

NAME:.....CLASS:...

.....

SECOND TERM E-LEARNING NOTES
JS3 (BASIC 9)

SUBJECT: MATHEMATICS

SCHEME OF WORK

WEEK TOPIC

- 1. Revision of first term work**
- 2. Simple Equations involving Fractions:** Simple Equations involving fractions.
Word problems leading to simple equations involving fractions
- 3. Simultaneous Linear equations:** Compilations of tables of values. Graphical solution of simultaneous linear equations in two variables.
- 4. Solution of simultaneous linear equations:** Method by elimination and method by substitution.
- 5. Similar shapes:** identification of similar figures-triangles, rectangles, squares, cubes and cuboids. Enlargement and scale factor
- 6. Use of scale factor in calculating lengths, areas and volumes of similar figures.**
- 7. Trigonometry:** the sine, cosine and tangent of an acute angle. Applications of Tri gonometrical ratios to finding distances and lengths
- 8. Area of plane Figures:** Area of plane figures-Triangles, parallelogram, trapezium and circle. Calculations of the areas of lands
- 9. Construction:** Bisection of a segment-using a pair of compasses and a ruler. Bisection of an angle. Construction of angles 90° , 45° , 60° and 30° . Copying giving angles. Solving quantitative reasoning aptitude problems on construction.
- 10. Revision.**
- 11. Examination.**

WEEK 1

Revision of Last Term's Work. The teacher should do a thorough revision of last term's work. Topics that were not well understood by the students or areas that were not well covered due to shortage of time or other reasons should be treated and class exercises, class activities, assignments, e.t.c should be given within this week.

WEEK 2.

TOPIC: EQUATIONS WITH FRACTIONS.

CONTENTS:

- ☐ Solving simple equations with fractions
- ☐ Word problems leading to fractions
- ☐ Word problems leading to equations with fractions

SIMPLE EQUATIONS WITH FRACTIONS

Equations such as $\frac{2}{3} + x = 1$, $\frac{3x}{2} + \frac{3}{5} = 10$, $\frac{2y}{5} = \frac{3}{5}$ e.t.c are equations involving fractions.

To solve any of these equations, we consider the L.C.M of the denominators and multiply each term of the equation by the L.C.M to clear the fractions and solve the equation as usual.

Example 1

Solve the equation: $\frac{x-4}{5} = 2 - \frac{x}{2}$

Solution

The denominators of the fractions in this equation are; 5 and 2.
L.C.M of 5 and 2 is 10.

We multiply through by this L.C.M. i.e
 $10 \times \frac{(x-4)}{5} = (2 \times 10) - (10 \times \frac{x}{2}) = 2$
 $(x - 4) = 20 - 5x$
Opening brackets we have,
 $2x - 8 = 20 - 5x$

Collecting like terms we have,

$$5x + 2x = 20 + 8$$

$$7x = 28.$$

$$\text{Thus, } x = \frac{28}{7} = 4$$

Example 2

$$\text{Solve } \frac{2x}{3} + \frac{4}{5} = \frac{17}{15}$$

Solution

The denominators are 3, 5 and 15.

Their L.C.M is 30

$$\text{Thus, } 30 \times \frac{2x}{3} + 30 \times \frac{4}{5} = 30 \times \frac{17}{15}$$

$$\text{? } 10 \times 2x + 6 \times 4 = 2 \times 17$$

$$\text{? } 20x + 24 = 34$$

$$\text{? } 20x = 34 - 24 = 10$$

$$\text{? } 20x = 10$$

$$\text{? } x = \frac{10}{20} = \frac{1}{2}$$

EVALUATION

Solve

$$(a) \frac{1}{y} + \frac{1}{5} = \frac{1}{3} \quad (b) \frac{5}{8} = \frac{10}{x} \quad (c) \frac{y-5}{4} = 8$$

NOTE: Sometimes it is not necessary to find the L.C.M of the denominators. If the equation has a single denominator on both sides of the equation, we simply solve the equation by cross multiplying as illustrated in the next example below.

Example 3 Solve i. $\frac{2}{t} = \frac{14}{21}$ ii. $\frac{1}{3r} - \frac{1}{24} = 0$

Solution

i. $\frac{2}{t} = \frac{14}{21}$

cross multiplying, we have,

$$2 \times 21 = t \times 14$$

$$42 = 14t$$

$$t = \frac{42}{14} = 3$$

ii. $\frac{1}{3r} - \frac{1}{24} = 0$

Rearranging the terms,

$$\frac{1}{3r} = \frac{1}{24}$$

Cross multiplying, we have,

$$1 \times 24 = 3r \times 1,$$

$$24 = 3r$$

$$r = \frac{24}{3} = 8$$

Example 4

Solve $\frac{2}{m-3} = \frac{3}{2m-1}$

Solution

Cross multiplying, we have,

$$2(2m - 1) = 3(m - 3)$$

Opening brackets, we have,

$$4m - 2 = 3m - 9$$

Collecting like terms

$$4m - 3m = 2 - 9$$

$$m = -7$$

WORD PROBLEMS INVOLVING FRACTIONS.

When solving word problems, identify the unknown and represent it by any letter of the alphabet. Form an equation in terms of the unknown based on the given information and solve the equation. Study the table below

S.N	word problem	unknown	mathematical form
1	a number added to triple the number	k	$k + 3K$
2	Three-quarter of a number subtracted from twice the number	r	$2r - \frac{3r}{4}$
3	The product of 5 and one-quarter of a number added to ten	m	$5 \times (\frac{m}{4} + 10)$
4	eight subtracted from three times a number is the same as twice the number	z	$3z - 8 = 2z$
5	Two-third of a number subtracted from thrice the number gives five	d	$3d - \frac{2d}{3} = 5$
6	The positive difference between nine and five	-----	$9 - 5$
7	one-quarter of w subtracted from twenty	w	$20 - (\frac{1}{4} \times w)$

EVALUATION

Write the following in mathematical form

- The positive difference between $2a$ and $5b$
- One-third of a number subtracted from n gives ten
- Five times m plus twice n gives ten z
- The product of two-fifth of k and two

- v. one-fourth of the product of eight and five
- vi. Three-fifth of the difference between four and y

Example 4

A fisherman had 30 fish in his net. He ate some of them and discovered that there are 19 fish left. How many did he eat?

Solution

Total number of fish in the net = 30.

Let number of fish eaten = x

Number of fish left in the net = 19

Hence, $30 - x = 19$

Or $x = 30 - 19 = 11$. Hence the man ate 11 fish

Example 5

The sum of the ages of a man and his son is 56 years. In 8 years time, the ratio of their ages will be 13:5. (a) How old are they now (b) find the difference between their ages.

Solution

(a) Let the age of the man be m years.

Therefore the son's age will be $(56 - m)$ years.

In 8 years time, their ages will be $(8 + m)$ years (man)

and $(8 + 56 - m)$ years (son).

The ratio of their ages at this time is 13:5.

Hence, $\frac{8+m}{8+56-m} = \frac{13}{5}$

Cross multiplying,

we have $5(8 + m) = 13(64 - m)$

Opening brackets,

$40 + 5m = 832 - 13m$

Collecting like terms,

$5m + 13m = 832 - 40$

$18m = 792$

$$m = \frac{792}{18} = 44.$$

Hence, the man's age is 44 years and the son's age is $56 - 44 = 12$ years.

(b) The difference between their ages is $44 - 12 = 32$

Example 6

When a certain number is subtracted from 56 and the result divided by 5, it is the same as if 14 is added to the number and the result divided by 2. Find the number

Solution

Let the number be n

The number subtracted from 56 is $56 - n$

The result divided by 5 is $\frac{56-n}{5}$

14 added to the number is $n + 14$

The result divided by 2 is $\frac{n+14}{2}$

Hence, according to the given information,

$$\frac{56-n}{5} = \frac{n+14}{2}$$

Cross multiplying, we have,

$$2(56 - n) = 5(n + 14)$$

Opening brackets,

$$112 - 2n = 5n + 70$$

Collecting like terms,

$$2n - 5n = 70 - 112$$

$$- 3n = - 42$$

$$n = \frac{42}{3} = 14$$

$$n = 14$$

EVALUATION:

- Subtract 8 from 78, then find one-seventh of the result
- if I add 8 to a certain number and I double the result, my final answer is 36.
What is the number?

WEEKEND ASSIGNMENT.

Refer to the following text books and do the following exercises

- (1) (Work book, Nelson Functional Mathematics for Junior secondary school, book 3). Exercise E, Pages 9 and 10 questions 1 – 5.
- (2) (New General Mathematics for West Africa, for junior secondary school, U.B.E Edition Book 3). Exercise 2f, Page 26 Questions 1 – 5.
- (3) (Nelson Functional Mathematics. Book 3) Exercise 2.6 pages 34 and 35. Questions 1, 4 , 14 , 19 , 24

WEEKEND READING ASSIGNMENT.

New General Mathematics for West Africa, for junior secondary school, U.B.E Edition Book 3), Pages 102-108

WEEK 3

TOPIC: SIMULTANEOUS LINEAR EQUATIONS

CONTENT:

- ☐ Concept of simultaneous equations
- ☐ Preparing table of values for variables
- ☐ Graphical approach to the solution of simultaneous equations

SUB TOPIC: CONCEPT OF SIMULTANEOUS EQUATIONS

Equations such as $x - y = -1$ i

$x + y = 3$ ii

With two variables x and y to be solved at the same time are called simultaneous equations.

SUB TOPIC: PREPARING TABLE OF VALUES.

The above equations $x - y = -1$, $x + y = 3$ are both linear equations with two variables. For a certain value of x , y has a corresponding value. For example, considering the equation $x - y = -1$, making y the subject of the formula, $y = x + 1$.

When x takes the value 0, $y = 0 + 1 = 1$, When x takes the value 1, $y = 1 + 1 = 2$. When x takes the value 2, $y = 2 + 1 = 3$. When x takes the value 3, $y = 3 + 1 = 4$. The result can be displayed in a table as shown below

x	0	1	2
y	1	2	3

The table above is known as table of specification. The table can be extended if we assign more values to x

EVALUATION

- Prepare table of specification for the equation $y = 3 - x$ for $x = 0$ to 2
- Copy and complete the table for the relation $y = \frac{1}{2} + x$

x	-2	-1	0	1	2
y	-1.5	?	0.5	?	?

SUB TOPIC: GRAPHICAL METHOD OF SOLVING SIMULTANEOUS EQUATIONS

This method involves creating tables of values for both variables or unknowns. y is usually made the subject of the formula and referred to as the dependent variable while x is the independent variable. Simple values can be assigned to x in order to determine the corresponding values of y . Although two plotted points are enough to draw the lines of both equations, it is advisable to plot at least three points. After plotting, the points are expected to lie on the same straight line for both equations. The lines should intersect at a point or this can be achieved by

extending one or both lines with a ruler. The point of intersection of the lines gives the solutions of the simultaneous equations. If the lines are parallel, it means the simultaneous equations have no solution.

NOTE: Educator should guide the students on the choice of simple and reasonable scales for plotting the points.

Example 1

Solve using graphical approach:

$$x - y = -1 \dots\dots\dots i$$

$$x + y = 3 \dots\dots\dots ii$$

Solution

We assign four values to x (0, 1, 2 and 3) and use them to find the corresponding values of y for both equations.

Considering equation 1 ($y = x + 1$).

When $x = 0$, $y = 0 + 1 = 1$.

When $x = 1$, $y = 1 + 1 = 2$.

When $x = 2$, $y = 2 + 1 = 3$.

When $x = 3$, $y = 3 + 1 = 4$.

$$y = x + 1$$

x	0	1	2	3
y	1	2	3	4

Similarly, from equation ii,

$$y = 3 - x.$$

When $x = 0$, $y = 3 - 0 = 3$.

When $x = 1$, $y = 3 - 1 = 2$.

