SYNDICATE EXAMINATION FEBRUARY 2024

ADVANCED MATHEMATICS 2 MARKING SCHEME

1 $(a)(i)$ *for no restrictions number of ways*

$$= {}^7C_4$$

$$= 35 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 01\tfrac{1}{2}\ marks$$

(ii) *two parents and* 2 *children*:

number of ways $= {}^2C_2 \times {}^5C_2$

$$= 1 \times 10$$

$$= 10 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 01\tfrac{1}{2}\ marks$$

(iii) *there should be* 3 *children and* 1 *parent or* 4 *children*

number of ways $= {}^5C_3 \times {}^2C_1 + {}^5C_4$

$$= 10 \times 2 + 5$$

$$= 25 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 01\tfrac{1}{2}\ marks$$

$1(b)(i)$ *total probabilities* $= 1$

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\int_0^2 kx\ dx + \int_2^4 k(4-x)dx = 1$$

$$\left[\frac{kx^2}{2}\right]_0^2 + \left[k\left(4k - \frac{x^2}{2}\right)\right]_2^4 = 1$$

$$2k - 0 + k\left[(16-8)-(8-2)\right] = 1$$

$$2k + 2k = 1$$

$$k = \frac{1}{4} \ldots\ldots\ldots\ldots\ldots\ldots\ldots 02\ marks$$

(ii) $P(1 \le x \le 3) = P(1 \le x \le 2) + P(2 \le x \le 3)$

$$= \int_1^2 \frac{x}{4}dx + \int_2^3 \frac{(4-x)dx}{4}$$

$$= \left[\frac{x^2}{8}\right]_1^2 + \frac{1}{4}\left[4x - \frac{x^2}{2}\right]_2^3$$

$$= \frac{4}{8} - \frac{1}{8} + \frac{1}{4}\left(12 - \frac{9}{2} - (8-2)\right)$$

$$= \frac{3}{8} + \frac{1}{4}\left(\frac{15}{2} - 6\right)$$

$$= \frac{3}{4} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .02\ marks$$

$$1(b)(iii)\ E(x)=\int_{-\infty}^{\infty} xf(x)dx$$

$$=\int_0^2 x\left(\frac{x}{4}\right)dx+\int_2^4 \frac{x}{4}(4-x)dx$$

$$=\left[\frac{x^3}{12}\right]_0^2+\frac{1}{4}\left[2x^2-\frac{x^3}{3}\right]_2^4$$

$$=\frac{8}{12}+\frac{1}{4}\left(32-\frac{64}{3}-\left(8-\frac{8}{3}\right)\right)$$

$$=\frac{8}{12}+\frac{4}{3}$$

$=2$ ……………………………………………….02 *marks*

1(c) (i) $P(x < 153) = P\left(Z < \frac{153-150.3}{5}\right)$

$= P(z < 0.54)$

$= \emptyset(0.54)$

$= 0.7054$ ……………………………….01.5 marks

(ii) $P(x > 158) = P\left(z > \frac{158-150.3}{5}\right)$

$= P(z > 1.54)$

$= \emptyset(1.54)$

$= 0.0618$ ……………………………….01.5 marks

(iii) $P(147 < x < 149.5) = P\left(\frac{147-150.3}{5} < z < \frac{149.5-150.3}{5}\right)$

$= P(-0.66 < z < -0.16)$

$= \emptyset(-0.16) - \emptyset(-0.66)$

$= 0.2454 - 0.0636$

$= 0.1818$ ……………………………….$01.5\ marks$

$$2(a)\ (p \leftrightarrow q) \to (p \to q) \equiv [(p \to q) \wedge (q \to p)] \to (p \to q) \text{..............} definition$$
$$\equiv \sim [(\sim p \vee q) \wedge (\sim q \vee p)] \vee (\sim p \vee q) \text{.........} defiition$$
$$\equiv [\sim (\sim p \vee q) \vee \sim (\sim q \vee p)] \vee (\sim p \vee q) \text{.......} demorgan's\ law$$
$$\equiv (\sim p \vee q) \vee [\sim (\sim p \vee q) \vee \sim (\sim q \vee p)] \text{......} commutative\ law$$
$$\equiv [(\sim p \vee q) \vee \sim (\sim p \vee q)] \vee \sim (\sim q \vee p) \text{.....} associative\ law$$
$$\equiv T \vee \sim (\sim q \vee p) \text{....................................} complement\ \ law$$
$$\equiv T \qquad \text{....................................} identity\ \ law$$

………03 marks

2(b)

p	q	$\sim p$	$\sim q$	$p \to q$	$(p \to q) \wedge \sim q$	$[(p \to q) \wedge \sim q] \to \sim p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

Hence it is tautology.

