning Outcomes

- *At the conclusion of this unit, students will be able to:*
    - Use probability to describe the likelihood that an event will occur.
    - Relate probability to mathematical fairness.
    - Understand theoretical probability and how it can be used.
    - Use theoretical probability to predict an outcome.
    - Compare theoretical and experimental probability.
    - Use experimental probability to make predictions.
    - Explain the differences between theoretical and experimental probability.
    - Develop a probability model.

- Use a probability model to evaluate a situation.
- Use a probability model to make an estimate.
- Use a tree diagram, a table, or an organized list to represent the sample space for a compound event.
- Organize information about a compound event on a tabe, a tree diagram, or an organized list.
- Find the probability of a compound event.
- Use different tools to simulate a compound event.
- Model a real-world situation involving a compound event and predict its outcome using a situation.

Mathematics Standards

- 7.RP.A.2c Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as .
- 7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar inches long in the center of a door that is inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.
- 7.SP.C.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
- 7.SP.C.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.
- 7.SP.C.7a Develop a uniform probability model by assigning equal probability to all outcomes and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.
- 7.SP.C.7b Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?
- 7.SP.C.8a Understand that, just as with simple events, the probability of a compound

event is the fraction of outcomes in the sample space for which the compound event occurs.
- 7.SP.C.8b Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.
- 7.SP.C.8c Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

Suggested Supplemental Resources
- Illustrative Mathematics Performance Tasks:
  - Stay or Switch?
  - Heads or Tails
  - Rolling Dice
  - Tossing Cylinders
  - Red, Green, or Blue?
  - Rolling Twice
  - Waiting Times
  - Sitting across from Each Other
  - Tetrahedral Dice
- CMP3 Investigations:
  - What Do You Expect?: 1.1 Choosing Cereal: Tossing Coins to Find Probabilities, 1.2 Tossing Paper Cups: Finding More Probabilities, 1.3 One More Try: Finding Experimental Probabilities, 1.4 Analyzing Events, 2.1 Predicting to Win: Finding Theoretical Probabilities, 2.2 Choosing Marbles: Developing Probability Models, 2.3 Designing a Fair Game: Pondering Possible and Probable, 2.4 Winning the Bonus Prize: Using Strategies to Find Theoretical Probabilities, 3.1 Designing a Spinner to Find Probabilities, 3.2 Making Decisions: Analyzing Fairness, 3.3 Roller Derby: Analyzing a Game, 3.4 Scratching Spots: Designing and Using a Simulation

Integration of Financial Literacy Standards (NJSLS 9.1)
- 9.1.8.FP.6: Compare and contrast advertising messages to understand what they are trying to accomplish.

Integration of Career Awareness Standards (NJSLS 9.2)
- 9.2.8.CAP.4: Explain how an individual’s online behavior (e.g., social networking, photo exchanges, video postings) may impact opportunities for employment or advancement

Integration of Diversity, Equity, and Inclusion (N.J.S.A. 18A:35-4.36a): (See Appendix A)
- AAPI: Students explore probability models in traditional AAPI games and inventions, such as the mathematical structure of the Japanese game “Go” or early Chinese probability theory. They simulate events to understand strategic and logical thinking.

Interdisciplinary Connections
Cultural Studies / History:

- **Example:** Examine how mathematical concepts from Asian civilizations have influenced global understanding of chance and probability.

Unit 8 - Solve Problems Involving Geometry
Timeframe: **April-May**
Overview

Using scale drawings, students will extend their understanding of ratio reasoning to understand that the scale factor represents a constant of proportionality between the scale drawing and the actual object. Students will explore the relationship between the circumference and diameter of a circle, and find the surface area of three-dimensional objects as the sum of the areas of each of their surfaces. Students will draw figures with given conditions and explore the relationship between angles that are formed by intersecting lines, segments, and rays. Students will draw geometric figures with protractors, rulers, and geometry software given certain conditions.

