Applications of Rational Equations

Table of Contents

Sanitizer Cases

You have a large bucket holding 5L of sanitizer spray, which is 40% alcohol and 60% water. You want to make sure it's strong enough to kill COVID19.

(a) Look up online what % of a sanitizer must be alcohol in order to kill COVID19 reliably.

(b) If you are able to remove water from the bucket, what is the minimal amount of water (L) you can remove to meet that % standard?

(c) If you are able to add pure alcohol to the bucket, what is the minimal amount of alcohol (L) you must add to meet that % standard?

(d) If you have a solution that is 95% alcohol and 5% water and you add that to your bucket, how much should you add?

(e) What will happen to the concentration of alcohol in the bucket as you add more and more of the 95% alcohol solution to the bucket? Is the relationship linear or nonlinear? What is the domain and range? Explain using words, a formula, and a graph. Assume that once you add the new solution, all liquid is thoroughly mixed.

The Salt Water Problem

You are researching if high concentrations of salt in water will kill COVID19. You currently have a 30kg saline (salt water) solution which is, by weight, 42% salt and 58% water. For your research, you need it to be 20% salt. Here are some ways you could do it.

(a) Remove salt. If you do this, how much salt (kg) should you remove?
(b) Add water. If you do this, will your answer be the same as (a)? Why or why not? Answer that first, then show by calculations exactly how much water (kg) you should add. Explain your results.
(c) Add ocean water. (Look up its salinity on the internet.) If you do this, how much ocean water should you add?

After your experiments with 20% salinity, you get another 30kg bucket of 42% salt and 58% water. You decide to experiment with 40% salinity.
(d) If you want 12kg of solution that is 40% salt, how much of your original solution and how much ocean water should you mix together?
(e) You put all of your original saline in a giant bucket and begin pouring in ocean water. Create an equation that relates the salinity of your new mixture to the amount of ocean water poured in. Is this relation linear or nonlinear? What happens in the long-run? Explain.

ABC’s Legal Custodian

You are the lawyer for ABC Inc, owned by Al, Bob, and Cindy. On January 1st, the ownership structure (AKA “Capitalization Table” AKA “Cap Table”) is: Al owns 250 shares, Bob owns 600 shares, and Cindy owns 150 shares.

1. What % of the business does each of them own on January 1st? Organize your results into a table as follows.

JANUARY 1st Cap Table		
Name	**Shares (absolute)**	**Shares (%)**
Al		
Bob		
Cindy		
Total		

2. On February 1st, Cindy invests more money into the business which issues her 300 more shares, tripling the number of shares she owns. She says: “After this investment, I will triple my number of shares and triple my % ownership of the business.” Create the February 1st cap table and respond to Cindy’s statement.
3. On March 1st, Bob says “I want to cash out through stock buybacks and still be a 25% owner.” What should happen to meet his request? Comment then create a new ownership table.
4. On April 1st, more shares are issued to Al. Create an equation and a graph relating the number of new shares issued to him to the % ownership he has. Will it be linear or non-linear? How do you know?

Work Rate Problems

For each of the following problems, assume that all work done occurs at the same rate whether working individually or together.

WORK RATE PROBLEM 1
You can mow a park in 4 hours.
Your friend can mow it in 5 hours.
If you mow simultaneously on separate mowers, how long will it take to mow the park?

WORK RATE PROBLEM 2
A normal hose can fill a small pool in 16 hours.
A hose from a fire hydrant can fill the pool in 2 hours.
How long to fill the pool if both hoses are on?

WORK RATE PROBLEM 3
You can mow a park in 4 hours.
You need to get the job done in 1h30m.
The park board assigns a second employee to help you.
In how many hours must that employee be able to mow the lawn on their own?

WORK RATE PROBLEM 4
A normal hose can fill a small pool in 16 hours.
You need it filled in 3 hours.
You can attach a second hose to a fire hydrant.
In how many hours must that second hose be able to fill the pool on its own?

WORK RATE PROBLEM 5A
A 2020 model of an autonomous vacuum cleaner can vacuum a convention room in 3 hours.
A 2021 model of an autonomous vacuum cleaner can vacuum a convention room in 1h45m.
There are 6 identical rooms to vacuum.
How long will it take?