………..03 marks

2(c) let p : I use anti malaria drugs

q: I will get malaria

r: I use mosquito net

The argument in symbols $p \to \sim q, \sim r \to p, q \therefore r$

The argument is valid if $[(p \to \sim q) \wedge (\sim r \to p) \wedge q] \to r$ is a tautology.

………..01 mark

p	q	r	$\sim q$	$\sim r$	A $p \to \sim q$	B $\sim r \to p$	C $A \wedge B \wedge q$	$C \to r$
T	T	T	F	F	F	T	F	T
T	T	F	F	T	F	T	F	T
T	F	T	T	F	T	T	F	T
T	F	F	T	T	T	T	F	T
F	T	T	F	F	T	T	T	T

F	T	F	F	T	T	F	F	T
F	F	T	T	F	T	T	F	T
F	F	F	T	T	T	F	F	T

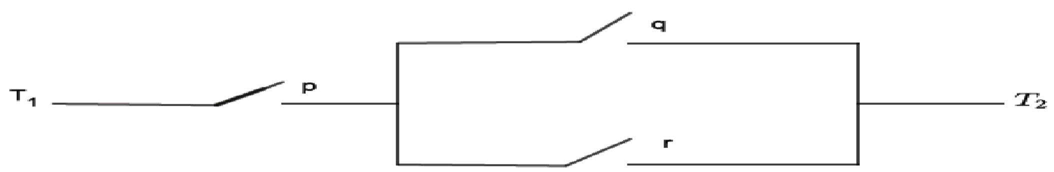
The argument is VALID since the last column has T throughout.

………03 marks

2(d) $K \equiv (p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r) \vee (p \wedge \sim q \wedge r)$ ....01 *mark*

$$
\begin{aligned}
\textit{simplifying}\quad K &\equiv [(p \wedge q) \wedge (r \vee \sim r)] \vee (p \wedge \sim q \wedge r) \text{ ....}\textit{distributive law} \\
&\equiv [(p \wedge q) \wedge T] \vee (p \wedge \sim q \wedge r) \text{ ....}\textit{complement law} \\
&\equiv (p \wedge q) \vee (p \wedge \sim q \wedge r) \text{ ....}\textit{identity law} \\
&\equiv p \wedge [q \vee (\sim q \wedge r)] \text{ ....}\textit{distributive law} \\
&\equiv p \wedge [(q \vee \sim q) \wedge (q \vee r)] \text{ ....}\textit{distributive law} \\
&\equiv p \wedge [T \wedge (q \vee r)] \text{ ....}\textit{complement law} \\
&\equiv p \wedge (q \vee r) \text{ ....}\textit{identity law}
\end{aligned}
$$

………..02 *marks*



………02 marks

$$3(a)\quad \underline{F}_1=\frac{21(6\underline{i}+2\underline{j}+3\underline{k})}{\sqrt{36+4+9}}=\frac{21(6\underline{i}+2\underline{j}+3\underline{k})}{7}=18\underline{i}+6\underline{j}+9\underline{k}$$

$$\underline{F}_2=\frac{6(2\underline{i}-2\underline{j}+\underline{k})}{\sqrt{4+4+1}}=\frac{6(2\underline{i}-2\underline{j}+\underline{k})}{3}=4\underline{i}-4\underline{j}+2\underline{k}$$

....01 *mark*

$$\textit{resultant force } \underline{F}=\underline{F}_1+\underline{F}_2$$
$$=18\underline{i}+6\underline{j}+9\underline{k}+4\underline{i}-4\underline{j}+2\underline{k}$$
$$=22\underline{i}+2\underline{j}+11\underline{k}$$

....01 *mark*

$$\textit{displacement } \underline{d}=(4,-3,1)-(2,1,-3)$$
$$=2\underline{i}-4\underline{j}+4\underline{k}$$

....01 *mark*

$$\textit{work done }=\underline{F}\bullet\underline{d}$$
$$=(22\underline{i}+2\underline{j}+11\underline{k})\bullet(2\underline{i}-4\underline{j}+4\underline{k})$$
$$=22\times 2+2\times -4+11\times 4$$
$$=80\ \textit{units.}$$

....01 *mark*

$$3(b)\quad \overrightarrow{AB}=\underline{b}-\underline{a}=(4\underline{i}-2\underline{j}+3\underline{k})-(3\underline{i}+\underline{j}+\underline{k})=\underline{i}-3\underline{j}+2\underline{k}$$

$$\overrightarrow{AC}=\underline{c}-\underline{a}=(\underline{i}+2\underline{j})-(3\underline{i}+\underline{j}+\underline{k})=-2\underline{i}+\underline{j}-\underline{k}$$