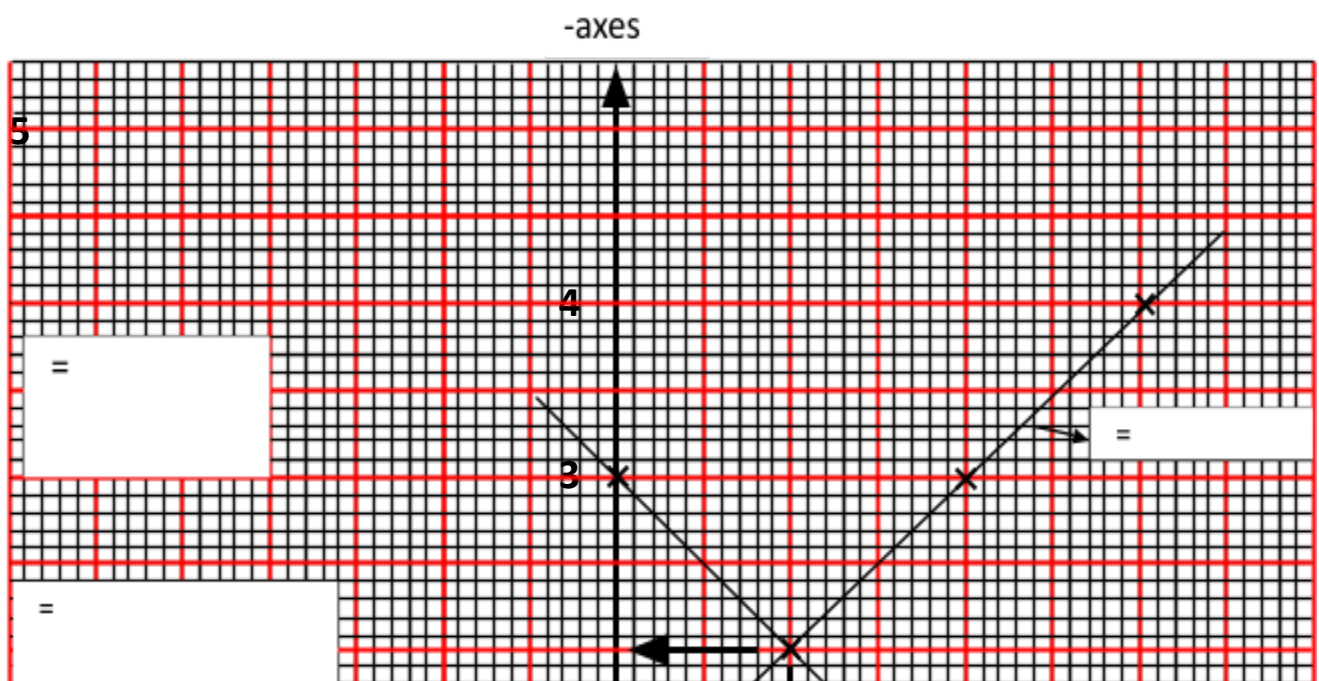
When $x = 2$, $y = 3 - 2 = 1$.

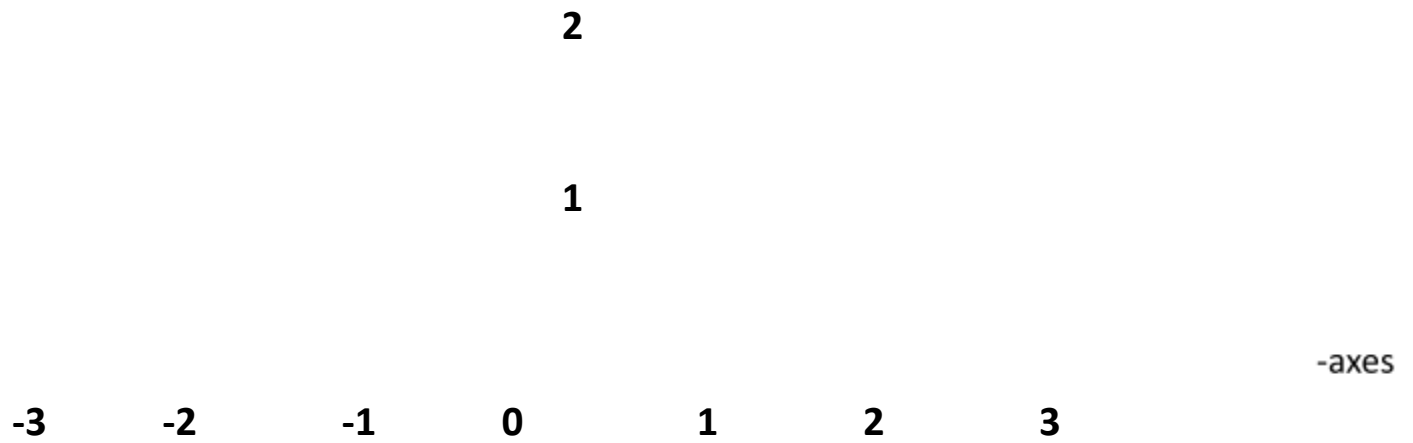
When $x = 3$, $y = 3 - 3 = 0$

$$y = 3 - x$$

x	0	1	2	3
y	3	2	1	0

Using a scale of 2cm = 1 unit for both axes, we plot a graph of y against x for both equations. See the graph below





E

Example 2.

Solve graphically the simultaneous equations

$$2x - y = -1 \quad \text{and} \quad x - 2y = 4 \quad \text{for } -1 \leq x \leq 3$$

Solution

Considering equation $2x - y = -1$,

We make y the subject of the formula

$$\text{Thus, } y = 2x + 1$$

$$\text{When } x = -1, \quad y = 2(-1) + 1 = -2 + 1 = -1$$

$$\text{When } x = 0, \quad y = 2(0) + 1 = 0 + 1 = 1$$

When $x = 1$, $y = 2(1) + 1 = 2 + 1 = 3$

When $x = 2$, $y = 2(2) + 1 = 4 + 1 = 5$

When $x = 3$, $y = 2(3) + 1 = 6 + 1 = 7$

Similarly, considering equation $x - 2y = 4$,

$$y = \frac{x-4}{2}$$

When $x = -1$, $y = \frac{-1-4}{2} = -\frac{5}{2} = -2.5$

When $x = 0$, $y = \frac{0-4}{2} = -\frac{4}{2} = -2$

When $x = 1$, $y = \frac{1-4}{2} = -\frac{3}{2} = -1.5$

When $x = 2$, $y = \frac{2-4}{2} = -\frac{2}{2} = -1$

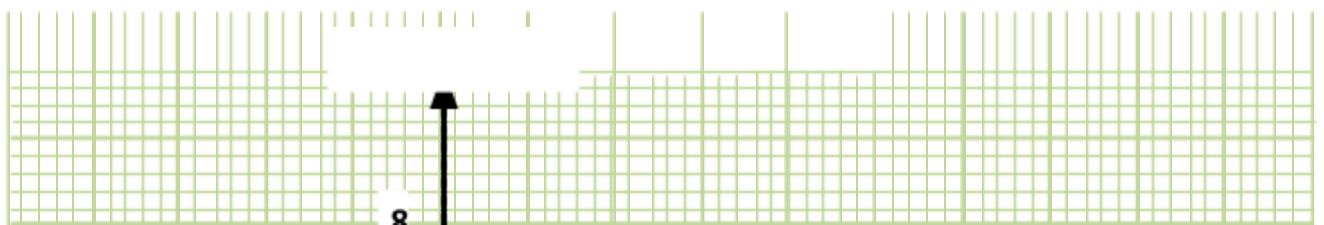
When $x = 3$, $y = \frac{3-4}{2} = -\frac{1}{2} = -0.5$

The results are tabulated below.

$$y = 2x + 1 \quad y = \frac{x-4}{2}$$

x	-1	0	1	2	3
y	-1	1	3	5	7
x	-1	0	1	2	3
y	-2.5	-2	-1.5	-1	-0.5

Using a scale of 1cm = 1unit on y-axes and 2cm = 1unit on x-axes, we plot a graph of y against x for the two equations as shown in the graph below.



-2-1

1

2

3

4

Solution:

Considering the equation, $3x + 2y = 4$,

we make y the subject of the formula.

$$2y = 4 - 3x$$

$$\text{or } y = \frac{4-3x}{2}.$$

$$\text{When } x = -2, \quad y = \frac{4-3(-2)}{2} = \frac{4+6}{2} = \frac{10}{2} = 5.$$

$$\text{When } x = -1, \quad y = \frac{4-3(-1)}{2} = \frac{4+3}{2} = \frac{7}{2} = 3.5 \quad .$$

$$\text{When } x = 0, \quad y = \frac{4-3(0)}{2} = \frac{4}{2} = 2$$

$$\text{When } x = 1, \quad y = \frac{4-3(1)}{2} = \frac{1}{2} = 0.5$$

Similarly, for $2x + 3y = 1$,

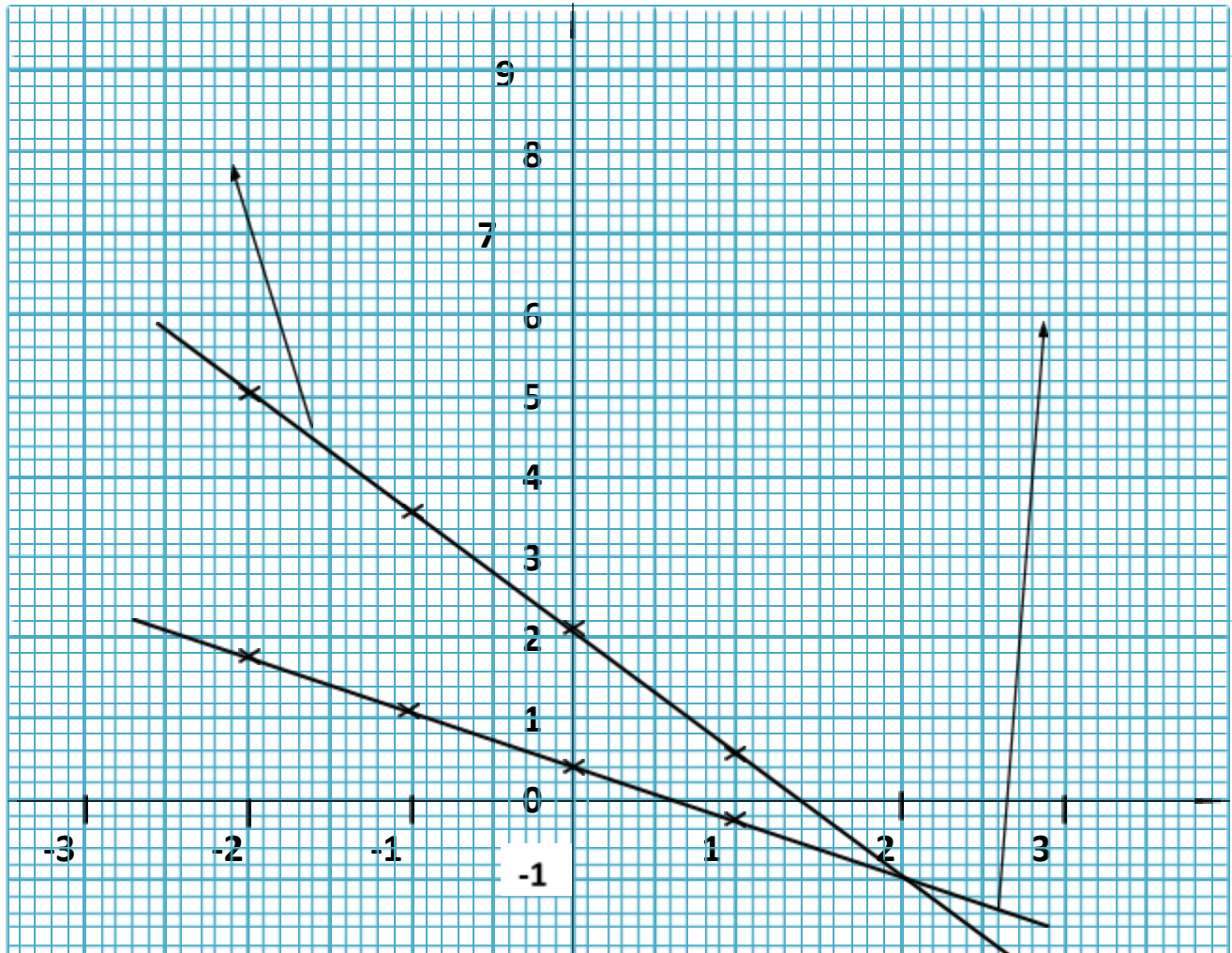
$$y = \frac{1-2x}{3}$$

$$\text{When } x = -2, \quad y = \frac{1-2(-2)}{3} = \frac{1+4}{3} = \frac{5}{3} = 1.67$$

$$\text{When } x = -1, \quad y = \frac{1-2(-1)}{3} = \frac{1-2(-1)}{3} = \frac{3}{3} = 1$$

$$\text{When } x = 0, \quad y = \frac{1-2(0)}{3} = \frac{1}{3}$$

$$\text{When } x = 1, \quad y = \frac{1-2(1)}{3} = -\frac{1}{3}$$



The two lines intersect at the coordinates (2, -1). Hence is the solution of the simultaneous equations.

