PDM: Chapter 6, Section 2: Proving Identities

In the last section, we found that we can show that an equation is an identity by graphing each side of the equation to see if the graphs overlap. In this section, we will be proving identities algebraically instead.

To prove an identity, there are many techniques:

- Rewrite one side using definitions, known identities, and algebraic properties until it equals the other side
- Rewrite each side independently until expressions are obtained that are known to be equal.
- Begin with a known identity and transform it using reversible steps until the desired identity appears
- Transform both sides of the equation to be proved using reversible steps until an equation known to be an identity appears

Common identities that we can use to start:

$$sin^2(x) + cos^2(x) = 1 \qquad csc(x) = \frac{1}{sin(x)}$$

$$sec(x) = \frac{1}{cos(x)}$$

$$tan(x) = \frac{sin(x)}{cos(x)} \qquad cot(x) = \frac{1}{tan(x)} = \frac{cos(x)}{sin(x)}$$

Prove the identity and specify its domain.

1. $sec^2(x) + csc^2(x) = sec^2(x)csc^2(x)$

2. $sin^4(x) - cos^4(x) = \frac{tan\,(x) - cot(x)}{sec\,(x)\,csc(x)}$

3. $tan\,(x) \;+\; cot\,(x) \;=\; sec\,(x)\,csc(x)$

4. $cot\,(2x)\,tan\,(2x) \;=\; sin\,(2x)\,csc(2x)$

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Homework

1. For each of the following, prove the identity and identify the domain:
   a. $cot^2(x) + 1 = csc^2(x)$

   b. $sin(x)\,cot(x) = cos(x)$

   c. $cos(x)\,tan(x) = sin(x)$

d. $tan(x)\, cot(x) = cos^2(x) + sin^2(x)$

e. $csc^2(x)\, sin(x) = \frac{sec^2(x) - tan^2(x)}{sin(x)}$

f. $tan(x) + cot(x) = sec(x)\, csc(x)$