Integrales trigonométricas

$$\int sen\,x\ dx = -\cos x + C$$

$$\int sen\,u \cdot u'\,dx = -\cos u + C$$

$$\int \cos x\ dx = \operatorname{sen} x + C$$

$$\int \cos u \cdot u'\,dx = \operatorname{sen} u + C$$

$$\int \frac{1}{\cos^2 x}\,dx = \int \sec^2 x\ dx = \int \left(1+tg^2 x\right) dx = tg\ x + C$$

$$\int \frac{u'}{\cos^2 u}\,dx = \int \sec^2 u \cdot u'\,dx = \int \left(1+tg^2 u\right) \cdot u'\,dx = tg\ u + C$$

$$\int \frac{1}{sen^2 x} \cdot dx = \int cosec^2 x\ dx = \int \left(1+cotg^2 x\right) dx = -cotg\ x + C$$

$$\int \frac{u'}{sen^2 u}\,dx = \int cosec^2 u \cdot u'\,dx = \int \left(1+cotg^2 u\right) \cdot u'\,dx = -cotg\,u + C$$

Ejercicios

$$\int \left(\cos x - senx\right) dx$$

$$\int \left(\cos x - senx\right) dx = senx + \cos x + C$$

$$\int \left(3x^2 - \sec^2 x\right) dx$$

$$\int \left(3x^2 - \sec^2 x\right) dx = x^3 - tg\,x$$

$$\int e^x \cos e^x\ dx$$

$$\int e^x \cos e^x\ dx = sen\ e^x + C$$

$$\int x\,\mathrm{sen}\left(x^2+5\right)dx$$

$$\int x\,sen\left(x^2+5\right)dx=\frac{1}{2}\int sen\left(x^2+5\right)\,2x dx=-\frac{1}{2}\cos\left(x^2+5\right)\;+\;C$$

$$\int\frac{sen\left(\ln x\right)}{x}dx$$

$$\int\frac{sen\left(\ln x\right)}{x}dx=\int sen\left(\ln x\right)\frac{1}{x}dx=-\cos\left(\ln x\right)+C$$

$$\int\cos^3 x\,dx$$

$$\int\cos^3 x\,dx=\int\cos^2 x\;\cos x\,dx=\int\left(1-\mathrm{sen}^2 x\right)\cos x\,dx=$$

$$\int\left(\cos x-\mathrm{sen}^2 x\cos x\right)dx=\int\cos x\,dx-\int\mathrm{sen}^2 x\cos x dx=$$

$$\int\cos x\,dx-\frac{1}{3}\int 3\,\mathrm{sen}^2 x\cos x dx=\mathrm{sen}\,x-\frac{1}{3}\mathrm{sen}^3 x+\mathrm{C}$$

$$\int\mathrm{sen}^4 x\,dx$$

$$\int\mathrm{sen}^4 x\,dx=\int\left(\frac{1-\cos 2x}{2}\right)^2 dx=\int\frac{1-2\cos 2x+\cos^2 2x}{4}dx=$$

$$=\frac{1}{4}\int dx-\frac{1}{4}\int 2\cos 2x\,dx+\frac{1}{4}\int\cos^2 2x\,dx=$$

$$=\frac{1}{4}x-\frac{1}{4}\sin 2x+\frac{1}{4}\int\frac{1+\cos 4x}{2}dx=$$

$$=\frac{1}{4}x-\frac{1}{4}sen\;2x+\frac{1}{8}x+\frac{1}{32}sen\,4x+C$$

$$=\frac{3}{8}x-\frac{1}{4}sen\;2x+\frac{1}{32}sen\;4x+C$$

$$\int\mathrm{sen}^5 x\;\cos^2 x\,dx$$

$$\int sen^5 x \cos^2 x\, dx = \int sen\, x\ sen^4 x \cos^2 x\, dx =$$

$$= \int \left(1-\cos^2 x\right)^2 sen\, x \cos^2 x\, dx =$$

$$= \int \left(1-2\cos^2 x+\cos^4 x\right) sen\, x \cos^2 x dx$$

$$= \left(\int \cos^2 x\ sen\, x - 2\cos^4 x\ sen\, x + \cos^6 x\, sen\, x\right)dx$$

$$= -\frac{1}{3}\cos^3 x + \frac{2}{5}\cos^5 x - \frac{1}{7}\cos^7 x + C$$

$$\int \frac{dx}{\operatorname{sen} x \cos x}$$

$$= \int \frac{\operatorname{sen}^2 x + \cos^2 x}{\operatorname{sen} x \cos x} dx = \int \frac{\operatorname{sen} x}{\cos x} dx + \int \frac{\cos x}{\operatorname{sen} x} dx =$$

$$= -\ln(\cos x) + \ln(\operatorname{sen} x) + C = \ln\left(\frac{\operatorname{sen} x}{\cos x}\right) + C = \ln(\operatorname{tg} x) + C$$

$$\int sen^2 4x\, dx$$

$$\int sen^2 4x\, dx = \int \frac{1-\cos 8x}{2} dx = \frac{1}{2}x - \frac{1}{16} sen\, 8x + C$$

$$\int \cos^5 x\ dx$$

$$\int \cos^5 x\ dx = \int \cos^4 x \cos x\, dx = \int \left(1-sen^2 x\right)^2 \cos x\, dx =$$

$$= \int \cos x\, dx - 2\int sen^2 x \cos x\, dx + \int sen^4 x \cos x\, dx =$$

$$= sen\, x - \frac{2}{3} sen^3 x + \frac{1}{5} sen^5 x + C$$

$$\int \sec^4 x\, dx$$

$$\int \sec^4 x\,dx = \int \sec^2 x \ \sec^2 x\,dx = \int \left(1+tg^2 x\right) \sec^2 x\,dx =$$

$$\int \left(\sec^2 x + \sec^2 x \ tg^2 x\right)dx = tg\ x + \frac{1}{3} tg^3 x \ + C$$

$$\int \mathrm{tg}^2\, x \ dx$$

$$\int \mathrm{tg}^2\, x \ dx = \int \left(1+\mathrm{tg}^2\, x - 1\right)dx = \int \left(1+\mathrm{tg}^2\, x\right)dx - \int dx = \mathrm{tg}\, x - x + \mathrm{C}$$

$$\int \mathrm{cos}ec^2 \left(3x+1\right) dx$$

$$\int \mathrm{cos}ec^2 \left(3x+1\right) dx = \frac{1}{3}\int \mathrm{cos}ec^2 \left(3x+1\right) 3\,dx = -\frac{1}{3} \mathrm{cot}\, g\left(3x+1\right) \ + C$$

$$\int \mathrm{cosec}^4\, x \ dx$$

$$\int \mathrm{cosec}^4\, x \ dx = \int \mathrm{cosec}^2\, x \ \mathrm{cosec}^2\, x\,dx =$$

$$\int \left(1+\mathrm{cotg}^2 x\right) \mathrm{cosec}^2\, x\,dx = \int \mathrm{cosec}^2\, x + \mathrm{cotg}^2 x \ \mathrm{cosec}^2\, x\,dx =$$

$$= -\mathrm{cotg}\, x - \frac{1}{3}\mathrm{cotg}^3 x + \mathrm{C}$$

$$\int \mathrm{cot}\, g^2 x\,dx$$

$$\int \mathrm{cot}\, g^2 x\,dx = \int \left[\left(1+\mathrm{cot}\, g^2 x\right) - 1\right]dx = -\,\mathrm{cot}\, g\ x - x \ + C$$