....01 *mark*

$$\textit{area of the triangle }=\frac{\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|}{2}$$ ....0.5 *marks*

$$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\underline{i} & \underline{j} & \underline{k}\\ 1 & -3 & 2\\ -2 & 1 & -1\end{vmatrix}$$
$$=\underline{i}\,(3-2)-\underline{j}\,(-1+4)+\underline{k}\,(1-6)$$
$$=\underline{i}-3\underline{j}-5\underline{k}$$

....01 *mark*

$$\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|=\sqrt{1+9+25}=\sqrt{35}$$ ....0.5 *marks*

$$\therefore\ \textit{area}=\frac{\sqrt{35}}{2}\ \textit{square units}.$$ ....0.5 *marks*

$3(c)\ |\underline{a} \times \underline{b}| = |\underline{a}|\,|\underline{b}| \sin\theta$

$$\underline{a} \times \underline{b} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & 1 & -2 \\ 1 & -2 & 1 \end{vmatrix}$$

$$= \underline{i}(1-4) - \underline{j}\,(2+2) + \underline{k}\,(-4-1)$$

$$= -3\underline{i} - 4\underline{j} - 5\underline{k}$$

.... 01 *mark*

$$|\underline{a} \times \underline{b}| = \sqrt{9+16+25} = \sqrt{50} = 2\sqrt{5}$$

$$|\underline{a}| = \sqrt{4+1+4} = 3$$

$$|\underline{b}| = \sqrt{1+4+1} = \sqrt{6}$$

....01.5 *marks*

$$\sin\theta = \frac{|\underline{a} \times \underline{b}|}{|\underline{a}|\,|\underline{b}|}$$

$$= \frac{2\sqrt{5}}{3\sqrt{6}}$$

$$\theta = 74.2^{\circ}$$

.... 01 *mark*

$3(d)\quad \underline{r}(t) = 3t^2\underline{i} + (t^2 - 2t)\underline{j} + t^3\underline{k}$

$$\underline{v} = \frac{d\underline{r}}{dt} = 6t + (2t-2)\underline{j} + 3t^2\underline{k}$$

when $t = 2,\ \underline{v} = 12\underline{i} + 2\underline{j} + 12\underline{k}$

$$|\underline{v}| = \sqrt{144+4+144} = \sqrt{292} = 2\sqrt{73}$$

....02 *marks*

$$\underline{a} = \frac{d\underline{v}}{dt} = 6\underline{i} + 2\underline{j} + 6t\underline{k}$$

when $t = 2,\ \underline{a} = 6\underline{i} + 2\underline{j} + 12\underline{k}$

$$|\underline{a}| = \sqrt{36+4+144} = \sqrt{184} = 2\sqrt{46}$$

.... 02 *marks*

$4(a) \Rightarrow \left|\sqrt{3}+i\right| = \sqrt{3+1} = 2$

$$\alpha = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$

$$\Rightarrow \left|\sqrt{3}-i\right| = \sqrt{3+1} = 2$$

$$\alpha = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \frac{-\pi}{6}$$

.... 01 *mark*

$$\Rightarrow \left(\sqrt{3}+i\right)^n + \left(\sqrt{3}-i\right)^n = \left[2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)\right]^n + \left[2\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)\right]^n$$

$$= 2^n\left(\cos\frac{n\pi}{6} + i\sin\frac{n\pi}{6}\right) + 2^n\left(\cos\frac{n\pi}{6} - i\sin\frac{n\pi}{6}\right)$$

$$= 2^n\left(2\cos\frac{n\pi}{6}\right)$$

$$= 2^{n+1}\cos\frac{n\pi}{6} \quad \textit{proved}$$

.... .02 *marks*

$4(b)\ z - \frac{1}{z} = 2i\sin\theta$

$$\left(z - \frac{1}{z}\right)^5 = (2i\sin\theta)^5$$

$$z^5 + 5z^4\left(\frac{-1}{z}\right) + 10z^3\left(\frac{-1}{z}\right)^2 + 10z^2\left(\frac{-1}{z}\right)^3 + 5z\left(\frac{-1}{z}\right)^4 + \left(\frac{-1}{z}\right)^5 = 32i^5\sin^5\theta \text{........01}\textit{mark}$$

$$z^5 - 5z^3 + 10z - \frac{10}{z} + \frac{5}{z^3} - \frac{1}{z^5} = 32i\sin^5\theta$$