Curricular Sequence

- Looking Back
    - *In 6th grade, students learned to:*
        - Draw polygons in the coordinate plane with given coordinates of the vertices.
        - Use coordinates to find distances between points on the coordinate plane, to find lengths, and to analyze polygons.
        - Solve problems involving the area of triangles and quadrilaterals.
        - Find the area of a variety of polygons by decomposing them into rectangles and triangles.
        - Find the volume of right rectangular prisms.
        - Find the surface area of three-dimensional objects.
- Looking Ahead
    - *In 8th grade, students will:*
        - Learn how to find the surface area of three-dimensional figures such as cylinders, cones, and spheres.
        - Understand the mathematical relationship between surface area and volume to solve real-world problems.
        - Learn the formulas to calculate the volume of cylinders, cones, and spheres.

Essential Questions

1. How can geometry be used to solve problems?

Essential Learning Outcomes

- *At the conclusion of this unit, students will be able to:*
    - Use a scale drawing as a representation of actual lengths and area.
    - Sketch quadrilaterals with given conditions.
    - Name and classify quadrilaterals according to their properties.
    - Construct triangles with given conditions.
    - Conclude whether or not a triangle is formed and what type of triangle it is.
    - Calculate the measures of angles by using angle relationships.
    - Calculate the circumference, radius, or diameter of a circle.
    - Recognize the relationship between the circumference and the diameter of a circle and π.
    - Find the area of a circle.
    - Use the area to find the radius and diameter.
    - Solve problems involving the area of a circle.
    - Describe cross sections of right rectangular prisms and pyramids.
    - Solve problems involving cross sections.
    - Find the surface area of two-dimensional composite shapes.
    - Calculate the volume of various three-dimensional figures.
    - Solve problems involving the volume of three-dimensional figures.

Mathematics Standards

- 7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers. (Clarification: Computations with rational numbers extend the rules for manipulating fractions to complex fractions.) 
- 7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar inches long in the center of a door that is inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation. 
- 7.EE.B.4a Solve word problems leading to equations of the form and , where , , and are specific rational numbers. Solve equations of these forms with accuracy and efficiency. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?
- 7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
- 7.G.A.2 Draw (with technology, with ruler and protractor, as well as freehand) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one

triangle, or no triangle.
- 7.G.A.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
- 7.G.B.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
- 7.G.B.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
- 7.G.B.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms 

Suggested Supplemental Resources

- Illustrative Mathematics Performance Tasks:
  - Approximating the area of a circle
  - Circumference of a Circle
  - Floor Plan
  - Map distance
  - Rescaling Washington Park
  - Scaling angles and polygons
  - Drinking the Lake
  - The Circumference of a Circle and the Area of the Region it Encloses
- CMP3 Investigations:
  - Stretching and Shrinking: 1.1 Solving a Mystery: An Introduction to Similarity, 1.2 Scaling Up and Down: Corresponding Sides and Angles, 2.1 Drawing Wumps: Making Similar Figures, 2.2 Hats Off to the Wumps: Changing a Figure's Size and Location, 2.3 Mouthing Off and Nosing Around: Scale Factors, 4.1 Ratios Within Similar Parallelograms, 4.2 Ratios Within Similar Triangles, 4.3 Finding Missing Parts: Using Similarity to Find Measurements, 4.4 Using Shadows to Find Heights: Using Similar Triangles
  - Filling & Wrapping: 1.1 How Big Are Those Boxes?: Finding Volume, 1.2 Optimal Containers I: Finding Surface Area, 1.3 Optimal Containers II: Finding the Least Surface Area, 1.4 Compost Containers: Scaling Up Prisms, 2.1 Folding Paper: Surface Area and Volume of Prisms, 2.2 Packing a Prism: Calculating Volume of Prisms, 2.3 Slicing Prisms and Pyramids, 3.1 Going Around in Circles: Circumference, 3.2 Pricing Pizza: Connecting Area, Diameter, and Radius, 3.3 Squaring a Circle to Find Its Area, 3.4 Connecting Circumference and Area

Integration of Financial Literacy Standards (NJSLS 9.1)

- 9.1.8.CP.1: Compare prices for the same goods or services.

Integration of Career Awareness Standards (NJSLS 9.2)

- 9.2.8.CAP.12: Assess personal strengths, talents, values, and interests to appropriate jobs and careers to maximize career potential.

Integration of Civics (NJSLS - Social Studies)

- 6.3.8.CivicsPR.4: Use evidence and quantitative data to propose or defend a public policy related to climate change.