WEEK END ASSIGNMENT:

Nelson Functional Mathematics for Junior Secondary Schools, Book 3. Page 87. Questions 1-5.

WEEK END READING ASSIGNMENT:

New General Mathematics for West Africa, for junior secondary school, U.B.E Edition Book 3). Pages 133 – 136

TOPIC: SOLVING SIMULTANEOUS EQUATIONS USING METHOD BY SUBSTITUTION AND METHOD BY ELIMINATION

CONTENT:

I. METHOD BY SUBSTITUTION

II. METHOD BY ELIMINATION

SUB TOPIC: METHOD BY SUBSTITUTION.

This is a method in which one variable is made the subject of the formula and substituted in the second equation. This will lead to a third equation with one variable. The equation is then solved for the value of the variable. This value can be substituted in any of the first two equations for the value of the second variable. See examples below.

Example 1.

Solve $2x + y = 4$1

$$3x - y = -1\frac{1}{2} \text{2}$$

using method by substitution

Solution

$$2x + y = 4 \text{Eqn i}$$

$$3x - y = -\frac{3}{2} \text{Eqn ii}$$

Considering eqn i ($2x + y = 4$),

$$y = 4 - 2x.$$

Now put $y = 4 - 2x$ into Eqn ii.

$$\text{Thus } 3x - (4 - 2x) = -\frac{3}{2} \text{Eqn iii.}$$

$$3x - 4 + 2x = -\frac{3}{2}$$

$$5x - 4 = -\frac{3}{2}$$

$$5x = 4 - \frac{3}{2} = \frac{8-3}{2} = \frac{5}{2}$$

$$5x = \frac{5}{2}$$

$$10x = 5$$

$$x = \frac{10}{5} = \frac{1}{2}$$

Substitute $x = \frac{1}{2}$ in Eqn i.

$$\text{This gives } 2\left(\frac{1}{2}\right) + y = 4$$

$$1 + y = 4,$$

$$y = 4 - 1 = 3.$$

$$\text{Thus } x = \frac{1}{2} \text{ and } y = 3$$

$$\text{Check } 2\left(\frac{1}{2}\right) + 3 = 1 + 3 = 4$$

.....Eqn 1

$$3\left(\frac{1}{2}\right) - 3 = \frac{3}{2} - 3 = \frac{3-6}{2} = -\frac{3}{2}$$

.....Eqn 2

Example 2

$$\text{Solve } 3x - 4y = -6 \text{.....1}$$

$$7x - 2y = 8 \text{.....2}$$

Using method by substitution.

Solution

$$3x - 4y = -6$$

.....eqn 1

$$7x - 2y = 8$$

.....eqn 2

From equation 1, make x the subject of the formula

$$3x = 4y - 6 \text{ Thus } x = \frac{4y-6}{3}$$

Substitute $x = \frac{4y-6}{3}$ into eqn 2

$$\text{Thus } 7\left(\frac{4y-6}{3}\right) - 2y = 8$$

.....eqn 3

Multiplying through by 3 the L.C.M, we have,

$$3 \times 7\left(\frac{4y-6}{3}\right) - 3 \times 2y = 3 \times 8$$

$$\text{Or } 7(4y - 6) - 6y = 24$$

$$\text{Opening brackets, } 28y - 42 - 6y = 24$$

Collecting like terms,

$$28y - 6y = 42 + 24$$

$$\text{Or } 22y = 66$$

$$\text{Or } y = \frac{66}{22} = 3$$

Now, put $y = 3$ into eqn1

$$\text{Thus, } 3x - 4(3) = -6$$

$$3x - 12 = -6$$

$$\text{Or } 3x = 12 - 6$$

$$3x = 6$$

$$\text{Thus, } x = \frac{6}{3} = 2$$

Hence $x = 2$, $y = 3$ is the solution of the simultaneous equations

$$\text{Check: } 3(2) - 4(3) = 6 - 12 = -6$$

.....eqn 1

$$7(2) - 2(3) = 14 - 6 = 8$$

.....eqn 2

EVALUATION

Using method by substitution, solve $x - y = 0$, $y - 3x = 1$

METHOD BY ELIMINATION

Example 3.

Solve $2x + y = 4$, $3x - y = -1\frac{1}{2}$ using elimination method

Solution

$$2x + y = 4 \dots\dots\dots 1$$

$$3x - y = -1\frac{1}{2} \dots\dots\dots 2$$

Adding equations 1 and 2, we have,

$$2x + 3x + y + (-y) = 4 + (-1\frac{1}{2})$$

$$5x + \cancel{y} - \cancel{y} = 4 - \frac{3}{2}$$

$$5x = \frac{8-3}{2} = \frac{5}{2}$$

$$x = \frac{5}{2} \times \frac{1}{5} = \frac{1}{2}$$

Substitute $x = \frac{1}{2}$ into eqn 1

$$2 \times \frac{1}{2} + y = 4$$

$$1 + y = 4$$

$$y = 4 - 1 = 3$$

$$\text{Hence } x = \frac{1}{2} \quad y = 3$$

Example 4

$$\text{Solve } 7x - 2y = 8 \dots\dots\dots 1$$

$$3x - 4y = -6 \dots\dots\dots 2$$

Solution:

Multiply eqn 1 by 4 and eqn 2 by 2

$$7x - 2y = 8 \dots\dots\dots \text{eqn 1} \times 4$$

$$3x - 4y = -6 \dots\dots\dots \text{eqn 2} \times 2$$

$$4 \times 7x - 4 \times 2y = 8 \times 4 \dots\dots\dots 1$$

$$2 \times 3x - 2 \times 4y = -6 \times 2 \dots\dots\dots 2$$

$$28x - 8y = 32 \dots\dots\dots 1$$

$$6x - 8y = -12 \dots\dots\dots 2$$

Subtract eqn 2 from eqn 1

$$28x - 6x - 8y - (-8y) = 32 - (-12)$$

$$22x - 8y + 8y = 32 + 12$$

$$22x = 44$$

$$x = \frac{44}{22} = 2$$

Substitute $x = 2$ into eqn 1

$$7(2) - 2y = 8$$

$$14 - 2y = 8$$

$$2y = 14 - 8 = 6$$

$$y = \frac{6}{2} = 3.$$

Therefore $x = 2, y = 3$

EVALUATION

Solve the equations $3x + 2y = 7, 4x - 2y = 7$ using method by elimination

WEEKEND ASSIGNMENT

(a) Solve the equation $\frac{p-5q}{2} = p - 3, 5p - 10q = 16$ simultaneously using any method of your choice

(b) Solve $2x + 3y - 1 = 3, 2x - 2y = 2$ Using method by graph, method by elimination and method by substitution

WEEKEND READING ASSIGNMENT

Nelson Functional Mathematics for Junior Secondary school, book 3 Pg87– 91.

WEEK 5.

TOPIC: SIMILAR FIGURES AND ENLARGEMENT

CONTENT:

- i. Concept of similar figures
- ii. Examples of similar figures
- iii. Enlargement and scale factor

SUB TOPIC: CONCEPT OF SIMILAR FIGURES

Two figures are similar, if they look alike and one is an enlargement or reduction of the other. Mathematically, if two shapes are similar, the ratio of their corresponding sides should be constant.

EXAMPLES OF SIMILAR FIGURES



x	-1	0	1	2	3
y	-1	1	3	5	7
x	-1	0	1	2	3
y	-2.5	-2	-1.5	-1	-0.5
x	-2	-1	0	1	
y	5	3.5	2	0.5	
x	-2	-1	0	1	
y	1.67	1	0.33	-0.33	



(a) Similar photographs

(b) Similar cylinders



(c) Similar Cuboids

Look at the two photographs in (a) above. Although the two photographs are not of the same size, they look alike. The same applies to (b) and (c). We say that the pictures in (a), (b) and (c) are similar to each other.

ENLARGEMENT AND SCALE FACTOR

The photograph in fig. a above is enlarged such that the sides are proportional. In this case, the ratio of the corresponding sides will be constant. Look at the example below.

Example 1

- Measure the length and breadth of the picture in fig. (a)
- Find the ratio length : breadth
- What do you notice?

Solution

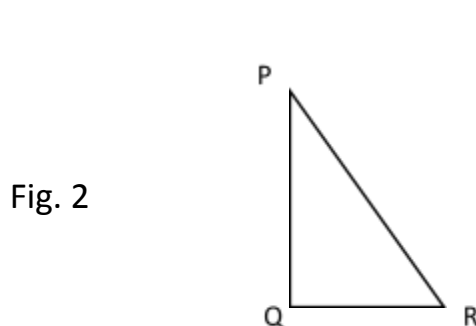
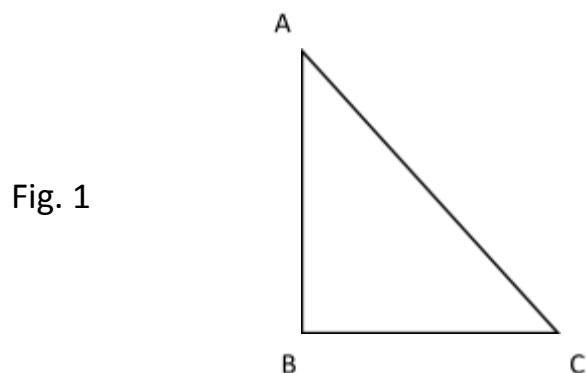
- Length = 2cm, Breadth = 4cm (smaller picture)
Length = 2.4cm Breadth = 4.8cm (larger picture)

(b) Length : breadth = $4 : 2 = 4/2 = 2$ (smaller picture)
Length : breadth = $4.8 : 2.4 = 4.8/2.4 = 2$ (larger picture)

(c) The ratio of length to breadth for each picture is 2
Length : breadth is constant for both pictures in (a) this means that the first picture is an enlargement of the second picture. Hence when a shape is reduced or enlarged to give another shape such that the ratio of their corresponding sides are the same, we say, the two shapes are similar mathematically. This ratio is what we call the scale factor. The scale factor for the pictures in (a) is 2

Example 2

Consider the shapes below and indicate whether they are similar or not



Solution

If the two shapes are similar, the ratio of their corresponding sides should be equal. i.e. $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

Considering Fig. 1,

AB = 3.2cm

BC = 2.9cm

AC = 4.3cm

Similarly, in fig. 2,

PQ = 2.4cm

QR = 1.8cm

PR = 3.0cm

$$\frac{AB}{PQ} = \frac{3.2}{2.4} = 1.33$$

$$\frac{BC}{QR} = \frac{2.9}{1.8} = 1.61$$

$$\frac{AC}{PR} = \frac{4.2}{3.0} = 1.40$$

From the above calculations, it is clear that the ratios of the corresponding sides are not equal i.e. $\frac{AB}{PQ} \neq \frac{BC}{QR} \neq \frac{AC}{PR}$. We therefore conclude that the two triangles are not similar.

NOTE: Educator should print out the above example(fig.1 and fig. 2), make a copy available to students and instruct them to carry out the measurements. Do not temper with the dimension of the shapes. The students should not carry out the measurement directly on the screen of their laptops. The diagrams in the evaluation question below should also be printed out for the students.

ORAL QUESTIONS.

State whether the following statements are **true** or **false**

- i. Two figures are similar, if one is a reduction of the other
- ii. Two figures are not similar, if one is an enlargement of the other
- iii. All isosceles triangles are similar
- iv. All cuboids are similar
- v. All pictures or images in the television are similar to the original bodies
- vi. All cubes are similar

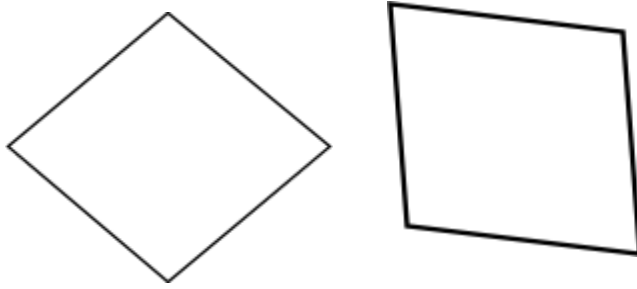
EVALUATION

Consider the following shapes and indicate whether they are similar or not. Use measurement if you are not sure.