WORK RATE PROBLEM 5B
A 2026 model must cut the time to clean all 6 rooms by 2 hours when working with the 2021 model.
What must be true about the 2026 model?

ANSWERS

1. $(\frac{1}{4} + \frac{1}{5})x = 1 \Rightarrow x \approx 2\ hours\ 12\ minutes$
2. $(\frac{1}{16} + \frac{1}{2})x = 1 \Rightarrow x \approx 1\ hour\ 47\ minutes$
3. $(\frac{1}{4} + \frac{1}{x})(1.5) = 1 \Rightarrow x \approx 2\ hours\ 24\ minutes$
4. $(\frac{1}{16} + \frac{1}{x})(3) = 1 \Rightarrow x \approx 3\ hours\ 42\ minutes$
5. $\left(\frac{1}{3} + \frac{1}{1.75}\right)x = 6 \Rightarrow x \approx 6\ hours\ 38\ minutes$
6. $\left(\frac{1}{1.75} + \frac{1}{x}\right)(4.63) \approx 6 \Rightarrow$

Rate, Distance, and Time Problems

PROBLEM 1
In still water, a motor boat moves 4km/h.
The current in a river is moving at 1km/hour.
How long will it take for the boat to go downstream 45km and then back?

PROBLEM 2
In still water, a motor boat moves 8km/hour.
A river's current is 2km/hour.
How long will it take for the boat to go downstream 30km and then back?

PROBLEM 3
In still water, a motor boat moves 10km/hour.
The boat goes downstream 48km then returns, all in 10 hours.
What is the speed of the current?

PROBLEM 4
In still water, a motorboat moves 20km/hour.
The boat goes 75km upstream and back in 8 hours.
What is the speed of the current?

PROBLEM 5
In still water, a motorboat moves 25km/hour.
It takes the boat 1 hour longer to go upstream 50km than downstream 50km.
What is the speed of the current?

PROBLEM 6
The current in a river is 3km/h.
A boat goes 45km downstream and back.
The downstream time is 10 hours shorter than the upstream time.
How fast is the boat in still water?

PROBLEM 7
The wind velocity is 80km/hour West.
A plane flies East 2400km and back in 8 hours.
How fast would the plane fly in still air?

PROBLEM 8
In still air, a plane goes 120km/hour.
It needs to go 600km North.

a) Make a table of values with various wind velocities going South (including zero, positives, and negatives) vs travel time.
b) Make and check an equation relating travel time and wind velocity.
c) Sketch a graph of the relationship. Label key characteristics.
d) Enter the table and your equation into Desmos to check your work. Adjust the window and scale so whatever you see on the screen tells the story.
e) In your own words, describe the relationship between wind velocity and travel time.
f) Starting from your equation, how would you infer the graph's key characteristics?

PROBLEM 9

A boat can move 10km/h in still-water. It is travelling along a man-made canal, so the water can change direction. Checking out of the docks takes 15 minutes and so does checking in.
(a) Docks A and B are 40km apart. If the canal water is still, how long does the trip take, including checking in and out, to go from one dock to the other?
(b) If the current is going 3km/h against the boat, how long will the trip from one dock to the other take?
(c) If the current is going 3km/ with the boat, how long will the entire trip take?
(d) Create a table and equation relating river current velocity to total trip time.
(e) What will the key properties of the graph of that equation be? And how does that make sense in terms of the story?
(f) Imagine a boat wants to go from Dock A to Dock B and back. Will the trip be faster if the river is still or if there is a current? Explain. Use a table and an equation, then sketch a graph showing relevant velocities and time.

Answers

1. $\frac{45}{5} + \frac{45}{3} = 24$ hours
2. $\frac{30}{6} + \frac{30}{10} = 8$ hours
3. $\frac{48}{10+x} + \frac{48}{10-x} = 10;\ x = 2$km/h
4. $\frac{75}{20+x} + \frac{75}{20-x} = 8;\ x = 2$km/h
5. $\frac{50}{25+x} + 1 = \frac{50}{25-x}\ ;\ x \approx 5.9$km/h
6. $\frac{45}{x+3} + 10 = \frac{45}{x-3}\ ;\ x = 6$km/h
7. $\frac{2400}{x+80} + \frac{2400}{x-80} = 8\ ;\ x \approx 610$km/h
8.

Pricing Problems [coming soon]