Integration of Diversity, Equity, and Inclusion (N.J.S.A. 18A:35-4.36a): (See Appendix A)

- LGBTQ+: Students study the geometric art of **LGBTQ+ mathematician and artist Emma Haruka Iwao** (who calculated trillions of digits of π). They explore the geometry of circles, circumference, and area while celebrating representation in STEM.

Interdisciplinary Connections
STEM / Art Integration:

- **Example:** Connect geometry with digital art, coding, and inclusivity in technology professions.

PLTW Unit - Automation & Robotics
Timeframe: May - June
Overview

In the Automation and Robotics (AR) unit, students trace the history, development, and influence of automation and robotics as they learn about mechanical systems, energy transfer, machine automation, and computer control systems. Students use the VEX Robotics® platform to design, build, and program real-world objects such as traffic lights, toll booths, and robotic arms.

Resources

1. www.pltw.org

Essential Questions

- Why is it important for an engineer to document their work in an engineering notebook?
- How are our lives impacted by engineers?
- What inventions and innovations do you think are essential to your life?
- How does the use of technology affect the way that you live?
- How is the design process different from how you solve problems in the past?
- Why is brainstorming important when modifying or improving a product?
- Why do people work in teams when solving design problems?
- Why are design elements considered when engineers and designers invent or innovate a product?
- What limitations do you think should be placed on the use of robots?
- What type of robot do you think makes the most significant contribution to our lives today and why?

- What is the greatest concern that should be considered before converting a factory from human workforce to robotic workforce?
- What impact do you think robots will have on your life in 10 years and in 50 years?
- Why is it important for you to learn about mechanisms?
- What is the purpose of being able to change speed, force, torque, direction and types of motion with a mechanism?
- Describe where you see mechanisms used in three real-life applications, explain the purpose of using a mechanism for that application.
- How does automation enhance our daily life?
- How can you apply troubleshooting skills that you developed in this lesson to your daily life?
- How do comments improve a computer program?
- Why is good communication and teamwork important when solving technological problems?

Essential Learning Outcomes

It is expected that students will…

- Demonstrate an ability to identify, formulate, and solve engineering problems.
- Demonstrate an ability to design a system, component, or process to meet desired needs within realistic constraints such as economic, environmental, social, political, ethical, health and safety, manufacturability, and sustainability.
- Demonstrate an ability to design and conduct experiments, as well as to analyze and interpret data.
- Demonstrate an ability to apply knowledge of mathematics, science, and engineering.
- Demonstrate an ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.
- Pursue the broad education necessary to understand the impact of engineering solutions in a global, economic, environmental, and societal context.
- Demonstrate an understanding of professional and ethical responsibility.\
- Demonstrate an ability to function on multidisciplinary teams.
- Demonstrate an ability to communicate effectively.
- Gain knowledge of contemporary issues.
- Recognize the need for, and develop an ability to engage in life-long learning.

Standards

- 8.1.8.CS.1 Recommend improvements to computing devices in order to improve the ways users interact with the devices.
- 8.1.8.CS.2: Design a system that combines hardware and software components to process data.
- 8.1.8.CS.3: Justify design decisions and explain potential system trade-offs.
- 8.1.8.CS.4 Systematically apply troubleshooting strategies to identify and resolve hardware and software problems in computing systems.