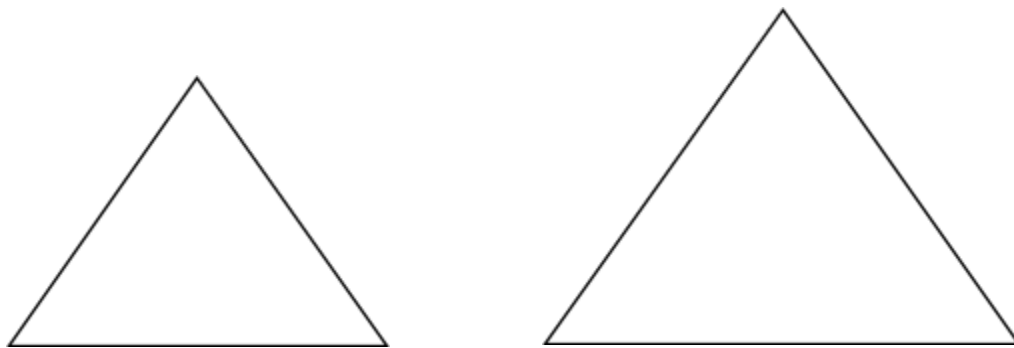
(1)



(2)



(3)



WEEKEND ASSIGNMENT

(Work book, Nelson Functional Mathematics for Junior secondary school, book 3).
Page 44 and 45. Exercise A

WEEK 6

TOPIC: USE OF SCALE FACTOR IN CALCULATING LENGTHS, AREAS AND VOLUME OF SIMILAR FIGURES.

CONTENT:i. Scale Factor (length ratio)

ii. Area of similar shapes

iii. Volume of similar shapes

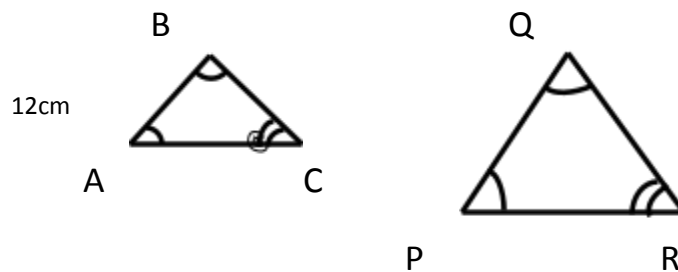
SUB – TOPIC 1: SCALE FACTOR

When two shapes are similar, the ratio of their corresponding sides are the same.
This ratio is called scale factor or length ratio.

Thus, the scale factor of two shapes is the ratio of two corresponding lengths.

Example

1. The scale factor of two similar triangles ABC and PQR shown below is 2.5. If AB = 12 cm and AC = 10 cm, find the length of (a) PQ (b) PR.



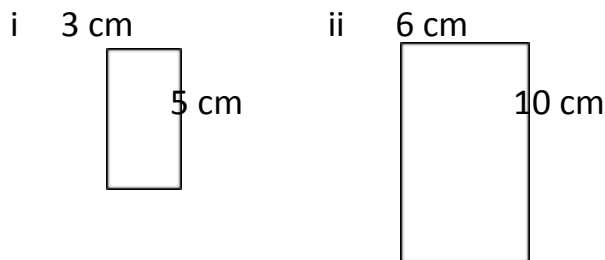
Solution

The scale factor of 2.5 means each corresponding side on triangle PQR is 2.5 times the corresponding length on triangle ABC.

(a) $PQ = AB \times 2.5 = 12 \times 2.5 = 30 \text{ cm}$

(b) $PR = AC \times 2.5 = 10 \times 2.5 = 25 \text{ cm}$

2. Determine the scale factor of the figures below.



Solution

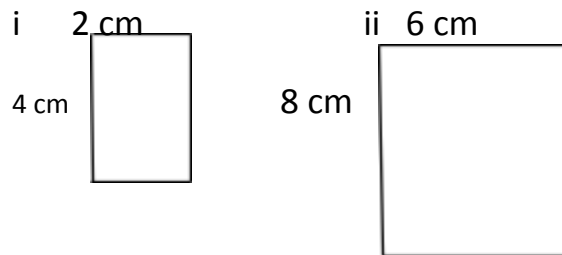
Scale factor of the big rectangle to the small rectangle = ratio of their corresponding sides.

$$\frac{10 \text{ cm}}{5 \text{ cm}} \text{ or } \frac{6 \text{ cm}}{3 \text{ cm}} = 2 \text{ cm}$$

$\therefore$ scale factor = 2 in both cases.

EVALUATION

Find the scale factor of the big rectangle.

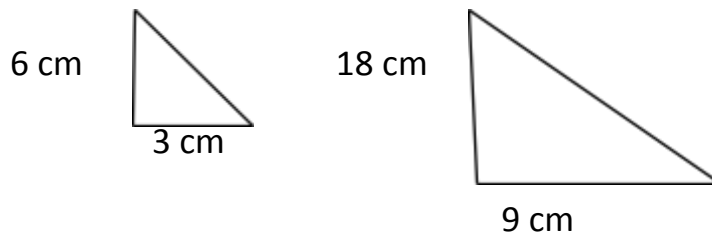


SUB-TOPIC 2 : AREA OF SIMILAR SHAPES

The ratio of the area of a figure to the area of the other similar figure is known as the area factor.

Examples

1. Determine :
 - (a) The scale factor
 - (b) The area factor of the triangles below.



Solution

(a) Scale factor = $\frac{18}{6} = \frac{9}{3} = 3$

(b) Area factor = $\frac{9 \times 9}{3 \times 3} = 3 \times 3 = 9$

2. A fashion designer measured and instructed that a particular material 6 m by 4.5 m will make good birthday shirts for two brothers. If the elder brother is 1.5m, how tall is the younger brother?

Solution

$$\text{Area factor} = \frac{6m^2}{4.5m^2} = \frac{6}{4.5} = \frac{60}{45} = \frac{2}{\sqrt{3}}$$

But the height of the elder brother is 1.5m. Therefore, the height of the younger brother would be if $2 = 1.5$

$$\sqrt{3} = ?$$

$$\frac{2}{\sqrt{3}} = \frac{1.5}{x}$$

$$x = \frac{\sqrt{3} \times 1.5}{2} = \frac{2.598}{2}$$

$$x = 1.3 \text{ (one decimal place)}$$

3. Two similar cones have corresponding slant heights of 8 cm and 12 cm.

a. Find the ratio of their areas.

b. The area of the smaller cone is 102 cm^2 .

Calculate the area of the larger cone.

Solution

$$\frac{\text{slant height of smaller cone}}{\text{slant height of larger cone}} = \frac{8 \text{ cm}}{12 \text{ cm}} = \frac{2}{3}$$

$$\text{a. } \frac{\text{area of smaller cone}}{\text{area of laeger cone}} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$\text{b. } \frac{\text{area of smaller cone}}{\text{area of larger cone}} = \frac{4}{9}$$

$$= \frac{102 \text{ cm}^2}{\text{area of larger cone}} = \frac{4}{9}$$

$$\text{area of larger cone} = \frac{102 \times 9}{4}$$

$$= 229.5 \text{ cm}^2$$

NOTE : The ratio of the areas of two similar shapes is the square of the scale factor of the two shapes.

EVALUATION

1. If the scale factor of a picture 15 cm long is 1 : 10, what is the length of the original object ?
2. Given that the ratio of the radii of two circles is $\frac{3}{4}$:

- a. Find the ratio of their areas.
 - b. If the smaller circle has a radius of 12 cm, find the radius of the bigger circle.
3. Two similar triangles have corresponding sides of length 4 cm and 7 cm. find the ratio of their areas.

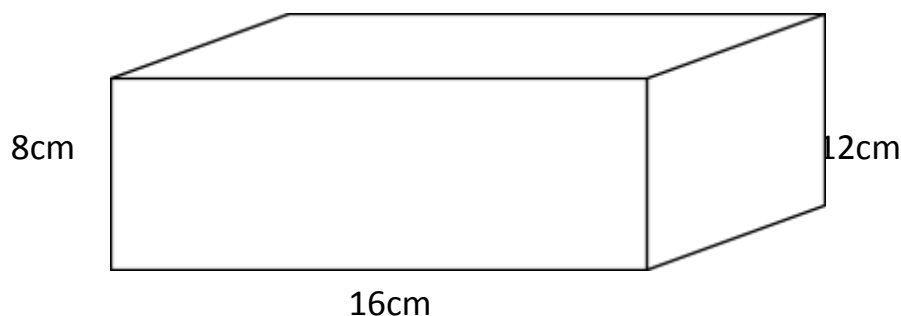
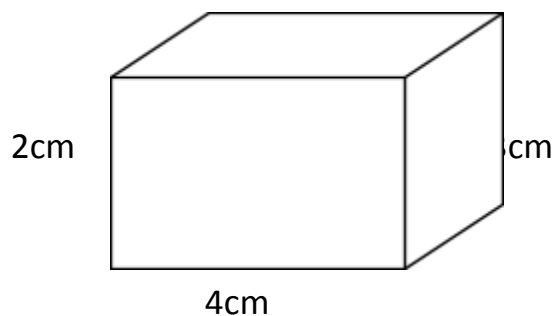
SUB-TOPIC 3: VOLUME OF SIMILAR SHAPES.

The ratio of the volumes of similar shapes could be compared to find the unknown parameters. This is possibly done using **scale factor**. The factor could be their edges, height or volume.

When the volumes of similar shapes are compared, ***the ratio of the volumes of the similar solids is the cube of the scale factor of the two solids.***

Examples

1. Using the dimensions of the similar shapes below, find their scale factor and the volume factor.



Solution:

Take the ratio of their corresponding sides

$$\text{Scale factor} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{1}{4}$$

$$\text{Therefore, Volume factor} = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$$

2. Two cylindrical pots similar in shape are respectively 5cm and 25cm high. If the smaller pot holds 1.5litres, find the capacity of the larger one.

Solution:

$$\text{Scale factor} = \frac{5}{25} = \frac{1}{5}$$

$$\text{Hence, the volume factor} = \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} = \frac{1}{125}$$

Since the smaller pots holds 1.5litres, the bigger pot will holds $1.5 \times 125 = 187.5$ litres.

3. Two similar blocks have corresponding edges of lengths 9cm and 27cm.

a). Find the ratio of their masses.

b). If the mass of the larger block is 216g, find the mass of the smaller block.

Solution:

$$\text{Their linear scale factor} = \frac{9}{27} = \frac{1}{3}$$

$$\text{Therefore, their volume factor} = \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$$

a). Since mass = volume $\times$ density,

hence, mass is proportional to volume.

$$\text{Thus, mass factor} = \frac{1}{27}$$

Hence, ratio of their masses = 1 : 27

b). Mass of the larger block = 216g

Therefore, mass of the smaller block = $216 \times \frac{1}{27} = 8\text{g}$.

EVALUATION

The sides of two cuboids are in the ratio 3:5. What is the ratio of their volumes?

READING ASSIGNMENT:

Functional Mathematics BK 3, pg 162-164.

New General Mathematics BK 3, pg 179-180.

WEEKEND ASSIGNMENT:

Functional Mathematics BK 3, pg 163, Ex. 10.3, No. 1-5.

New General Mathematics BK 3, pg 181, Ex. 18b, No. 1-5.

WEEK 7

TOPIC: TRIGONOMETRY

CONTENT:i. The trigonometric ratios

ii. Using Sine and Cosine Tables

iii Applications of sine and cosine

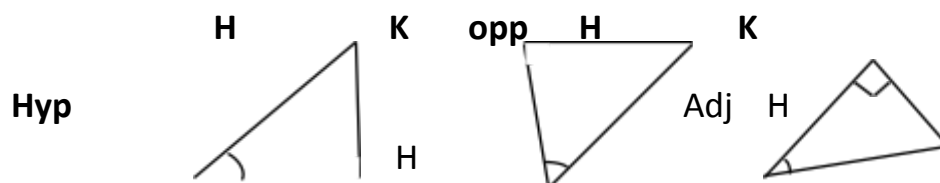
iv. The tangent of an acute angle

v. Applications of Trigonometric ratios to finding distances and lengths.