$$z^5 - \frac{1}{z^5} - 5\left(z^3 - \frac{1}{z^3}\right) + 10\left(z - \frac{1}{z}\right) = 32i\sin^5\theta \text{ ....01 } \textit{mark}$$

$$2i\sin 5\theta - 5(2i\sin 3\theta) + 10(2i\sin\theta) = 32i\sin^5\theta$$

$$2(\sin 5\theta - 5\sin 3\theta + 10\sin\theta) = 2(16\sin^5\theta)$$

$$\sin^5\theta = \frac{(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)}{16} \text{ ....01 } \textit{mark}$$

$$\int(16\sin^6\theta - 10\sin\vartheta)d\theta = \int(\sin 5\theta - 5\sin 3\theta)d\theta$$

$$= \frac{-\cos 5\theta}{5} - \left(\frac{-5\cos 3\theta}{3}\right) + c$$

$$= \frac{5\cos 3\theta}{3} - \frac{\cos 5\theta}{5} + c \text{ ....01 } \textit{mark}$$

$4(c)\quad z^4=-16$

$r=|-16|=16$

$\alpha=\tan^{-1}\left(\frac{0}{-16}\right)=180^{\circ}$ ……………………………………………………………..01 *mark*

$\Rightarrow z^4=r\left(\cos(\alpha+360^{\circ}k)+i\sin(\alpha+360^{\circ}k)\right)$

$z=16^{1/4}\left[\cos(180^{\circ}+360^{\circ}k)+i\sin((180^{\circ}+360^{\circ}k))\right]^{1/4}$

$=2\left[\cos\left(\frac{180^{\circ}+360^{\circ}k}{4}\right)+i\sin\left(\frac{180^{\circ}+360^{\circ}k}{4}\right)\right]$

$=2\left[\cos(45^{\circ}+90^{\circ})+i\sin(45^{\circ}+90^{\circ})\right]$ …………………………………………….01 *mark*

$k=0,\ z_1=2(\cos 45^{\circ}+i\sin 45^{\circ})=2\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)=\sqrt{2}+i\sqrt{2}$

$k=1,\ z_2=2(\cos 135^{\circ}+i\sin 135^{\circ})=2\left(\frac{-\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)=-\sqrt{2}+i\sqrt{2}$

……………………………………………………………………………………………………….01 *mark*

$$k=2,\ z_3=2\left(\cos 225\ \ +i\sin 225^{\circ}\right)=2\left(\frac{-\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}\right)=-\sqrt{2}\ -i\sqrt{2}$$

$$k=3,\ z_4=2\left(\cos 315^{\circ}+i\sin 315^{\circ}\right)=2\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)=\sqrt{2}-i\sqrt{2}$$

....01 *mark*

$$4(d)\frac{z+i}{z+2}=\frac{x+yi+i}{x+yi+2}=\frac{x+(y+1)i}{x+2+yi}$$

$$=\frac{x+(y+1)i}{x+2+yi}\times\frac{x+2-yi}{x+2-yi}$$

$$=\frac{x(x+2)-xyi+(x+2)(y+1)i+y(y+1)}{(x+2)^2+y^2}$$

$$=\frac{x^2+2x+y^2+y}{(x+2)^2+y^2}+\frac{(x+2)(y+1)-xy}{(x+2)^2+y^2}i$$ ....01 *mark*

