Cost Effective Airport Hub Location

Course/Section

Math 240 Section E1

Name

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Problem # Done

1

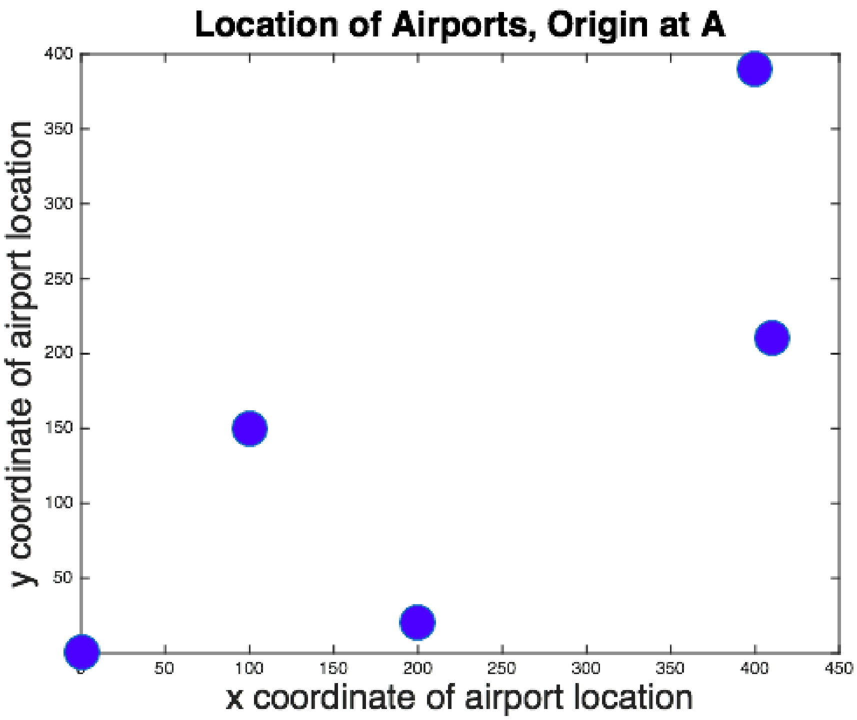
Problem # Attempted

2

Date Due

Noon, Friday, June 19, 2015

<u>Problem</u>:
The goal is to serve 5 airports located in different cities. The government is willing to pay for the construction of a hub either located at one of the cities or any other tangible location. The cost of fuel per mile is $10. The maximum capacity for 1 airplane is 50 passengers. The planes must start from the hub, go to the city, return to the hub, and depart to the location. Given below are the locations of the cities and the number of passengers needed to get from current location to destination. Note: All passengers trips are roundtrips. **<u>Find an ideal location of the hub that will result in the least amount of fuel consumption.</u>**



	A	B	C	D	E
A	*************	75	35	20	110
B	75	*************	40	90	20
C	35	40	*************	130	15
D	20	90	130	*************	40
E	110	20	15	40	*************

Solution:

A rough eyeball estimate is (250, 150) by assuming the hub should be near the center of area of the area enclosed by straight line segments bounding the region.

When understanding the data, there are 240 people initially located at A, 150 people initially located at B, 145 people initially located at C, 40 people initially located at D, and 0 people initially located at E.

If 50 passengers maximum can travel on 1 plane, then I'll need 5 planes headed towards A, 3 planes headed towards B, 3 planes headed towards C, 1 plane headed towards D, and zero planes to E.

Example calculation for determining number of planes to original location:

240 passengers * (1 plane/ 50 passengers) = 4.8 planes. → 5 total planes to A.

150 passengers * (1 plane/ 50 passengers) = 3.0 planes. → 3 total planes to B.

145 passengers * (1 plane/ 50 passengers) = 2.9 planes. → 3 total planes to C.

40 passengers * (1 plane/ 50 passengers) = 0.8 planes. → 1 total plane to D.

0 passengers * (1 plane/ 50 passengers) = 0 planes. → 0 total planes to E.

Sum of Airplanes: 12 Airplanes Total

Since I do not have functional planes that operate when cut into pieces, and since I do not want to leave any customers behind, I must round to the larger whole number.

All in all, I need 12 planes to travel to the original locations. When these planes return to the hub, I'll need 13 planes due to the distribution of people to different locations.

0 passengers * (1 plane/ 50 passengers) = 4.8 planes. → 0 total planes to A.

75 passengers * (1 plane/ 50 passengers) = 1.5 planes. → 2 total planes to B.

75 passengers * (1 plane/ 50 passengers) = 1.5 planes. → 2 total planes to C.

240 passengers * (1 plane/ 50 passengers) = 4.8 planes. → 5 total plane to D.

185 passengers * (1 plane/ 50 passengers) = 3.7 planes. → 4 total planes to E.

Sum of Airplanes: 13 Airplanes Total

The parameters are the number of planes used and the total distance traveled by the planes. The total distance traveled will be minimized by an ideal location of the hub, h.

I now need an equation to help me view total distance traveled by the planes.

$$5H(a) + 3H(b) + 3H(c) + 1H(d) + 0H(e) = d$$

d = the distance traveled between the hub to the original equation.

3*d = the total distance traveled from the hub to the original, the original to the hub, and the final last trip from the hub to the original location.

The coefficients represent the number of planes needed to travel to the location such that the total distance of all of the planes are accounted for.

$$0H(a) + 2H(b) + 2H(c) + 5H(d) + 4H(e) = dd$$

dd = the distance traveled between the hub to the destination equation.