- 8.1.8.NI.4 Explain how new security measures have been created in response to key malware events.
- 8.1.8.DA.1 Organize and transform data collected using computational tools to make it usable for a specific purpose.
- 8.1.8.DA.4 Transform data to remove errors and improve the accuracy of the data for analysis.
- 8.1.8.AP.2 Create clearly named variables that represent different data types and perform operations on their values.
- 8.1.8.AP.3 Design and iteratively develop programs that combine control structures, including nested loops and compound conditionals.
- 8.1.8.AP.4 Decompose problems and sub-problems into parts to facilitate the design, implementation, and review of programs.
- 8.1.8.AP.6 Refine a solution that meets users' needs by incorporating feedback from team members and users.
- 8.1.8.AP.7 Design programs, incorporating existing code, media, and libraries, and give attribution.
- 8.1.8.AP.8: Systematically test and refine programs using a range of test cases and users.
- 8.1.8.AP.9: Document programs in order to make them easier to follow, test, and debug.
- 8.1.8.IC.1: Compare the trade-offs associated with computing technologies that affect an individual's everyday activities and career options.
- 8.1.8.IC.2: Describe issues of bias and accessibility in the design of existing technologies
- 8.2.8.ED.1: Evaluate the function, value, and aesthetics of a technological product or system, from the perspective of the user and the producer.
- 8.2.8.ED.2: Identify the steps in the design process that could be used to solve a problem.
- 8.2.8.ED.3: Develop a proposal for a solution to a real-world problem that includes a model (e.g., physical prototype, graphical/technical sketch).
- 8.2.8.ED.4: Investigate a malfunctioning system, identify its impact, and explain the step-by-step process used to troubleshoot, evaluate, and test options to repair the product in a collaborative team
- 8.2.8.ED.5: Explain the need for optimization in a design process.
- 8.2.8.ED.6: Analyze how trade-offs can impact the design of a product.
- 8.2.8.ED.7: Design a product to address a real-world problem and document the iterative design process, including decisions made as a result of specific constraints and trade-offs (e.g., annotated sketches).
- 8.2.8.ITH.1: Explain how the development and use of technology influences economic, political, social, and cultural issues.
- 8.2.8.ITH.2: Compare how technologies have influenced society over time.
- 8.2.8.ITH.3: Evaluate the impact of sustainability on the development of a designed product or system.

- 8.2.8.ITH.4: Identify technologies that have been designed to reduce the negative consequences of other technologies and explain the change in impact.
- 8.2.8.ITH.5: Compare the impacts of a given technology on different societies, noting factors that may make a technology appropriate and sustainable in one society but not in another.
- 8.2.8.NT.1: Examine a malfunctioning tool, product, or system and propose solutions to the problem.
- 8.2.8.NT.2: Analyze an existing technological product that has been repurposed for a different function.
- 8.2.8.NT.3: Examine a system, consider how each part relates to other parts, and redesign it for another purpose.
- 8.2.8.EC.1: Explain ethical issues that may arise from the use of new technologies.
- 8.2.8.EC.2: Examine the effects of ethical and unethical practices in product design and development.
- 8.2.8.NT.4: Explain how a product designed for a specific demand was modified to meet a new demand and led to a new product.

Integrated Accommodations and Modifications

- **Special Education Students**
    - Provide graphic organizers for additional support or encourage students to create digital multimedia to showcase knowledge of mathematical concepts.
    - Provide extended time for revisions or opportunity to identify and develop areas of personal interest.
    - Unique Learning System enables educators in self-contained classes to deliver differentiated, standards-aligned content from one convenient, cloud-based platform enhanced by robust assessments, data tools, and evidence-based instructional support. Students from pre-K through transition have the advantage of consistent, high-quality instruction, a motivating interactive learning environment, engaging symbol support, and a path to independence.
- **English Language Learners**
    - Invite students to explore different points of view on a particular mathematical standard and compare.
    - Provided translated materials and audio versions of text when applicable.
- **Skills Fragile Students**
    - Encourage students to make transformations - use a common task or item in a different way
    - Provide graphic organizers for additional support or encourage students to create digital multimedia to showcase knowledge of mathematical concepts.
    - Provide extended time for revisions or opportunity to identify and develop areas of personal interest.
- **504 Students**
    - Encourage creative expression and thinking by allowing students to choose how to approach a problem or assignment.

        - Provide graphic organizers for additional support or encourage students to create digital multimedia to showcase knowledge of mathematical concepts.
        - Provide extended time for revisions or opportunity to identify and develop areas of personal interest
    - **Gifted & Talented Students**
        - Encourage students to explore concepts in depth and encourage independent studies or investigations.
        - Modeling or independent student led research

Assessments (Formative, Summative, Benchmark, Alternative)

1. Formal Assessments by Way of Tests and Quizzes
2. Accountable Talk
3. Vocabulary Quizzes
4. Formative assessments in the form of quizzes, class participation, discussion, topic blogging, and/or journaling

Integration of Career Awareness Standards (NJSLS 9.2)

- 9.2.8.CAP.10: Evaluate how careers have evolved regionally, nationally, and globally.
- 9.2.8.CAP.15: Present how the demand for certain skills, the job market, and credentials can determine an individual's earning power.