Sub Topic 1 : The Trigonometric ratios

A trigonometric ratio is a ratio of the lengths of **two sides of a right – angled triangle**. The three trigonometric ratios are sine (sin), cosine (cos) and tangent (tan).

The figure below show triangle AHK in various positions



A K A A

The sides of the triangles are as follows.

AH, the hypotenuse

KH, the side opposite to $\hat{A}$

AK, the side adjacent to $\hat{A}$

These are abbreviated to **hyp**, **opp**, **adj**, respectively, so that:

$$\sin A = \frac{\text{opp}}{\text{hyp}} \qquad \cos A = \frac{\text{adj}}{\text{hyp}} \qquad \tan A = \frac{\text{opp}}{\text{adj}}$$

USE OF SINE

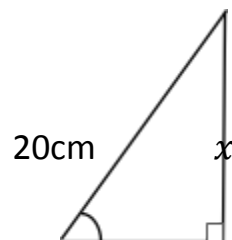
Sines and cosines of angles are used to find the lengths of unknown side in triangles. The table below gives the sines of some chosen angles.

Angle A	Sin A
30°	0.5000
35°	0.5736
40°	0.6428
45°	0.7071
50°	0.7660
55°	0.8192
60°	0.8660

The values in the table are given to 4 significant figures.

Example 1

Calculate the value of x in the figure below.



$$55^\circ$$

Solution

In the figure above, the hypotenuse is given and x is opposite the given angle.

Thus ,

$$\sin 55^\circ = \frac{x}{20}$$

$$\text{Thus, } x = 20 \times \sin 55^\circ$$

$$= 20 \times 0.8192$$

$$= 16.384 \text{ cm}$$

$$= 16 \text{ cm to 2 s.f.}$$

USE OF COSINE

Angle A	Cos A
30°	0.8660
35°	0.8192
40°	0.7660
45°	0.7071
50°	0.6428
55°	0.5736
60°	0.5000

Example 2 :

Calculate the value of y in the figure below.

17 cm

50°



y

Solution

The hypotenuse is 17 cm and y is adjacent to the given angle. Thus use the cosine of the given angle.

$$\cos 50^\circ = \frac{y}{17}$$

$$\text{Thus, } y = 17 \times \cos 50^\circ$$

$$= 17 \times 0.6428$$

$$= 10.93 \text{ cm}$$

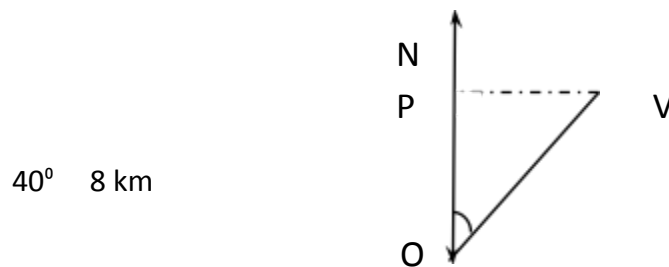
$$= 11 \text{ cm to 2 s.f}$$

Example 3:

A village is 8 km on a bearing of 040° from a point O. Calculate how far the village is north of O.

Solution

The figure below shows the position of the village, V in relation to O.



It is required to find the length of OP.

OP is adjacent to the known angle.

Use the cosine of 40° .

$$\cos 40^\circ = \frac{OP}{8}$$

$$\text{Thus, } OP = 8 \times \cos 40^\circ \text{ km}$$

$$= 8 \times 0.7660 \text{ km}$$

$$= 6.128 \text{ km}$$

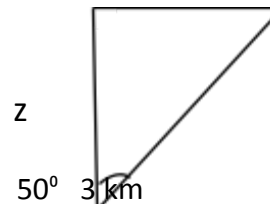
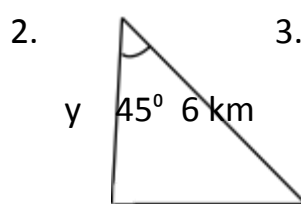
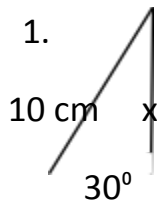
$$OP = 6.1 \text{ km to 2 s.f.}$$

The village is 6.1 km north of o.

EVALUATION:

Use the values in the tables above to answer the following questions. Give all answers correct to 2 s.f.

Find the values of x, y and z in each of the triangles bellow.



Sub Topic 2 : Using sine and cosine tables

Example 4:

Use tables to find the angles:

- whose sine is $= \frac{2}{7}$,
- whose cosine is 0.4478,
- whose sin is 0.649 2,
- whose cosine is 0.568 2,

Solution

- Let the angle be A, then $\sin A = \frac{2}{7}$.

Express $\frac{2}{7}$ as a decimal fraction correct to 4 d.p.

$$\frac{2}{7} = 0.2857$$

$$\sin A = 0.2857$$

Looking within the sine table entries,
0.2857 is opposite 16° and under 0.6°.

Thus, $A = 16.6^\circ$.

- b. Let the angle be B , then $\cos B = 0.4478$.

Looking within the cosine table entries,

0.447 8 is opposite 63° and under 0.4.

Thus, $B = 63.4^\circ$.

- c. Let the angle be x , then $\sin x = 0.6492$.

Looking within the sin table entries, the nearest value to 0.649 2 is 0.648

1. 0.648 1 is opposite 40° and under 0.4.

The difference between 6 492 and 6 481 is 11. Look for 11 in the difference column along the row of 40° . 11 is under 8.

Thus, $x = 40.40^\circ + 0.08^\circ$

$$X = 40.48^\circ$$

- d. Let the angle be y , then $\cos y = 0.5682$.

Looking within the cosine table entries, the nearest value to 0.568 2 is

0.567 8. 0.567 8 is opposite 55° and under 0.4. The difference between

5 682 and 5 678 is 4. Look for 4 in the difference column along the row of

55° . 4 is under 3. As the angles increase, their cosines decrease, therefore subtract the difference.

Thus, $y = 55.40^\circ - 0.03^\circ$

$$Y = 55.37^\circ$$

EVALUATION:

Use tables to find the angles whose

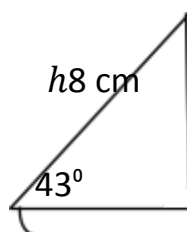
- a. Sines b. cosines are as follows.

1. 0.5878 2. 0.798 6 3. $\frac{2}{5}$ 4. 0.564 8 5. 0.632 7

SUB TOPIC 3 :APPLICATIONS OF SINE AND COSINE

Example 5:

Calculate the length of the hypotenuse of the triangle below.



Solution

$$\sin 43^\circ = \frac{8}{h}$$

$$h \times \sin 43^\circ = 8$$

$$h = \frac{8}{\sin 43^\circ} \text{ cm} = \frac{8}{0.6820} \text{ cm}$$

From reciprocal tables, $\frac{1}{0.6820} = 1.466$,

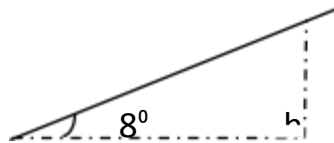
$$\begin{aligned} \text{Thus, } h &= 8 \times 1.466 \text{ cm} \\ &= 11.728 \text{ cm} \end{aligned}$$

$$h = 12 \text{ cm to 2 s.f}$$

Example 6:

A car travels 120 m along a straight road which is inclined at 8° to the horizontal. Calculate the vertical distance through which the car rises.

120 m



Solution

From the sketch of the road, h is the vertical distance.

$$\sin 8^\circ = \frac{h}{120}$$

$$h = 120 \times \sin 8^\circ$$

$$= 120 \times 0.1392$$

$$= 16.70 \text{ m}$$

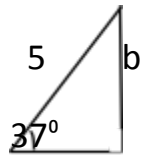
$$h = 17 \text{ m to 2 s.f}$$

NOTE: Educators are to solve more examples for students.

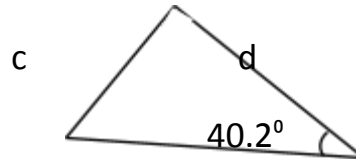
EVALUATION:

1. Calculate the lengths a, b, c, d, in the diagrams below. All lengths being in cm.

i



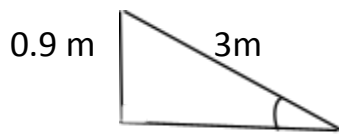
ii



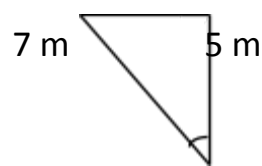
a 2

2. Calculate the angles α , β in the triangles below.

i



ii



Sub Topic 4: Tangent of an acute angle

Tangents of some chosen angles

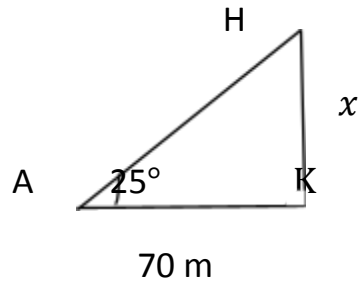
<i>Angle A</i>	<i>Tan A</i>
25°	0.4663
30°	0.5774
35°	0.7002
40°	0.8391
45°	1.000
50°	1.192
55°	1.428
60°	1.732
65°	2.145
70°	2.747

Example 7:

The angle of elevation of the top of a building is 25° from a point 70 m away on a level ground. Calculate the height of the building.

Solution

HK represents the height of the building; AK is on level ground.



$$\frac{HK}{KA} = \tan 25^\circ$$

Let HK be x cm. $KA = 70$ cm and, from table above, $\tan 25^\circ = 0.4663$. Hence, $\frac{x}{70}$

$$= 0.4663$$

$$x = 0.4663 \times 70$$

$$= 4.663 \times 7$$

$$= 32.641$$

$\therefore$ The height of the building is 33 m to 2 s.f.

Example 8:

Use tables to find the tangents of angles

- a. 32° b. 59.6°

Solution

- a. Looking within the table entries, the number opposite 32° and under 0.0° is 0.6249.

Thus, $\tan 32^\circ = 0.6249$.

- b. The number opposite 59° and under 0.6° is 1.704. Thus, $\tan 59.6^\circ$ is 1.704. Thus, $\tan 59.6^\circ = 1.704$.

Example 9:

Use tables to find the angles whose tangents are:

a. 0.9556 b. $\frac{7}{3}$

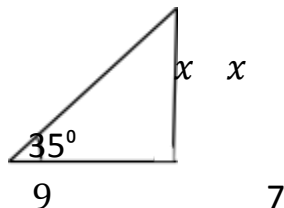
Solution

- a. Let the angle be A, then $\tan A = 0.9556$. Looking within the table entries, 0.9556 is opposite 43° and under 0.7°. Thus, $A = 43.7^\circ$
- b. Let the angle be B, then $\tan B = \frac{7}{3} = 2.333$ to 4 S.f. Thus, $B = 66.8^\circ$.

EVALUATION

1. Use table to find the value of x each triangles below.

a. b.



2. Use 4 figure table to find the tangent of the following:
a. 35° b. 23.1° c. 19.5°
3. Use 4 figure table to find the angles whose tangents are as follows.
a. 0.9325 b. 0.8847 c. $\frac{5}{8}$

SUB – TOPIC 5: APPLICATIONS OF TANGENTS

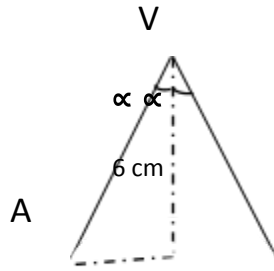
Tangent ratio can be used to solve real life problems such as angles of elevation and depressions and bearings.

Example 10:

A cone is 6 cm high and its vertical angle is 54° . Calculate the radius of its base.

Solution

The vertical angle is the angle between opposite slant heights VA and VB.



Thus, the vertical angle is 2α .

$$2\alpha = 54^\circ$$

$$\text{Thus } \alpha = 27^\circ$$

$$\text{In } \triangle AVO, \tan \alpha = \frac{r}{6}$$

$$r = 6 \tan 27^\circ$$

$$= 6 \times 0.5095$$

$$= 3.057$$

$$= 3.1 \text{ to 2 s.f.}$$

The radius of the base of the cone is 3.1 cm.

NOTE: More examples are to be solved for students by educators.

EVALUATION

1. A cone is 8 cm high and its vertical angle is 62° . Find the diameter of its base.
2. An isosceles triangle has a vertical angle of 116° , and its base is 8 cm long. Calculate its height.
3. Find the angle of elevation of the top of a flagpole 31.9 m high from a point 55 m away on level ground.

