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M.Sc. (Mathematics) (Sem: -3rd)
MATHEMATICAL METHODS
SUBJECT CODE: MMAT1-314
Paper ID: [19220514]

Time Allowed: 3 hrs.

Maximum Marks: 60

Instruction to Candidates:

1. **Section-A** is Compulsory.
2. Attempt any **three** questions from **Section-B**.
3. Attempt any **two** questions from **Section-C**.

Section –A

(4 marks each)

- Q1. Find the solution of integral equation by using successive approximation to the third order

$$\phi(x) = \left(\frac{3}{2}e^x - \frac{1}{2}xe^x - \frac{1}{2} \right) + \frac{1}{2} \int_0^1 \xi \phi(\xi) d\xi$$

- Q2. If a kernel is symmetric, then all its iterated kernels are also symmetric.

$$\int_0^\pi [(y')^2 + 2y \sin x] dx$$

- Q3. Find the curve which makes an extremum given that it passes through (0, 0) and (π , 0).

- Q4. Find the plane curve of fixed perimeter which encloses maximum area.

Section –B

(8 marks each)

- Q5. Classify the following integral equation $g(s) = \frac{1}{2} + s - \int_0^1 (s-t) g^2(t) dt$ as Fredholm or Volterra integral equation, linear or nonlinear and homogeneous or non-homogeneous.

$$y(x) = \lambda \int_a^b K(x, \xi) y(\xi) d\xi$$

- Q6. For the Fredholm integral equation with symmetric kernel, prove that : (i) The eigen functions corresponding to two different eigen values are orthogonal over (a, b). (ii) The eigen values are real.

$$\int_{x_1}^{x_2} [y^2 + (y')^2 + 2y \sec hx] dx.$$

- Q7. Find the extremal of the functional $\int_{x_1}^{x_2}$

- Q8. Find the curve passing through (x1, y1) and (x2, y2) which when rotated about the x-axis gives a minimum surface area.

Section –C

(10 marks each)

- Q9. (i) Solve the following Abel's problem $\frac{\pi}{2} x = \int_0^x \frac{1}{\sqrt{x-t}} g(t) dt.$

(ii) Reduce following initial value problem into Volterra integral equation: $y'' + xy = 1$, $y'(0) = 0 = y(0)$.

- Q10. (i) To find the equation of the plane curve down which a particle acted upon by gravity would descend from one fixed point to another fixed point in the shortest possible time.
 (ii) (a) Explain with suitable examples: Iterated and reciprocal functions, Nonlinear and Singular equations.
 (b) State and prove that Hadamard's theorem of integral equation.

$$\delta \int_1^2 \left[x^2 (y')^2 + 2y(x+y) \right] dx = 0$$

- Q11. (i) Solve the variation problem given that $y(1) = y(2) = 0$.
 (ii) (a) Write a short note on: Fredholm's solution of Dirichlet's problem and Neumann's problem.
 (b) Explain in brief Schwarz's inequality.