

OpenAI o1 Pro's Answers to the Putnam 2024 Exam

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2024 Putnam Exam Questions: <https://kskedlaya.org/putnam-archive/2024.pdf>

2024 Putnam Exam Solutions: <https://kskedlaya.org/putnam-archive/2024s.pdf>

Key

Correct (8)

Unsure (1)

Incorrect (3)

A1 (3m 47s): <https://chatgpt.com/share/675f86b6-467c-8000-9708-b506d80a8a69>

A2 (6m 25s): <https://chatgpt.com/share/675f8891-46a8-8000-8c0d-92ec42a2d887>

A3 (6m 18s): <https://chatgpt.com/share/675f90c1-3fb4-8000-b547-817b7b7b7759>

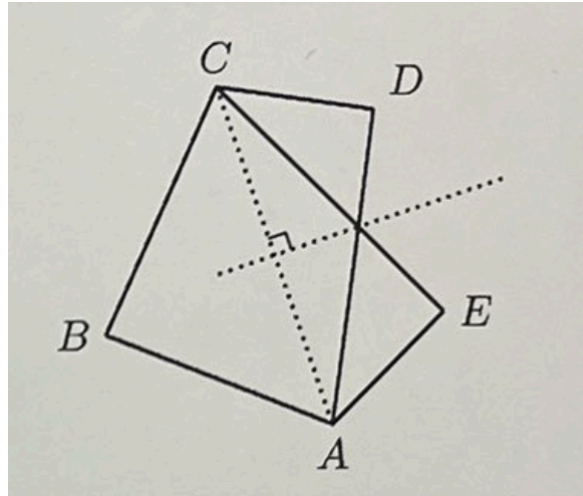
A4 (3m 41s): <https://chatgpt.com/share/675f90b0-0628-8000-b9d3-be6745218788>

A5 (9m 59s): <https://chatgpt.com/share/675f9be5-d89c-8000-9b7f-4cb8210eac59>

A6 (12m 1s) : <https://chatgpt.com/share/675fa0af-25b0-8000-a5df-c61f427c2153>

B1 (2m 41s): <https://chatgpt.com/share/675fa1cd-6a28-8000-be21-c1b560f3e471>

B2 (2m 31s): (Can't share chat because of diagram included in problem prompt):



Short Answer: Yes, it is possible to construct such an infinite sequence.

Detailed Explanation:

First, let's restate the problem in our own words to ensure clarity:

We have a notion of two convex quadrilaterals being "partners." Specifically, two convex quadrilaterals are called partners if they share three vertices and can be named $ABCD$ and $ABCE$ such that the fourth vertex E is the reflection of D across the perpendicular bisector of the diagonal AC .

The question asks whether one can construct an infinite sequence of convex quadrilaterals $(Q_1, Q_2, Q_3, \dots)$ where each quadrilateral is a partner of the next, and yet no two quadrilaterals in the entire sequence are congruent. In other words, starting from some quadrilateral Q_1 , we find a partner Q_2 , then from Q_2 we find another partner Q_3 , and so forth, generating infinitely many distinct quadrilaterals.

Key Insight:

1. Basic Partner Operation: Given three fixed points A, B, C , reflecting a point D across the perpendicular bisector of AC produces a unique point E . The quadrilaterals $ABCD$ and $ABCE$ are partners by definition. This reflection is involutive: if you start with $ABCD$ and apply the partner operation with respect to A, B, C , you get $ABCE$. Applying the same operation again with the same triple of points A, B, C would send E back to D . Thus, if we never change the triple of points, reflecting back and forth would produce only two quadrilaterals, not an infinite variety.

2. Changing the Triple of Common Vertices: The subtlety lies in the fact that the three vertices that remain fixed (the ones across whose chosen diagonal we reflect) do not have to be the same at every step. While one "partnership" step is defined by a specific labeling $ABCD \rightarrow ABCE$, in the next step we might choose a different triple of vertices from the newly formed quadrilateral to define the next reflection.

Consider that a quadrilateral has four vertices. After producing a partner quadrilateral, we now have a shape with vertices, say, A, B, C, E . For the next step, instead of using (A, B, C) as the fixed triple, we could use another combination, such as (B, C, E) or (A, C, E) , and reflect the remaining vertex to form the next partner quadrilateral. By carefully selecting these triples from step to step, we can continually "walk" around the configuration, altering which vertex is being reflected and against which diagonal.

3. Constructing an Infinite Sequence: By systematically choosing different triples, one can ensure that the resulting new vertex is never the same as one previously generated. Because the reflection is a geometric involution (it preserves distances but can produce infinitely many distinct positions as long as the axis of reflection changes), this process can yield a countably infinite set of distinct quadrilaterals. Each step changes the shape in a controlled but non-repetitive way.

For instance, start with a generic quadrilateral $ABCD$. First, reflect D about the perpendicular bisector of AC to get $ABCE$. Next, consider the new quadrilateral $ABCE$ and choose a different triple, say (A, C, E) , to reflect B and get a new partner quadrilateral $ACEF$. Continue this process, always selecting a triple that has not previously been used in the exact same configuration, ensuring that the resulting figure is not congruent to any earlier one. With care, one can avoid cycles and generate infinitely many distinct convex quadrilaterals.

Conclusion:

Yes, it is indeed possible. By varying the triple of vertices that remains fixed at each step and consistently choosing to reflect the appropriate "fourth vertex," we can produce an infinite, non-repetitive chain of quadrilaterals where each is a partner of the next, and no two are congruent.

B3 (6m 49s): <https://chatgpt.com/share/675faaa2-9d50-8000-8aa1-dfa3f639330b>

B4 (2m 13s): <https://chatgpt.com/share/675faf8a-d19c-8000-898f-4a366e90bda9>

B5 (4m 37s): <https://chatgpt.com/share/675fb203-c6c0-8000-9ad1-5778408309c7>

B6 (2m 19s): <https://chatgpt.com/share/675fb323-ae6c-8000-a3a1-a212706ed5d9>