2*dd = the total distance traveled from the hub to the destination, then from the destination back to the hub.

$$D = (3*d) + (2*dd)$$

D = the net distance traveled for the whole day.

Since I am trying to decrease the value of D based on two values that define the location, (x,y), I will convert the values of H( ) to an equation that is dependent on both x and y. This will allow me to assess the relationship between distance traveled by and the location of the hub.

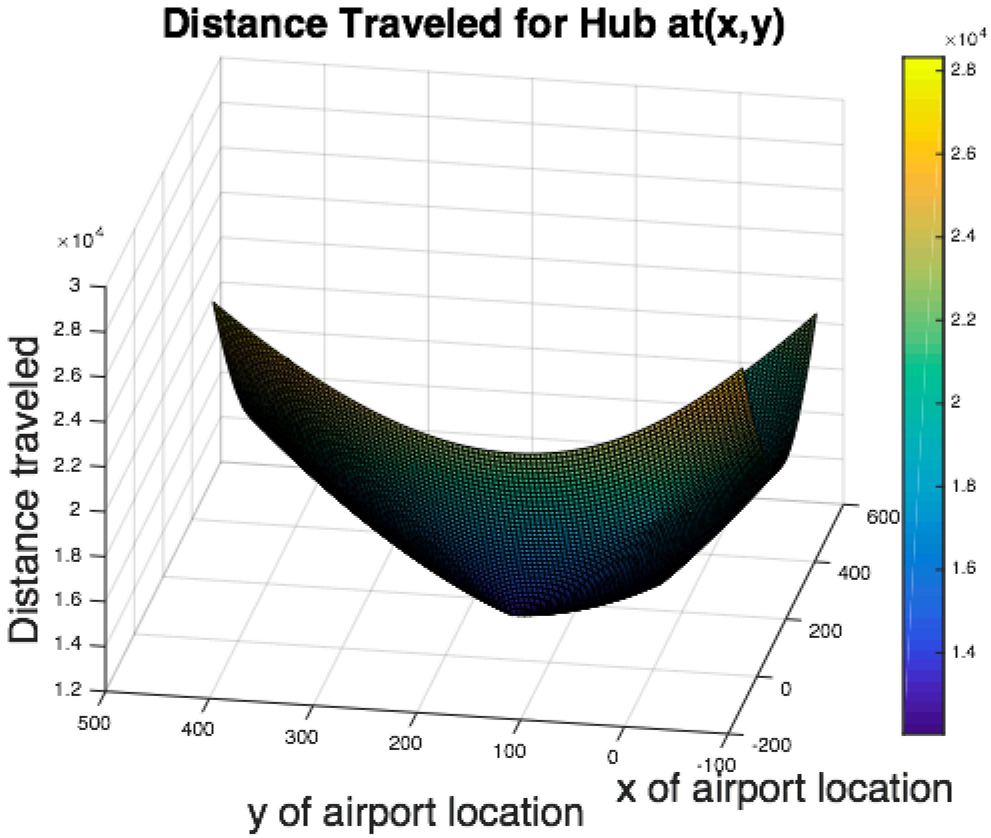
H(a) = sqrt(x^2)+(y^2))
H(b) = sqrt[((x-100)^2)+((y-150)^2)]
H(c) = sqrt[((x-200)^2)+((y-20)^2)]
H(d) = sqrt[((x-400)^2)+((y-390)^2)]
H(e) = sqrt[((x-410)^2)+((y-210)^2)]

<u>Note</u>: I use the pythagorean theorem to find the distance between a possible location of the hub to the actual location of airports A,B,C,D, and E.

Combining equations D and the set of equations of H(a), H(b), H(c), H(d), and H(e) gives Z.

Z = 15*sqrt(((x-100).^2)+((y-150).^2))+ 13*sqrt(((x-200).^2)+((y-20).^2))+ 13*sqrt(((x-400).^2)+((y-390).^2))+ 13*sqrt(((x-410).^2)+((y-210).^2))+ 8*sqrt((x.^2)+(y.^2))

Plotting the function gives:



Using MATLAB's built-in function fminsearch(fun, x), I found the minimum value of the function and the location of the minimum value:

Answer:

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>> [x,fval] = fminsearch(Z,[200,200])
x =
  192.0331  131.5125

fval =
   1.2048e+04
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What the Data means:

x(1) = x = 192.0331 miles from origin
x(2) = y = 131.5125 miles from origin

The location of the hub is (192.0331, 131.5125) relative to city A. 192.0331 is the number of miles east from A and 131.5125 is the number of miles of the hub north of A.

fval = total distance traveled at the ideal location of the hub = 1.2048*10^4 miles.

Conclusion:
Presented is a solution of the ideal location of a hub. The ideal location of the hub is at (192.0331, 131.5125). The answer is generated by first calculating the number of airplanes needed and finding an equation that relates distance traveled per plane as a function of any location of the hub. In doing so, we calculated the hypothetical total distance traveled at hypothetical locations of the hub. With the graph, we saw the smallest value of D, the total distance traveled, located at the minimum value of the function. So, finding the x and y values at the minimum value of D gives us the location (x,y) in which we travel the least amount needed. It results in fewer fuel costs since the total distance traveled by all of the planes is decreased to its minimum possible value of 12048 miles.

Problem 2:

Find the maximum ladder length that can reach across the corridor while minimizing the ladder length.

Solution:

I did not come up with a solution. I plugged and chugged trying to understand the question by its numerical constraints, but I did not understand the question. Next time, I will approach the problem a bit earlier and send email communication about any unclear parameters in the problem to verify the problem being posed.