if the complex number is purely imaginary, then the real part $=0$

$$\therefore\frac{x^2+2x+y^2+y}{(x+2)^2+y^2}=0$$ .... 01 *mark*

$$x^2+2x+y^2+y=0$$

$$(x+1)^2+\left(y+\frac{1}{2}\right)^2=1+\frac{1}{4}$$

$$(x+1)^2+\left(y+\frac{1}{2}\right)^2=\frac{5}{4}$$ .... 01 *mark*

it is a circle with centre $=\left(-1,-\frac{1}{2}\right)$

and radius $r=\sqrt{\frac{5}{4}}$

$r=\frac{\sqrt{5}}{2}$ *units. proved.* .... 01 *mark*

$5(a)\ 5\cos\theta - 2\sin\theta = 2$

$\Rightarrow \cos\theta = \dfrac{1-t^2}{1+t^2}$

$\sin\theta = \dfrac{2t}{1+t^2}$

.... 01 *mark*

$5\left(\dfrac{1-t^2}{1+t^2}\right) - 2\left(\dfrac{2t}{1+t^2}\right) = 2$

$5 - 5t^2 - 4t = 2 + 2t^2$

$7t^2 + 4t - 3 = 0$

$t = -1,\ t = \dfrac{3}{7}$ ....01 *mark*

$\tan\dfrac{\theta}{2} = -1 \ or \ \tan\dfrac{\theta}{2} = \dfrac{3}{7}$

$\Rightarrow \tan\dfrac{\theta}{2} = -1$

$\dfrac{\theta}{2} = 180^{\circ}n - 45^{\circ}$

$\theta = 360^{\circ}n - 90^{\circ}$

$\theta = -90^{\circ},\ 270^{\circ}$

$\Rightarrow \tan\dfrac{\theta}{2} = \dfrac{3}{7}$

$\dfrac{\theta}{2} = 180^{\circ}n + 23.2^{\circ}$

$\theta = 360^{\circ}n + 46.4^{\circ}$

$= 46.4^{\circ}, \ldots$

solutions $\theta = 46.4^{\circ},\ 270^{\circ}$

.... 02 *marks*

$5(b)\ 3\cos\theta - 4\sin\theta = R\cos(\theta+\alpha)$

$= R(\cos\theta\cos\alpha - \sin\theta\sin\alpha)$

$= R\cos\alpha\cos\theta - R\sin\alpha\sin\theta$ ....01 *mark*

comparing :

$R\sin\alpha = 4$

$R\cos\alpha = 3$

$\Rightarrow \dfrac{R\sin\alpha}{R\cos\alpha} = \dfrac{4}{3}$

$\tan\alpha = \dfrac{4}{3}$

$\alpha = 53.13^{\circ}$ ....01 *mark*

$\Rightarrow R^2(\sin^2\alpha + \cos^2\alpha) = 4^2 + 3^2$

$R^2 = 25$

$R = 5$ ....01 *mark*

$\therefore 3\cos\theta - 4\sin\theta = 5\cos(\theta + 53.13^{\circ})$ ....01 *mark*

$5(c)\ \cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$2\cos 3\theta = 8\cos^3\theta - 6\cos\theta$ .... 01 *mark*

$= (2\cos\theta)^3 - 3(2\cos\theta)$

$= \left(x+\dfrac{1}{x}\right)^3 - 3\left(x+\dfrac{1}{x}\right)$ ....01 *mark*

$= x^3 + 3x^2\left(\dfrac{1}{x}\right) + 3x\left(\dfrac{1}{x^2}\right) + \dfrac{1}{x^3} - 3x - \dfrac{3}{x}$

$= x^3 + 3x + \dfrac{3}{x} + \dfrac{1}{x^3} - 3x - \dfrac{3}{x}$

$= x^3 + \dfrac{1}{x^3}$ *proved* ....02 *marks*

$5(d)\ \dfrac{\sin A + \sin 3A + \sin 5A + \sin 7A}{\cos A + co3A + \cos 5A + \cos 7A} = \dfrac{2\sin 2A\cos A + 2\sin 6A\cos A}{2\cos 2A\cos A + 2\cos 6A\cos A}$

$= \dfrac{2\cos A(\sin 2A + \sin 6A)}{2\cos A(\cos 2A + \cos 6A)}$ ....02 *marks*

$= \dfrac{\sin 2A + \sin 6A}{\cos 2A + \cos 6A}$

$= \dfrac{2\sin 4A\cos 2A}{2\cos 4A\cos 2A}$

$= \dfrac{\sin 4A}{\cos 4A}$

$= \tan 4A$ *proved* .... 02 *marks*

$$5(e)\ \tan^{-1}\left(\frac{1-4x}{1+6x}\right)-\tan^{-1}\left(\frac{1+2x}{1-3x}\right)=\frac{\pi}{4}$$

$$let\ \ A=\tan^{-1}\left(\frac{1-4x}{1+6x}\right)\ \ \Rightarrow \tan A=\frac{1-4x}{1+6x}$$

$$B=\tan^{-1}\left(\frac{1+2x}{1-3x}\right)\ \ \Rightarrow \tan B=\frac{1+2x}{1-3x}\qquad \text{.......... .... .01}\ \ mark$$

$$\Rightarrow A-B=\frac{\pi}{4}$$

$$\tan(A-B)=\tan\frac{\pi}{4}$$

$$\frac{\tan A-\tan B}{1+\tan A\tan B}=1$$

$$\tan A-\tan B=1+\tan A\tan B$$

$$\frac{1-4x}{1+6x}-\frac{1+2x}{1-3x}=1+\frac{(1-4x)(1+2x)}{(1+6x)(1-3x)}\qquad \text{.......... .... 01}\ mark$$

