Mathematics, Grade 9, Factorisation

- In algebra, expressions also have factors.

For example:

The factors of the expression

2x + 2y are 2 and (x + y). We say, 2x + 2y = 2 (x + y)

Also we say (2x + 2y) ÷ 2 = x + y (2x + 2y) ÷ (x + y) = 2

- Factorisation is like splitting a number or an expression into a product of simpler expressions.

For example
Factor 4x + 20
Both 4x and 20 have a common factor of 4

4x is (4)(x)

20 is (4)(5)

So you can factor the whole expression into:
4x +20 = 4(x+5)

So, 4x+20 has been "factored into" 4 and x+5.

Using the Distributive property let us change the expression from a product to a sum by finding the GCF and Polynomial Factor.

For example,

- An expression such as 4m(x – b) tells you to multiply 4m by x – b.
- When you do that, you get the sum 4mx – 4mb. When you do that in reverse, by writing 4mx – 4mb as the product of the two factors 4m by x – b, you are factoring.
- Factoring is simply the process of finding the factors of a given product.

- **A Common Factor is a term that occurs as a factor in each of the expressions being added or subtracted.**
- **A common monomial factor is the product of the literal and numerical factor common to all terms of the polynomial.**
- **The Highest Common Factor is the highest among the common factors in each expressions being added or subtracted.**

Now study the following examples of factorisation of algebraic expression.

Example 1

Factorize $8x^2 + 28x$

Solution: Both 8 and 28 have a common factor of 4 and they also share a variable x.

Together that makes 4x which is the **HCF** of $8x^2$ and 28x.

So you can factor $8x^2 + 28x$ by dividing by 4x

Therefore, $8x^2 + 28x$ = **$4x(2x + 7)$**

Example 2

Factorize $2x^2 - 6x^3 + 8x^4$

Solution: $2x^2 - 6x^3 + 8x^4$ has $2x^2$ as a common factor,

So, $2x^2 - 6x^3 + 8x^4 = 2x^2(1 - 3x + 4x^2)$ divide each term by HCF

Therefore, the factors of $2x^2 - 6x^3 + 8x^4$ are $2x^2$ and $(1 - 3x + 4x^2)$.

Check using the distributive law of multiplication.

$$2x^2(1 - 3x + 4x^2) = 2x^2(1) - 2x^2(3x) + 2x^2(4x^2)$$

$$= 2x^2 - 6x^3 + 8x^4$$

Rules

- Take note of the following notes and shortcuts if they should arise in tests or exams

Special Products

- $(x + y)(x - y) = x^2 - y^2$
- $(x + y)^2 = x^2 + 2xy + y^2$
- $(x - y)^2 = x^2 - 2xy + y^2$
- $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
- $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

Factoring

- $x^2 - y^2 = (x + y)(x - y)$
- $x^2 + 2xy + y^2 = (x + y)^2$
- $x^2 - 2xy + y^2 = (x - y)^2$
- $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
- $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$