WEEKEND ASSIGNMENT

New General Mathematics for junior Secondary Schools 3 by M.F Macrae et al. Exercise 8h pages 84 – 85, Questions 1 (a – c), 2 (a – d).
Exercise 8d pages 79 & 80, Qs no 4 -6. Exercise 15b pages 145- 146, Qs no 4 – 7

READING ASSIGNMENT

New General Mathematics for junior Secondary Schools 2 by M.F Macrae et al. Pages 196 – 198. Pages 223 – 227 .

WEEK 8

TOPIC: AREA OF PLANE FIGURES

CONTENT:

- i. Area of basic shapes: Revision (triangles, rectangles, trapeziums, parallelogram and circles)
- ii. Using trigonometry in area problems
- iii. Area of circles and sectors
- iv. Land measure

SUB – TOPIC 1. AREA OF BASIC SHAPES

The formulae for areas of some basic shapes are given in below. These were previously found in book 1, Chapter 14.

- a. Area of rectangle = length x breadth = (l x b)

b. Area of trapezium = $\frac{1}{2}$ of (sum of parallel sides) x height = $\frac{1}{2}$ (a + b)h

c. Area of parallelogram = base x height = b x h

d. Area of circle = πr^2

e. Area of triangle = $\frac{1}{2}$ x product of sides containing the right angle.

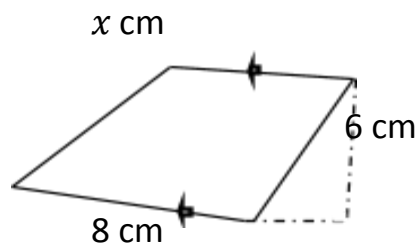
Examples:

1. Find the area of a triangle with base 15 cm and height 9 cm.

Solution

$$\begin{aligned}\text{Area of triangle} &= \frac{1}{2} \text{base} \times \text{height} \\ &= \frac{1}{2} \times 15 \times 9 \\ &= 67.5 \text{ cm}^2\end{aligned}$$

2. If the area of the trapezium below is $40\frac{1}{2} \text{ cm}^2$, find the value of x.



Solution

$$\begin{aligned}\text{Area of trapezium} &= \frac{1}{2} (x + 8) \times 6 \text{ cm}^2 \\ &= 3(x + 8) \text{ cm}^2\end{aligned}$$

Thus, $3(x + 8) = 40\frac{1}{2}$

$$x + 8 = 40 \frac{1}{2} \div 3 = 13 \frac{1}{2}$$

$$x = 13 \frac{1}{2} - 8 = 5 \frac{1}{2}$$

3. What is the diameter of a circle of area 3 850 m²?

Solution

$$\text{Area of circle} = \pi r^2 = 3\,850 \text{ m}^2$$

$$\text{Thus, } \frac{22}{7} r^2 = 3\,850$$

$$\begin{aligned} r^2 &= 3\,850 \times \frac{7}{22} \\ &= \frac{26950}{22} \end{aligned}$$

$$r^2 = 1225$$

$$r = \sqrt{1225}$$

$$r = 35\text{m}$$

$$\text{diameter } d = 2r$$

$$\therefore d = 2 \times 35 = 70\text{m}$$

EVALUATION

1. The seconds hand of a watch is 14 mm long. What area does it sweep through in 30 seconds?
2. A goat is tied by a rope $2 \frac{1}{2}$ m long to a peg in some grass. The goat eats 1 m² of grass in 28 min. How long will it take to eat all that it can reach?
3. A triangle has area of 35 cm² and a base of 10 cm. Find the height of the triangle.

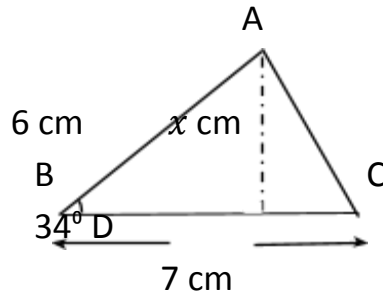
SUB – TOPIC 2: USING TRIGONOMETRY IN AREA PROBLEMS

Examples:

1. Find the area of $\triangle ABC$ to the nearest cm^2 if $BA = 6 \text{ cm}$, $BC = 7 \text{ cm}$ and $\hat{B} = 34^\circ$.

Solution

Let the height of the triangle be $x \text{ cm}$



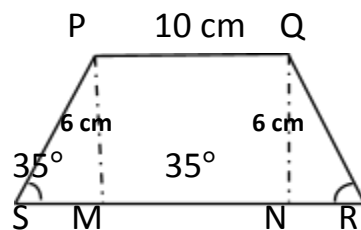
$$\text{In } \triangle ABD, \frac{x}{6} = \sin 34^\circ$$

$$\begin{aligned} x &= 6 \sin 34^\circ \\ &= 6 \times 0.5592 \\ &= 3.3552 \end{aligned}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \times BC \times AD$$

$$\begin{aligned} &= \frac{1}{2} \times 7 \times 3.3552 \text{ cm}^2 \\ &= \frac{1}{2} \times 23.4864 \text{ cm}^2 \\ &= 11.7432 \text{ cm}^2 \\ &= 12 \text{ cm}^2 \text{ to the nearest cm}^2. \end{aligned}$$

2. Find the area of the isosceles trapezium shown in the diagram below.



Solution

From the diagram, $SP = SM + MN + NR$

$PQ = MN = 10 \text{ cm}$ and $PS = QR$. (given)

Also, $SM = NR$ ($\Delta PSM \equiv \Delta QRN$)

Now in ΔSPM , the complement of 35° is 55° .

Using tan ratio:

$$\tan 55^\circ = \frac{SM}{6}$$

$$\therefore SM = 6 \tan 55^\circ$$

$$= 6 \times 1.428 = 8.569 \text{ cm.}$$

So, $SM = NR = 8.569 \text{ cm}$

$$\therefore SR = 8.569 + 10 + 8.569 = 27.138 \text{ cm}$$

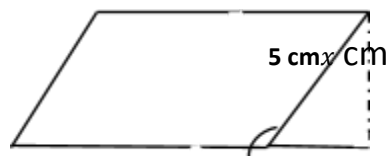
$$\begin{aligned} \text{Area of trapezium PQRS} &= \frac{1}{2}(PQ + SR)h \\ &= \frac{1}{2}(10 + 27.138) \times 6 \\ &= 3 \times 37.138 \\ &= 111.414 \text{ cm}^2 \end{aligned}$$

Area of trapezium = 110 cm^2 to 2 s.f.

3. Calculate the area of parallelogram PQRS if $QR = 5 \text{ cm}$, $RS = 6 \text{ cm}$, $\hat{QRS} = 118^\circ$.

P

Q



$$118^\circ$$

S 6 cm R D

Solution

Let QD be x cm.

$$\text{In } \triangle QRD, \hat{QRD} = 180^\circ - 118^\circ = 62^\circ$$

$$\frac{x}{5} = \sin 62^\circ$$

$$x = 5 \sin 62^\circ$$

$$= 5 \times 0.8829$$

$$= 4.4145$$

$$\text{Area of PQRS} = SR \times QD$$

$$= 6 \times 4.4145 \text{ cm}^2$$

$$= 26.487 \text{ cm}^2$$

$$= 26 \text{ cm}^2 \text{ to the nearest cm}^2.$$

N.B: For a triangle with sides a, b containing an angle θ , **area of triangle** $= \frac{1}{2} ab \sin \theta$

For a parallelogram with non – parallel sides a, b containing an angle θ .

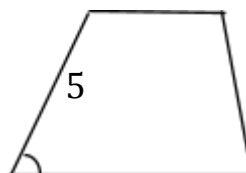
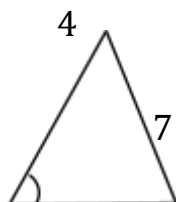
$$\text{Area of parallelogram} = ab \sin \theta$$

EVALUATION

1. The area of a parallelogram is 240 cm^2 . The angle between the two adjacent sides of the parallelogram is 145° . If the length of one of the adjacent side is 16 cm, find the length of the other side correct to the nearest cm.

2. Find the area of the following shapes. All dimensions are in cm.

a. b.



57°

6

63°

8

SUB – TOPIC 3: AREA OF CIRCLES AND SECTORS

A sector is part of a circle bounded by an arc and two radii.

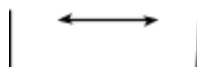
In the diagram bellow, O is the centre, AO and BO divide the inside of the circle into two parts called sectors. The smaller is called the minor sector (shaded), and the larger is called the major sector (unshaded)



$$\text{Area of sector} = \frac{\theta}{360} \times \pi r^2$$

Examples:

1. What is the area of a flat washer 4.8 cm in outside diameter, the hole being of diameter 2.2 cm?



$$2.2 \text{ cm } 22.2 \text{ cm } 2.2 \text{ cm. } 2$$

2

$$4.8 \text{ cm}$$

Solution

$$\text{Area} = \pi(2.4)^2 - \pi(1.1)^2 \text{ cm}^2$$

$$= \pi(2.4^2 - 1.1^2) \text{ cm}^2$$

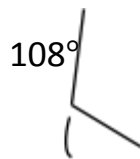
$$= \pi(2.4 + 1.1)(2.4 - 1.1) \text{ cm}^2$$

$$= \frac{22}{7} \times 3.5 \times 1.3 \text{ cm}^2$$

$$= 14.3 \text{ cm}^2$$

2. Find the area of a sector of radius 7 cm, the angle at the centre of the circle being 108° .

Solution



$$\text{Area of sector} = \frac{108}{360} \text{ of } \pi \times 7^2 \text{ cm}^2$$

$$= \frac{108}{360} \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2$$

$$= \frac{3}{10} \times 22 \times 7 \text{ cm}^2$$

$$= 46.2 \text{ cm}^2$$

EVALUATION

1. The area of a sector of a circle is 44 cm^2 . What is the radius of the circle if the angle subtended at the centre is 140° ?
2. A sector of a circle of radius 8 cm has an angle of 120° at the centre.
 - a. Find its perimeter
 - b. Find its area (take $\pi = 3.142$)
 Hint: perimeter = length of minor arc + radii.

$$\text{Length of arc} = \frac{\theta}{360} \times 2\pi r$$
3. **Find** the area between two circles with the same centre and of radius 5 cm and 9 cm respectively. Take $\pi = \frac{22}{7}$

SUB – TOPIC 4: LAND MEASURE

The hectare (ha) is a unit of area which is primarily used in the measurement of land. We can picture an hectare as the area about two times the size of a football pitch.

i.e, 1 hectare = $100 \text{ m} \times 100 \text{ m} = 10\,000 \text{ m}^2$.

Recall that $1 \text{ km}^2 = 1000 \text{ m} \times 1000 \text{ m} = 1\,000\,000 \text{ m}^2$.

So there are 100 hectares in a km^2 .

i.e $1 \text{ km}^2 = 100 \text{ hectares}$

or 1 hectare = 0.01 km^2 .

Examples

1. A village is roughly in the shape of a circle of diameter 400 m. Use the value 3 for π to find the approximate area of the village in hectares.

Solution

Radius of village = $\frac{1}{2}$ of 400 m = 200 m

Area of village $\approx \pi \times 200^2 \text{ m}^2$

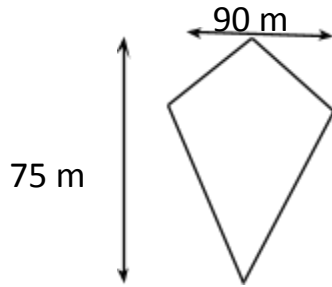
$\approx 3 \times 40\,000 \text{ m}^2$

$\approx 3 \times 4$ hectares

≈ 12 hectares

The area of the village is about 12 ha.

2. The diagram bellow shows a plot of land in the shape of a kite. Calculate the area of the land, giving your answer in hectares.



Solution

Recall that the diagonals of a kite bisect each other at right angles.