$$(1-4x)(1-3x)-(1+2x)(1+6x)=(1+6x)(1-3x)+(1-4x)(1+2x)$$

$$26x^2-16x-2=0\qquad \text{.......... .... 01}\ mark$$

$$13x^2-8x-1=0$$

$$x=-0.1066,\ x=0.7219$$

$$positive\ \ value\ \ of\ \ x=0.7219\qquad \text{.......... .... 01}\ mark$$

$$6(a)\ \ let\ m=x+2\quad \Rightarrow x=m-2\qquad \text{..01}\ mark$$

$$\frac{x^2+6x+5}{(x+2)^4}=\frac{(m-2)^2+6(m-2)+5}{m^4}$$

$$=\frac{m^2-4m+4+6m-12+5}{m^4}\qquad \text{...01}\ mark$$

$$=\frac{m^2+2m-3}{m^4}$$

$$=\frac{1}{m^2}+\frac{2}{m^3}-\frac{3}{m^4}$$

$$=\frac{1}{(x+2)^2}+\frac{2}{(x+2)^3}-\frac{3}{(x+2)^4}\qquad \text{...02}\ marks$$

$6(b) \Rightarrow when\ \ n=1,$

$9^1 - 1 = 8 \quad it\ is\ true$ ....01 *mark*

$\Rightarrow assume\ \ it\ is\ true\ \ when\ \ n = k$

$9^k - 1 = 8m\ where\ \ m = whole\ \ number$

$9^k = 1 + 8m$ ....01 *mark*

$\Rightarrow when\ \ n = k+1,\ we\ have$

$9^{k+1} - 1 = 9(9^k) - 1 \quad but \quad 9^k = 1 + 8m$

$= 9(1+8m) - 1$

$= 9 + 72m - 1$

$= 8 + 72m$

$= 8(1+9m)\ \ hence\ \ proved$ ....02 *marks*

$6(c)\ \alpha + \beta = \dfrac{-b}{a}$

$\alpha\beta = \dfrac{c}{a}$

....01 *mark*

$(\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ ....01 *mark*

$1 = \left(\dfrac{-b}{a}\right)^2 - 2\left(\dfrac{c}{a}\right)$

$1 = \dfrac{b^2}{a^2} - \dfrac{2c}{a}$

$a^2 = b^2 - 2ac$

$b^2 = a^2 + 2ac \quad proved$ ....02 *marks*

$6(d)\ |3-2x| < |4+x|$

$\Rightarrow |3-2x|^2 < |4+x|^2$

$(3-2x)^2 < (4+x)^2$

$9 - 12x + 4x^2 < 16 + 8x + x^2$

$3x^2 - 20x - 7 < 0$ .... 01 *mark*

$3x^2 - 21x + x - 7 < 0$

$3x(x-7) + 1(x-7) < 0$

$(x-7)(3x+1) < 0$ ....01 *mark*

Critical values $x = 7,\ x = \frac{-1}{3}$ ....*table* 01 *mark*

	$x-7$	$3x+1$	$(x-7)(3x+1)$
$x < \frac{-1}{3}$	-	-	+
$\frac{-1}{3} < x < 7$	-	+	-
$x > 7$	+	+	+

Solution=

$\frac{-1}{3} < x < 7$ ....01 *mark*

$6(e)\ given \left(\sqrt{x} - \frac{p}{x^2}\right)^{10}$

$\Rightarrow the\ general\ term = {}^{10}C_r\left(\sqrt{x}\right)^{10-r}\left(\frac{-p}{x^2}\right)^r$

$= {}^{10}C_r\left(x^{1/2}\right)^{10-r}(-p)^r\left(x^{-2r}\right)$

$= {}^{10}C_r(-p)^r(x)^{5-\frac{5r}{2}}$ ....01 *mark*

for a constant term , exponent of x is zero

$5 - \frac{5r}{2} = 0$

$5r = 10$

$r = 2$ ....01 *mark*

$constant\ term = {}^{10}C_2(-p)^2$

$405 = 45p^2$ ....01 *mark*

$p^2 = 9$

$p = \pm 3$ ....01 *mark*

$7(a)\quad y = e^{x}(A\cos 2x + B\sin 2x)$

$ye^{-x} = A\cos 2x + B\sin 2x$

$\Rightarrow -e^{-x}y + e^{-x}\dfrac{dy}{dx} = -2A\sin 2x + 2Bco2x$ ....01 *mark*

$\Rightarrow e^{-x}y - e^{-x}\dfrac{dy}{dx} - e^{-x}\dfrac{dy}{dx} + e^{-x}\dfrac{d^2y}{dx^2} = -4A\cos 2x - 4B\sin 2x$

$e^{-x}y - 2e^{-x}\dfrac{dy}{dx} + e^{-x}\dfrac{d^2y}{dx^2} = -4(A\cos 2x + B\sin 2x)$ ....01 *mark*

but $\quad A\cos 2x + B\sin 2x = ye^{-x}$

$e^{-x}y - 2e^{-x}\dfrac{dy}{dx} + e^{-x}\dfrac{d^2y}{dx^2} = -4ye^{-x}$ ....01 *mark*

$y - 2\dfrac{dy}{dx} + \dfrac{d^2y}{dx^2} = -4y$

$\dfrac{d^2y}{dx^2} - 2y + 5y = 0$ ....01 *mark*

$7(b)\ xy\frac{dy}{dx}=x^2+y^2$

$\frac{y}{x}\frac{dy}{dx}=1+\left(\frac{y}{x}\right)^2$

$let\ u=\frac{y}{x}\ \Rightarrow y=ux$

$\frac{dy}{dx}=u+x\frac{du}{dx}$ .... 01 *mark*

$\Rightarrow u\left(u+x\frac{du}{dx}\right)=1+u^2$

$u^2+ux\frac{du}{dx}=1+u^2$

$ux\frac{du}{dx}=1$ .... 01 *mark*

$udu=\frac{dx}{x}$

$\int udu=\int\frac{dx}{x}$

$\frac{u^2}{2}=\ln x+c$

$u^2=2\ln x+2c$

$\left(\frac{y^2}{x^2}\right)=2\ln x+A$

$y^2=2x^2\ln x+Ax^2$ ....02 *marks*

$OR\quad y^2=2x^2\ln Ax$

$$7(c)\ (1+x)\frac{dy}{dx}-2y=\frac{1}{1+x}$$

$$\frac{dy}{dx}-\frac{2}{1+x}y=\frac{1}{(1+x)^2}\ \textit{it is a linear equation}$$

$$\text{int}\,\textit{egrating factor}\quad R=e^{\int\frac{-2dx}{1+x}}=e^{-2\ln(1+x)}=(1+x)^{-2}\qquad \text{.....................01 mark}$$

$$\Rightarrow (1+x)^{-2}\frac{dy}{dx}-\frac{2}{(1+x)^3}y=\frac{1}{(1+x)^4}$$

$$\frac{d}{dx}\left(\frac{y}{(1+x)^2}\right)=(1+x)^{-4}\qquad \text{..01 mark}$$

$$\frac{y}{(1+x)^2}=\int(1+x)^{-4}dx$$

$$\frac{y}{(!+x)^2}=\frac{(1+x)^{-3}}{-3}+c$$

$$y=\frac{(1+x)^{-1}}{-3}+c(1+x)^2$$

$$y=\frac{-1}{3(1+x)}+c(1+x)^2\ \ \textit{proved}\qquad \text{..................................02 marks}$$

