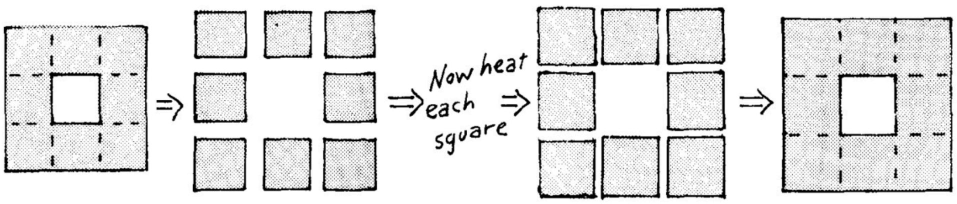
ANSWER: EXPANSION OF NOTHING

The answer is: a. The hole is nothing, but nothing expands, too. There is no way to avoid it. Every dimension of the ring expands in proportion. To visualize the expansion, suppose a photo of the ring were made and the photo was then enlarged one percent. Everything in the photo would be enlarged, even the hole.

Or look at it this way: uncurve the ring and open it up so that it forms a bar. It does become thicker when heated; but it also becomes longer. So when curved back into a ring we see that the inner circumference as well as the thickness is larger.

It's quite easy to see that the hole becomes larger upon expansion if we consider a square hole in a square piece of metal. Break it into square segments as shown, heat and expand them, then put them back together. The empty hole expands every bit as much as the solid metal.



Blacksmiths used to shrink fit iron rims on wooden wagon wheels by simply heating a rim that was initially a slight bit smaller than the wheel. When heated and expanded, the rim was simply slipped onto the wooden wheel. When cooled, a snug fit was provided that required no fasteners of any kind.

The next time you can't open the metal lid on a stubborn jar, heat the lid under hot water or by momentarily placing it on a hot stove. The lid, inner circumference and all, will expand and be easily loosened.

Thumbs-up explanations that worked well for you.

Ray's Commentary:

Several people have asked "okay, but, why do the iron particles in the middle ring *have* to get farther apart? isn't at least conceivable they get squished together? Shouldn't that happen for some materials?"

I'm not a physicist and am actually not that confident in the deep answer here (physics-y people feel free to add to this doc with more commentary), but some models that seem relevant to think about:

- *what is temperature?* The particles are heated up. Heated up implies that they are bouncing around more. (My current understanding is that this is what pushes them apart and expands the material).
- *Metal conducts heat effectively*, so the inner part of the ring should be roughly the same temperature as the outside. So the innermost particles should have some propensity to push each other apart, same as the outer particles.

- If somehow the particles *did* get squished closed together, that implies the inner part of the ring is under more pressure than the outer part of the ring. This doesn't seem like a stable equilibrium – if there's no outside pressure forcing the whole thing together, then if the innermost particulars have temperature pressuring them to expand, the only stable state is that they fully expand (meaning the particles in the center get farther apart from each other, meaning that the whole inner empty spot must expand in size)

Kurt's Comments:

- The given answer is consistent with the ball and stick model of solids. You can draw the metal annulus as a cluster of balls connected to their neighbors by sticks. If you uniformly heat the annulus while leaving it free to expand outward into space, then at least some of the sticks will extend, and some of the balls will become farther from their neighbors. This is most apparent at the outer edge of the object. The author's answer requires that this be true at the inner radius as well (and it is implied to be true at every intermediate radius too).
- If you disagree with the author's answer, ask yourself what would happen if you etched a portrait or a geometrical pattern (like concentric rings or a chessboard) onto the face of an annulus or disk before heating it.

Rudolf's comments:

- it is not entirely clear to me that the square model in the answer is unambiguously applicable, since in reality the squares expand against each other when they expand, and it's not entirely obvious clear the interaction forces don't change this
- I think there is a more principled answer:
    - Consider a one-atom-thick ring, modelled as a bunch of particles connected by springs; here it is obvious that if the "default length" of the springs gets longer, this must expand in diameter (e.g. consider balance of forces; it is outwards; or can also do an energy-based argument)
    - Now consider decomposing the ring into one-atom thick rings. Note that each ring, if it were on its own, would want to expand by 1%
    - But how do we prove that there are no interaction terms between the rings, that mean the solution is different than when each ring is on its own (after all, there are "springs" connecting particles in different rings)? Answer: if each ring has its circumference expand by 1%, each ring's diameter also expands by 1%, and since the rings further out have expanded by 1%, the net effect is an increase in the length of the springs between rings of 1% ((x + epsilon) * 1.01 - x * 1.01 = epsilon * 1.01).

    - So: *if every ring does what it wants to do if it were alone (expand by 1%),* the springs between rings also get 1% longer, and now every spring has gotten exactly 1% longer and the material has expanded without any non-uniform internal tension and *there's no reason why different rings would "fight it out" / interact with each other*
- lesson from this: the best decomposition is the one such that the answer can be very clearly expressed as a sum (in the general sense, not literal +) of answers to each component
    - concretely, above we have a reason why we don't expect cross-interactions between the rings to matter, so the rings are a good abstraction

People's Confusions

Do you not like any of the above explanations? Add things you are confused about here: