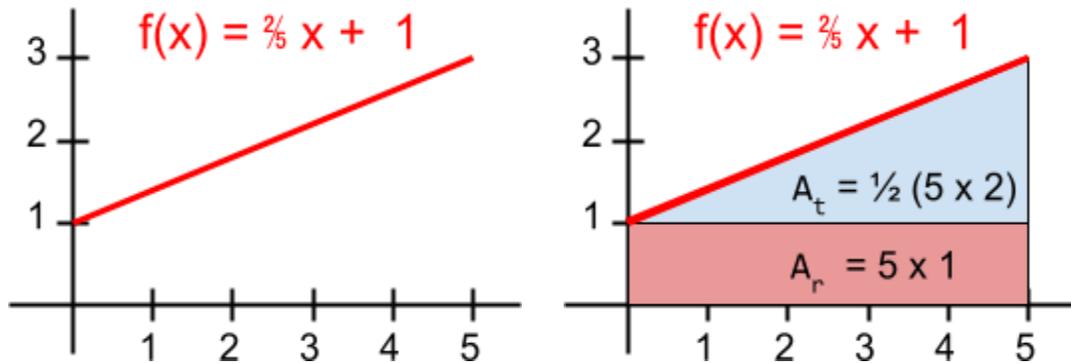


The area under a "curve" is the area bounded by a function and the x and y axis. For the function  $f(x) = \frac{2}{5}x + 1$  between  $x=0$  and  $x=5$ , this area is the sum of the area of the rectangle and the area of the triangle  $A = A_r + A_t = (5 \times 1) + \frac{1}{2}(5 \times 2) = 10$

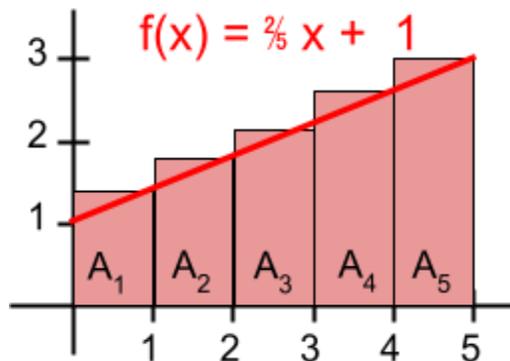



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The area under **any** curve can be **approximated** by dividing the area into vertical rectangles of equal widths and then adding up the area of all the rectangles.

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When using 5 rectangles for the area under  $f(x) = \frac{2}{5}x + 1$  between  $x=0$  and  $x=5$  the total area is given by  $A = A_1 + A_2 + A_3 + A_4 + A_5$  where the width of each rectangle is 1 and the height of each rectangle is given by evaluating the function  $f(x)$  at  $x=1, 2, 3, 4,$  and  $5$ .



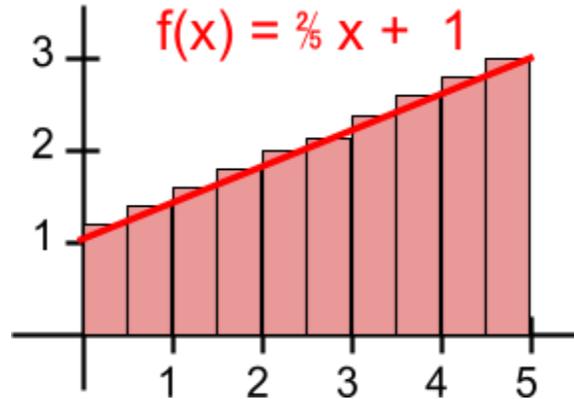
Calculating the total area of the 5 rectangles gives

$$\begin{aligned}
 A &= 1 \times f(1) + 1 \times f(2) + 1 \times f(3) + 1 \times f(4) + 1 \times f(5) \\
 A &= 1 \times \frac{7}{5} + 1 \times \frac{9}{5} + 1 \times \frac{11}{5} + 1 \times \frac{13}{5} + 1 \times \frac{15}{5} \\
 A &= \frac{55}{5} \\
 A &= 11
 \end{aligned}$$

---

Using **more rectangles** gives a better approximation of the actual area.

For example, with 10 rectangles the width of each rectangle becomes  $\frac{1}{2}$  and a more accurate area is calculated as



$$A = \frac{1}{2}f(0.5) + \frac{1}{2}f(1) + \frac{1}{2}f(1.5) + \frac{1}{2}f(2) + \frac{1}{2}f(2.5) + \frac{1}{2}f(3) + \frac{1}{2}f(3.5) + \frac{1}{2}f(4) + \frac{1}{2}f(4.5) + \frac{1}{2}f(5)$$

$$A = \frac{6}{10} + \frac{7}{10} + \frac{8}{10} + \frac{9}{10} + \frac{10}{10} + \frac{11}{10} + \frac{12}{10} + \frac{13}{10} + \frac{14}{10} + \frac{15}{10}$$

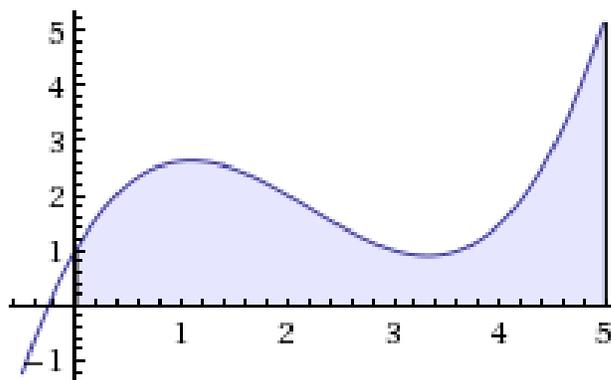
$$A = \frac{105}{10}$$

$$A = 10.5$$

For a given function **f(x)**

- write a **Python function**, that given a value for x **returns** the height of the rectangle (i.e. the value of **f(x)**).
- Use this new function in a **for loop** to approximate the area under the curve **between 0 and 5** by dividing the area into equal width rectangles and summing up the area of all the rectangles.
- Test your functions with  $f(x) = \frac{2}{5}x + 1$  used in the previous examples for 5 rectangles. Then try it for 10 & 100 rectangles.

- **Finally use the function**  $f(x) = \frac{17x^3}{55} - \frac{45x^2}{22} + \frac{369x}{110} + 1$  **graphed below**



- use another **for loop** to calculate the area under by using 10,100,1000,10000, 100000, and 1000000 rectangles

**for rectangles in [10,100,1000,10000,100000, 1000000]:**

FYI, in Calculus you will learn how to calculate this area using the definite integral<sup>1</sup>

$$\int_0^5 \left( \frac{17x^3}{55} - \frac{45x^2}{22} + \frac{369x}{110} + 1 \right) dx = 10$$

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<sup>1</sup> Nov 11, 1675 – Gottfried Leibniz demonstrated integral calculus for the first time to find the area under the graph of good ol  $y=f(x)$ .