Area of a kite = $\frac{1}{2}$ of the product of the diagonals

$$= \frac{1}{2} \times 90 \times 75 = 3375 \text{ m}^2$$

$$\text{Now } 10\,000 \text{ m}^2 = 1 \text{ ha}$$

$$3375 \text{ m}^2 = \frac{1}{10\,000} \times 3375 \text{ ha}$$

$$= 0.3375 \text{ ha}$$

EVALUATION

1. A hockey pitch measures 90 m by 55 m. Express its area as a fraction of a hectare. Would you say that this is roughly half of a hectare?
2. A town is roughly in the shape of an equilateral triangle of side 600 m. What is the approximate area of the town in hectares? (Use the value 0.9 for $\sin 60^\circ$)
3. A football pitch is 110 m long and 75.5 m wide. Calculate its area, to the nearest m^2 ,
 - a. In square metres
 - b. In hectares.

NOTE: Educators should solve more examples from Nelson Functional Mathematics, for JS BK 3.

WEEKEND ASSIGNMENT

New General Mathematics for junior secondary school 3 by M.F Macrae et al.
Chapter 17 pg 166, exercise 17a questions no 1a – d, 2a – b, 3a – c, 4a – d.
Exercise 17 b questions 1a & b, 2a – c, 17c questions 3a – d.

READING ASSIGNMENT

New General Mathematics for Junior Secondary Schools by M.F Macrae et al.
Chapter 14 page 112- 113, 116- 119.

TOPIC: GEOMETRICAL CONSTRUCTIONS

CONTENT:

- i. Bisection of a segment
- ii. Bisection of an angle
- iii. Construction of special angles (90° , 45°)
- iv. Construction of special angles (60° , 30°)
- v. Construction of shapes
- vi. Copying given angles.
- vii. Quantitative reasoning

SUB – TOPIC 1: BISECTION OF A SEGMENT

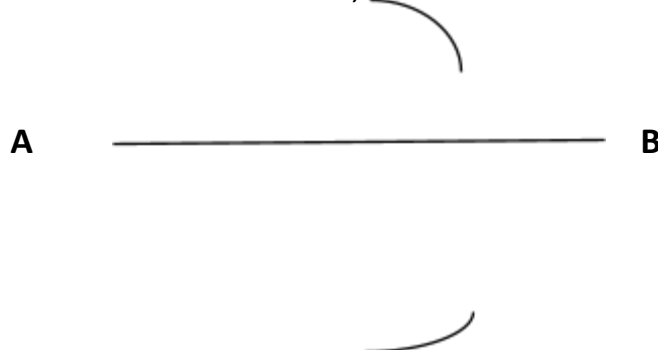
To bisect a straight line segment



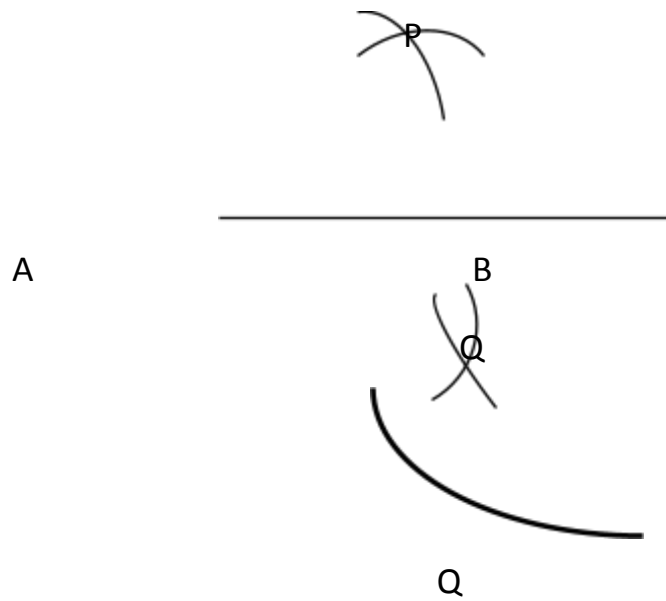
The **line segment** AB is the part of the line between A and B, including the points A and B.

To **bisect** the line segment AB means to divide it into two equal parts.

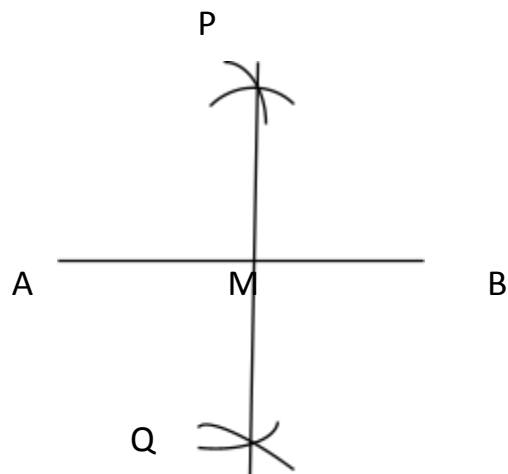
- a. **Open** a pair of compasses so that the radius is about $\frac{3}{4}$ of the length of AB.
- b. Place the sharp point of the compasses on A. Draw two arcs, one above, the other below the middle of AB, as shown below.



- c. Keep the same radius and place the sharp point of the compasses on B. Draw two arcs so that they cut the first arcs at P and Q as shown below.



d. Draw a straight line through P and Q so that it cuts AB at M.



M is the mid – point of AB. PQ meets AB perpendicularly. PQ is the perpendicular bisector of AB. Use a ruler and protractor to check that $AM = MB$ and $\hat{AMP} = \hat{BMP} = 90^\circ$.

EVALUATION

1. Draw any line segment AB. Use the above method to find the mid – point of AB. Check by measurement that your answer is correct.
2. Draw the following line segments and construct their perpendicular bisectors:
 - a. A 6 cm B
 - b. H 4.5 cm G

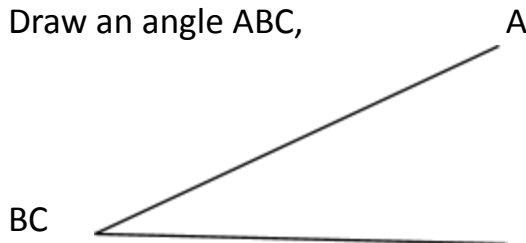
SUB – TOPIC 2 : BISECTION OF ANGLES

Given any angle ABC.

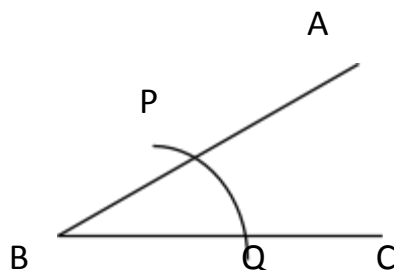
Required to construct a line BY which bisects angle ABC such that angle ABY = angle CBY.

CONSTRUCTION

1. Draw an angle ABC,

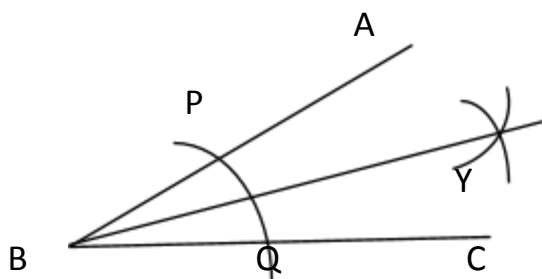


2. With B at the centre and using any convenient radius, construct an arc that cuts AB at P and BC at Q as shown below.



3. Place the needle point of the compass at P, with a convenient radius. Construct an arc in – between $\overline{AB}$ and $\overline{BA}$, then place the point at Q. with the same radius,

cut the last arc at Y. Draw line BY. The line BY bisects $\hat{ABC}$. Use a protractor to check if angle $ABY = \text{angle } CBY$.



EVALUATION

1. Draw any angle PQR, then bisect it and use a protractor to check if both angles are equal.
2. Use a protractor to draw angle PQR such that $\text{PQR} = 150^\circ$, then:
 - a. bisect angle $\hat{PQR}$.
 - b. bisect each of the angles in (a)
 - c. measure the angles.

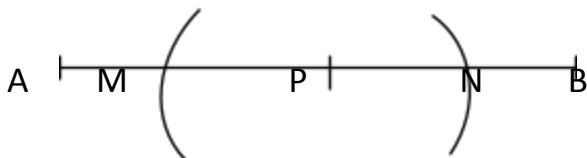
SUB – TOPIC 3: CONSTRUCTION OF SPECIAL ANGLES(90° AND 45°)

i. Construction of angle 90°

Given a point on a straight line AB. Required to construct a line PT through P such that $\angle TPA = \angle TPB = 90^\circ$.

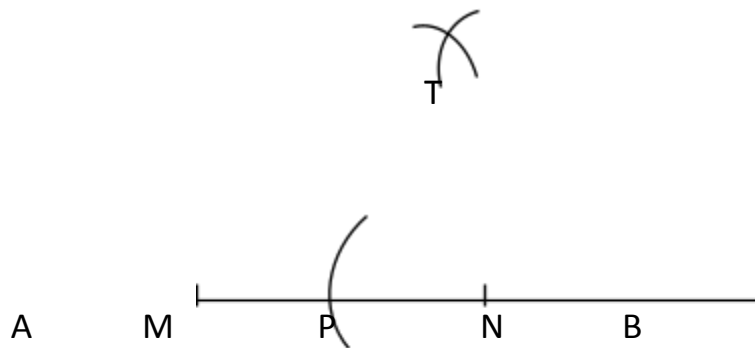
Construction

Draw line AB with a point P on it. With P as the centre and using any convenient radius, inscribe an arc cutting AP at M and PB at N.

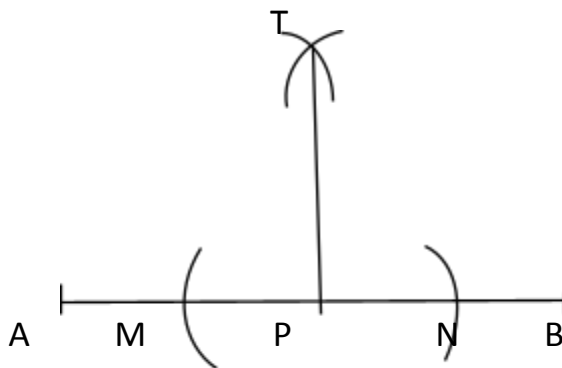


Now, place the needle point of the compass at M. Using any convenient radius (bigger than the former radius) inscribe an arc at T, then, place the needle point at

N and using the same radius, inscribe an arc, cutting the previous arc at T. See the figure below.



Finally, join PT.



PT is perpendicular to AB. Thus, $\angle TPA = \angle TPB = 90^\circ$. Use a protractor to check the result.

ii. Construction of angle 45°

$$\text{Since } 45^\circ = \frac{90^\circ}{2}$$

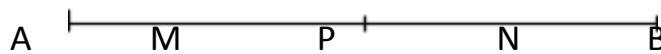
Then, to construct 45° , we may first construct an angle 90° and then bisect it.

Given: a point P on a straight line AB.

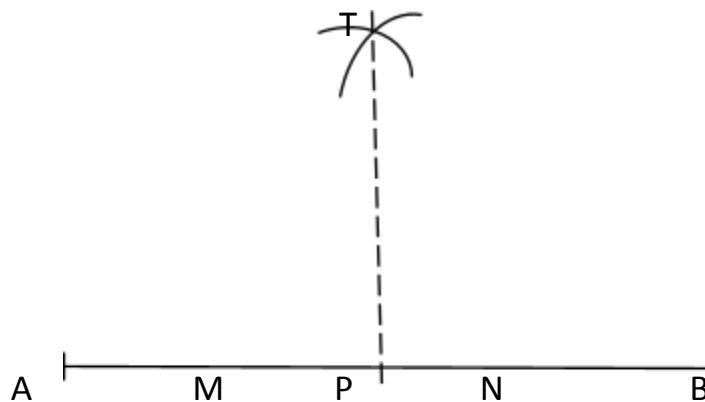
Required: to construct a line PD such that $\angle DPB = 45^\circ$.

Construction

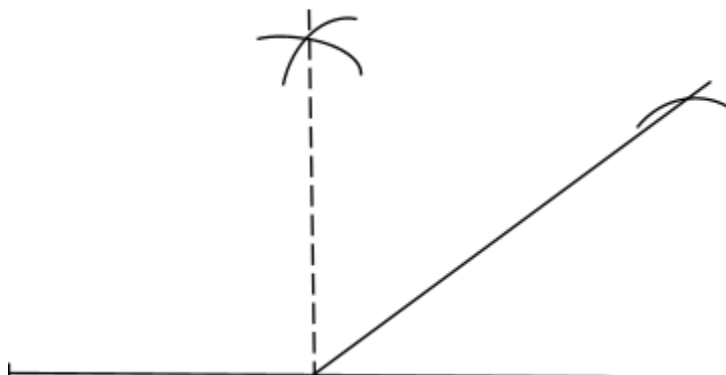
Draw a line AB with a point on it. With P as the centre and using any convenient radius, construct a semicircle cutting AP at M and PB at N as shown below.