Integration of Diversity, Equity, and Inclusion (N.J.S.A. 18A:35-4.36a): (See Appendix A)

- Persons with Disabilities: Students design and test assistive robotic devices that could support persons with disabilities, applying mathematical concepts of measurement, scale, and motion.

Interdisciplinary Connections
Engineering / Design Thinking:

- **Example:** Use the engineering design process to model inclusive technologies that address real-world accessibility challenges.

Science:

- Engaging in the practices of science helps students understand how scientific knowledge develops; such direct involvement gives them an appreciation of the wide range of approaches that are used to investigate, model, and explain the world. Engaging in the practices of engineering likewise helps students understand the work of engineers, as well as the links between engineering and science. Participation in these practices also helps students form an understanding of the crosscutting concepts and disciplinary ideas of science and engineering; moreover, it makes students' knowledge more meaningful and embeds it more deeply into their worldview. The actual doing of science or engineering can also pique students' curiosity, capture their interest, and motivate their continued study; the insights thus gained help them recognize that the work of scientists and engineers is a creative endeavor—one that has deeply affected the world they live in. Students may then recognize that science and engineering can contribute to meeting many of the major challenges that confront society today, such as

generating sufficient energy, preventing and treating disease, maintaining supplies of fresh water and food, and addressing climate change (APPENDIX F – Science and Engineering Practices in the NGSS)

Computer Science

- Students learn about computing systems, networks and the internet, the impacts of computing, data & analysis, and algorithms & programming.

English Language Arts

- **Thinking Aloud-** Students use language to articulate their thinking process.
- **Partner Talk-** Students have discussions with a partner to explain their thinking to a partner while teachers listen for particular vocabulary and linguistic structure.
- **Vocabulary Support-**Learning, understanding, and using mathematical vocabulary allows students to attach their ideas to specific math words and phrases. Students must understand mathematical terminology and key words to gain access to any math problem.
- **Written Expression-** Use journals, quick writes, and outlines to provide students with opportunities to write in the mathematics classroom.

DEI Integration Chart

Unit	Description	Example
1	Amistad	Students explore the **ancient African origins of mathematical reasoning**—such as the Ishango Bone (early counting tool from the Congo Basin)—and compare early numeric systems to modern integers and rational numbers.
2	Amistad	Through proportional reasoning, students analyze data on **the transatlantic trade economy** to understand the mathematical impact of supply, demand, and human labor systems. Students create ratio tables to model population and economic change.
3	Persons with Disabilities	Students analyze how **accessibility costs and modifications** (ramps, elevators, adaptive technologies) can be represented as percentages of overall budgets. They calculate percent increases in inclusive design initiatives for public buildings.
5	Amistad / African- American History	Students learn about **Katherine Johnson**, the African American mathematician whose calculations helped NASA send astronauts into orbit and safely return. Using equations and inequalities, students solve rate-time-distance problems to simulate

		aspects of space travel—such as determining when two spacecraft meet in orbit or comparing ascent paths.
6	Holocaust Education	Students investigate how bias in sampling can lead to inaccurate or unfair conclusions—mirroring how prejudice and stereotypes distort understanding of groups in society. I. Students are given several small "data samples" that describe fictional populations (e.g., students' interests, participation, or talents). II. Some samples are representative (diverse, balanced), while others are biased (missing or overrepresenting certain groups). III. Using measures of center (mean, median, mode) and variability, students analyze how the biased samples give misleading results compared to the full population. IV. Students then discuss: How can collecting or reporting biased data reinforce stereotypes or unfair judgments?
7	AAPI (Asian American and Pacific Islander Heritage)	Students explore **probability models in traditional AAPI games and inventions**, such as the mathematical structure of the Japanese game "Go" or early Chinese probability theory. They simulate events to understand strategic and logical thinking.
8	LGBTQ+ History	Students study the geometric art of **LGBTQ+ mathematician and artist Emma Haruka Iwao** (who calculated trillions of digits of π). They explore the geometry of circles, circumference, and area while celebrating representation in STEM.
9	Persons with Disabilities	Students design and test **assistive robotic devices** that could support persons with disabilities, applying mathematical concepts of measurement, scale, and motion.