$$7(d)\ \frac{d^2y}{dx^2}-5\frac{dy}{dx}=18e^{3x}+35$$

complementary solution

$$m^2-5m=0$$

$$m(m-5)=0$$

$$m=0\ ,\ m=5$$

$$y_c=A+Be^{5x}\qquad \text{..01 mark}$$

particular integral

$$\textit{let}\ y_p=Cx+De^{3x}$$

$$y_p'=C+3De^{3x}$$

$$y_p''=9De^{xx}\qquad \text{..01 mark}$$

subtitute in the equation

$$9De^{3x}-5(C+3De^{3x})=18e^{3x}+35$$

$$-6De^{3x}-5C=18e^{3x}+35$$

$$\textit{comparing}:\ -6D=18\ \Rightarrow D=-3$$

$$-5C=35\ \Rightarrow C=-7$$

$$y_p=-7x-3e^{3x}\qquad \text{..01 mark}$$

$$\textit{solution}\ y=y_c+y_p$$

$$y=A+Be^{5x}-7x-3e^{3x}\qquad \text{..01 marks}$$

$7(e)$ let m = mass of salt in the tank at any time t

$$\frac{dm}{dt} = \text{mass rate in} - \text{mass rate out}$$

$$\frac{dm}{dt} = 0 - \text{flow rate out} \times \text{concentration out}$$

$$\frac{dm}{dt} = -20\left(\frac{m}{V}\right) \quad \text{where } V = 1000\,l$$

$$\frac{dm}{dt} = -20\left(\frac{m}{1000}\right)$$

$$\frac{dm}{dt} = \frac{-m}{50}$$ .... 01 *mark*

$$\frac{dm}{m} = \frac{-dt}{50}$$

$$\int_{m_0}^{m} \frac{dm}{m} = \int_0^t \frac{-dt}{50}$$

$$[\ln m]_{m_0}^{m} = \frac{-t}{50}$$

$$\ln m - \ln m_0 = \frac{-t}{50}$$

$$\ln\left(\frac{m}{m_0}\right) = \frac{-t}{50}$$ ....01 *mark*

$$\frac{m}{m_0} = e^{-t/50}$$

$$m = m_0 e^{\frac{-t}{50}} \quad \text{but initial mass } m_0 = 10\ kg$$

$$m = 10e^{\frac{-t}{50}}$$ .... 01 *mark*

when $t = 5$ min

$$m = 10e^{\frac{-5}{50}}$$

7 $m = 9.048\ kg$ ....01 *mark*

$8(a)\ x+\frac{y^2-12y+60}{24}-6=0$

$24x+y^2-12y+60-144=0$

$y^2-12y=84-24x$

$y^2-12y+36=-24x+84+36$

$(y-6)^2=-24(x-5)$

....02 *marks*

$\Rightarrow vertex=(h,\ k)=(5,\ 6)$ ....01 *mark*

$\Rightarrow focus=(h,\ k)+(a,\ 0)\ =(h+a,\ k)$

$where\ \ 4a=-24$

$a=-6$

$focus=(5-6,\ 6)=(-1,\ 6)$ ....01 *mark*

$\Rightarrow directrix\ \ x=-a+h$

$x=-(-6)+5$

$x=11$ ....01 *mark*

$8(b)\ \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$$\Rightarrow \frac{2x}{a^2}+\frac{2y}{b^2}\frac{dy}{dx}=0$$

$$2b^2x+2a^2y\ \frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{-2b^2x}{2a^2y}$$ ……………………………………………01 *mark*

at (x_1, y_1) $\quad \frac{dy}{dx}=\frac{-b^2x_1}{a^2y_1}$

gradient of the tan *gent* $m=\frac{-b^2x_1}{a^2y_1}$ ……………………………………………01 *mark*

eqution of tan *gent*

$$y-y_1=m(x-x_1)$$

$$y-y_1=\frac{-b^2x_1}{a^2y_1}(x-x_1)$$

$$a^2yy_1-a^2{y_1}^2=-b^2xx_1+b^2{x_1}^2$$ ……………………………………01.5

dividing by a^2b^2

$$\frac{yy_1}{b^2}-\frac{{y_1}^2}{b^2}=\frac{-xx_1}{a^2}+\frac{{x_1}^2}{a^2}$$

$$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=\frac{{x_1}^2}{a^2}+\frac{{y_1}^2}{b^2}\quad but\quad \frac{{x_1}^2}{a^2}+\frac{{y_1}^2}{b^2}=1$$

$$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1\ \ provd$$ ……………………………………………01.5

$8(a)\ \frac{x^2}{9}-\frac{y^2}{16}=1 \ \Rightarrow \frac{x^2}{3^2}-\frac{y^2}{4^2}=1$

$a=3, \quad b=4$ ....01 *mark*

Transverse axis is horizontal since *the term* x^2 *is positive*.

$$b^2=a^2\left(e^2-1\right)$$
$$16=9\left(e^2-1\right)$$
$$\frac{16}{9}=e^2-1$$
$$e^2=\frac{16}{9}+1$$
$$e^2=\frac{25}{9}$$
$$e=\frac{5}{3}$$

.... .01.5 *marks*

$\Rightarrow foci=(\pm\, ae,\ 0)$

$$=\left(\pm 3\left(\frac{5}{3}\right),0\right)$$
$$=(\pm 5\ ,0)$$

....01.5 *marks*

$\Rightarrow asymptotes \quad y=\pm\frac{b}{a}x$

$y=\pm\frac{4}{3}x$ .... 01 *mark*

$8(d)\ a=93{,}000{,}000$

$$b^2=a^2\left(1-e^2\right)$$
$$=93{,}000{,}000^2\left(1-\frac{1}{62^2}\right)$$
$$b\ =92{,}987{,}902.4$$

....01 *mark*

distance from center of the orbit to one focus

$$ae=c=\sqrt{a^2-b^2}$$
$$c=\sqrt{93{,}000{,}000^2-92987902.4^2}$$
$$c=1{,}500{,}000$$

....01 *mark*

$least\ distance=a-c$

$$=93{,}000{,}000-1{,}500{,}000$$
$$=91{,}500.000\ \ miles$$

....01.5 *marks*

$greatest\ distance=a+c$

$$=93{,}000{,}000+1{,}500{,}000$$
$$=94{,}500{,}000\ miles$$

....01.5 *marks*