Expand the former radius to a convenient size, then place the needle at M and inscribe an arc at T. Using the same radius, place the needle point at N and inscribe an arc cutting the previous arc at T. Join TP using dotted lines as shown below.



Place the needle point at N and inscribe an arc at D using a convenient radius. Place the needle point on the point of intersection of the semicircle and the dotted line TP, then inscribe an arc cutting the previous arc at D. Join DP with a thick line.



A M P N B

Thus, $\angle DPB = 45^\circ$.

Use a protractor to check the result in the diagram above.

EVALUATION

Construct angle $22\frac{1}{2}^\circ$.

Note : $22\frac{1}{2}^\circ = \frac{45^\circ}{2}$

Educators should guide students in this exercise.

Sub – Topic 4 : Construction of special angles ($60^\circ, 30^\circ$)

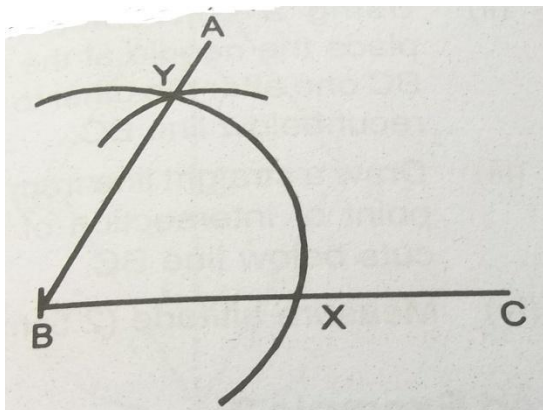
i. Construction of angle 60°

Given a line BC with B as the centre. Required to construct an angle $ABC = 60^\circ$.

Construction

Draw a line BC with B as the centre. Using any convenient radius, inscribe an arc that cuts BC at X, then place the needle point of the compass at X. Using the same radius, inscribe an arc that cuts the previous arc at Y.

Join YB and produce to A, $\angle ABC = 60^\circ$. With a protractor, measure angle ACB to check if it is really 60° .



ii. Construction of angle 30°

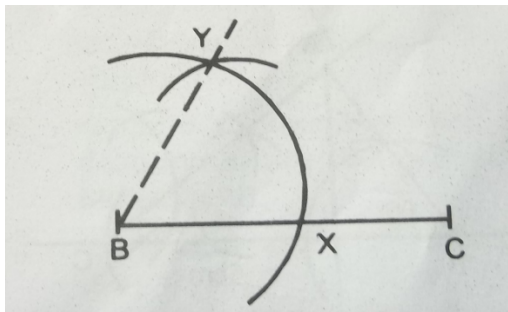
Since $30^\circ = \frac{60^\circ}{2}$, then to construct angle 30° , we may first construct an angle 60° and then bisect it.

Given a line BC with B at the centre. Required to construct an angle $ABC = 30^\circ$.

Construction

Draw a line BC with B at the centre. Inscribe an arc using any convenient radius that cuts BC at X.

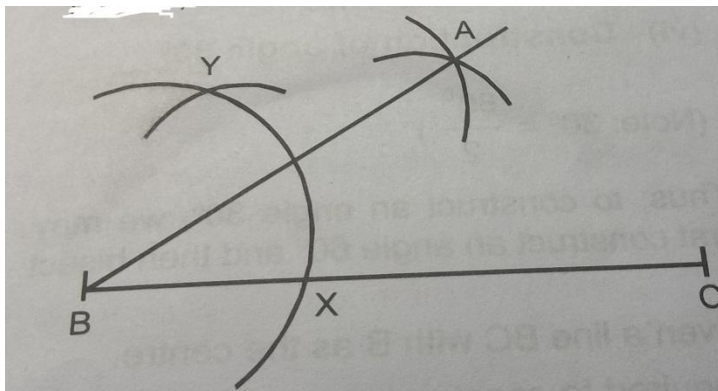
Place the needle point of the compass at X. Using the same radius, inscribe an arc that cuts the previous arc at Y. Using dotted lines, join YB.



Now, place the needle point at X. Using any convenient radius, inscribe an arc A in the space between line BC and line BY.

Place the needle point at Y. Using the same radius, inscribe an arc that cuts the previous arc A. Join AB with a thick line.

Thus, $\angle ABC = 30^\circ$. Use a protractor to check the result in the figure below.



Note : Educators should teach students how to construct angle 120°

Hint: $120^\circ = 60^\circ + 60^\circ$.

EVALUATION

Construct angle 15° .

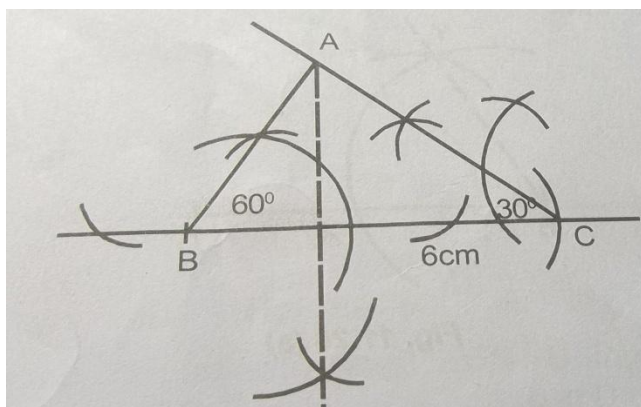
Hint: $15^\circ = \frac{30^\circ}{2}$

Sub – Topic v : Construction of shapes

Worked examples

Use a ruler and a pair of compasses only to construct a triangle ABC such that $BC = 6\text{ cm}$, $\angle ABC = 60^\circ$, $\angle ACB = 30^\circ$ and an altitude from A that will be perpendicular to line BC. Measure the altitude.

Solution



- Construct a line $BC = 6\text{ cm}$.
- Construct an angle $ABC = 60^\circ$.
- Construct an angle $ACB = 30^\circ$.
- Construct the altitude from A which is perpendicular to line BC as follows:
 - Place the needle point of the compass at A and cut line BC at two points using a convenient radius.
 - Using a convenient radius, place the needle at the cuts on BC one after the other to cut and recut below line BC.
 - Draw a straight line from A to the point of intersection of the two cuts below line BC.

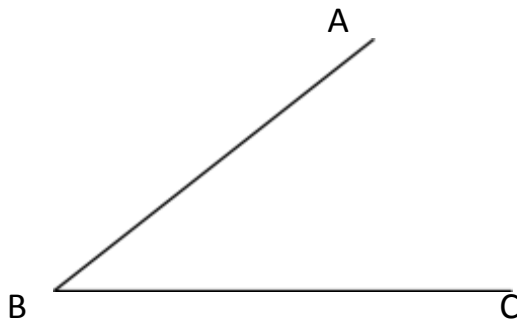
- iv. Measure altitude ($2.5 \text{ cm} \pm 0.1$)

EVALUATION

Construct an isosceles triangle PQR such that $QR = 8 \text{ cm}$ and $\angle PQR = \angle PRQ = 45^\circ$.

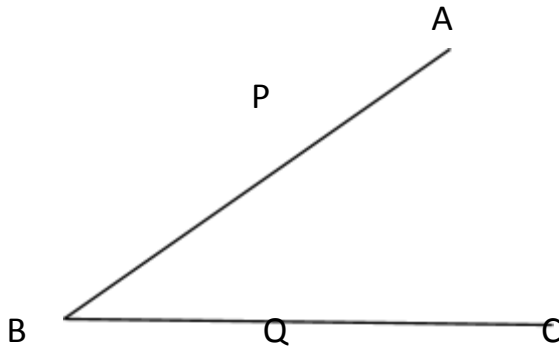
SUB – TOPIC VI: COPYING GIVEN ANGLE

Given any angle ABC:

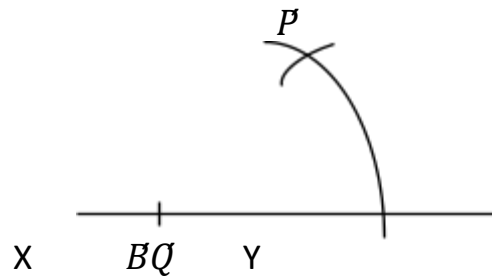
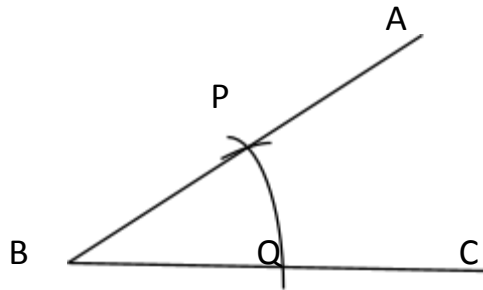


To make a copy of $\angle ABC$,

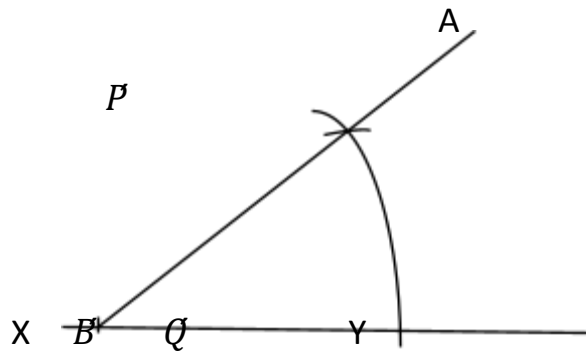
- a. Draw any line XY. Mark a point B' on XY. With centre B and any radius, draw an arc to cut BA, BC at P, Q. Then with centre B' and the same radius, draw an arc to cut XY at Q' .



- b. With centres Q , open the compasses until the radius = QP . Make an arc at P as a check. Then with centre Q' and the same radius, draw an arc to cut through Q at P .



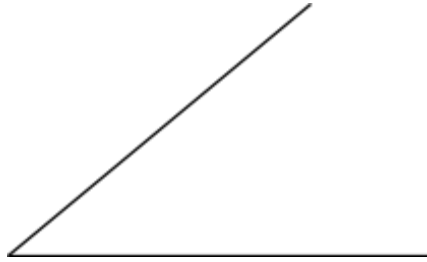
- c. Draw a line through B and P .



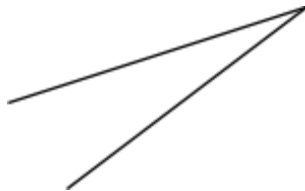
EVALUATION

Use the method given above to copy the following angle. Use a protractor to check your accuracy.

1.



2.



SUB – TOPIC VII: QUANTITATIVE REASONING

Sample A

$$100^\circ \rightarrow 50^\circ \rightarrow 25^\circ$$

$$10^\circ \leftarrow 20^\circ \leftarrow 40^\circ$$

$$5^\circ \leftrightarrow 20^\circ \leftrightarrow 80^\circ$$

Answer the following

1. $90^\circ \rightarrow 45^\circ \rightarrow ?$
2. $15^\circ \leftarrow 30^\circ \leftarrow ?$
3. $15^\circ \leftrightarrow 60^\circ \leftrightarrow ?$
4. $100^\circ \rightarrow 50^\circ \leftrightarrow ?$
5. $50^\circ \leftrightarrow 200^\circ \rightarrow ?$

Sample B

1. $15^\circ, 45^\circ, 30^\circ, 20^\circ$
(a) 15° (b) 45° (c) 30° (d) 20°

Answer = D (odd)

2. 20° , 70° , 80° , 90°

(a) 20° , (b) 70° (c) 80° (d) 90°

Answer = D (odd)

Answer the following

1. 15° , 45° , 120° , 30°

2. 120° , 150° , 135° , 30°

3. 130° , 30° , 150° , 120°

4. 135° , 120° , 180° , 50°

5. 15° , 30° , 60° , 105°

WEEKEND ASSINGMENT

Nelson Functional Mathematics for Junior Secondary Schools Book 3 by T. M. Asiru et al. Chapter 11, pg 28- 51.

READING ASSIGNMENT

New General Mathematics for Junior Secondary Schools book 3. Chapter 4 page 36- 44.

WEEK 10: REVISION

WEEK11: EXAMINATION

