1.

The following points

$$A(0, 1), \quad B(1, 6), \quad C(1.5, 7.75), \quad D(1.9, 8.79) \quad \text{and} \quad E(2, 9)$$

lie on the curve $y = \mathrm{f}(x)$. The table below shows the gradients of the chords AE and BE.

Chord	AE	BE	CE	DE
Gradient of chord	4	3		

(a) Complete the table to show the gradients of CE and DE. [2]

$$\text{gradient of CE} = \frac{7.75-9}{1.5-2} = 2.5$$
$$\text{gradient of DE} = \frac{8.79-9}{1.9-2} = 2.1$$

(b) State what the values in the table indicate about the value of $\mathrm{f}'(2)$. [1]

2.1

2.

Functions f and g are defined by

$$\mathrm{f} : x \mapsto 3x + 2, \quad x \in \mathbb{R},$$

$$\mathrm{g} : x \mapsto 4x - 12, \quad x \in \mathbb{R}.$$

Solve the equation $\mathrm{f}^{-1}(x) = \mathrm{gf}(x)$. [4]

$$\begin{aligned}
y &= 3x + 2 \\
x &= \frac{y-2}{3} \\
\mathrm{f}^{-1}(x) &= \frac{x-2}{3} \\
\mathrm{gf}(x) &= 4(3x+2) - 12 \\
&= 12x - 4 \\
\frac{x-2}{3} &= 12x - 4 \\
x - 2 &= 36x - 12 \\
x &= \frac{10}{35} = \frac{2}{7}
\end{aligned}$$

3.

An arithmetic progression has first term 7. The nth term is 84 and the $(3n)$th term is 245.

Find the value of n. [4]

$$\begin{aligned}u_n &= a+(n-1)d\\ 84 &= 7+(n-1)d\\ 77 &= nd-d\\ u_{3n} &= a+(3n-1)d\\ 245 &= 7+3nd-d\\ 238 &= 3(77+d)-d\\ d &= \frac{7}{2}\\ n &= 23\end{aligned}$$

4.

A curve has equation $y = \mathrm{f}(x)$. It is given that $\mathrm{f}'(x) = \dfrac{1}{\sqrt{x+6}} + \dfrac{6}{x^2}$ and that $\mathrm{f}(3) = 1$.

Find $\mathrm{f}(x)$. [5]

$$\begin{aligned}\mathrm{f}'(x) &= (x+6)^{-\frac{1}{2}} + 6x^{-2}\\ \mathrm{f}(x) &= 2(x+6)^{\frac{1}{2}} - 6x^{-1} + c\\ 1 &= 2(9)^{\frac{1}{2}} - 2 + c\\ c &= -3\\ \mathrm{f}(x) &= 2\sqrt{x+6} - \frac{6}{x} - 3\end{aligned}$$

5.

(a) The curve $y = x^2 + 3x + 4$ is translated by $\begin{pmatrix}2\\0\end{pmatrix}$.

Find and simplify the equation of the translated curve. [2]

$$\begin{aligned}y &= (x-2)^2 + 3(x-2) + 4\\ &= x^2 - 4x + 4 + 3x - 6 + 4\\ &= x^2 - x + 2\end{aligned}$$

(b) The graph of $y = \mathrm{f}(x)$ is transformed to the graph of $y = 3\mathrm{f}(-x)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [3]

Stretch by a scale factor of 3 parallel to $y-$axis

Reflects by the $y-$axis

6.

(a) Find the coefficients of x^2 and x^3 in the expansion of $(2-x)^6$. [3]

$$\text{coefficient of } x^2 = \binom{6}{2} 2^4 \cdot (-1)^2 = 240$$
$$\text{coefficient of } x^3 = \binom{6}{3} 2^3 \cdot (-1)^3 = -160$$

(b) Hence find the coefficient of x^3 in the expansion of $(3x+1)(2-x)^6$. [2]

$$\begin{aligned}\text{coefficient of } x^3 &= 240 \times 3 - 160 \times 1 \\ &= 560\end{aligned}$$

7.

(a) Show that the equation $1 + \sin x \tan x = 5\cos x$ can be expressed as

$$6\cos^2 x - \cos x - 1 = 0.$$ [3]

$$\text{Let } s = \sin x,\ c = \cos x$$
$$1 + \frac{s^2}{c} = 5c$$
$$c + 1 - c^2 = 5c^2$$
$$6c^2 - c - 1 = 0$$

(b) Hence solve the equation $1 + \sin x \tan x = 5\cos x$ for $0° \leqslant x \leqslant 180°$. [3]

$$(2c-1)(3c+1) = 0$$
$$c = \frac{1}{2}, -\frac{1}{3}$$
$$c = 60,\ 109.5°\ (1\text{dp})$$

8.

A curve has equation $y = \dfrac{12}{3-2x}$.

(a) Find $\dfrac{dy}{dx}$. [2]

$$y = 12(3-2x)^{-1}$$
$$\frac{dy}{dx} = 12(-1)(3-2x)^{-2}(-2)$$
$$= \frac{24}{(3-2x)^2}$$

A point moves along this curve. As the point passes through A, the x-coordinate is increasing at a rate of 0.15 units per second and the y-coordinate is increasing at a rate of 0.4 units per second.

(b) Find the possible x-coordinates of A. [4]

$$\frac{dx}{dt} = 0.15, \ \frac{dy}{dt} = 0.4$$
$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{0.4}{0.15} = \frac{8}{3}$$
$$\frac{24}{(3-2x)^2} = \frac{8}{3}$$
$$(3-2x)^2 = 9$$
$$3 - 2x = \pm 3$$
$$2x = 3 \pm 3$$
$$x = 0, 4$$

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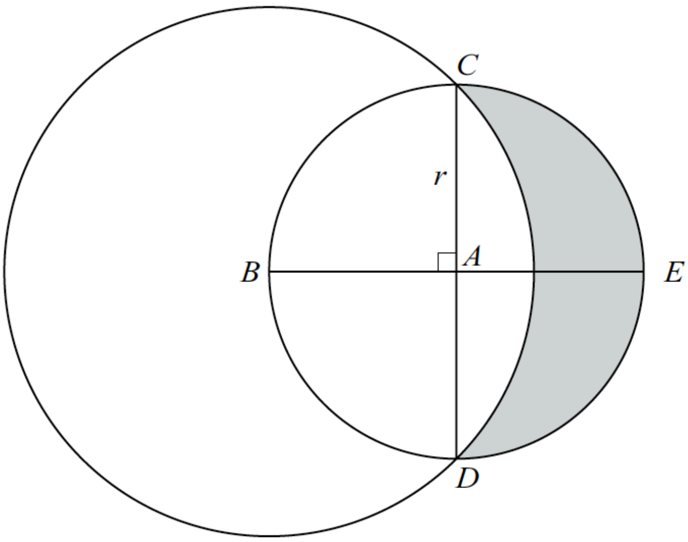
9.



The diagram shows a circle with centre A and radius r. Diameters CAD and BAE are perpendicular to each other. A larger circle has centre B and passes through C and D.

(a) Show that the radius of the larger circle is $r\sqrt{2}$. [1]

$$CA^2 + BA^2 = BC^2$$
$$BC = \sqrt{r^2 + r^2} = r\sqrt{2}$$

(b) Find the area of the shaded region in terms of r. [6]

$$\text{segment CADC} = \frac{1}{2}\left(r\sqrt{2}\right)^2\left(\frac{\pi}{2}\right) - 2 \times \frac{1}{2}r^2$$
$$= \frac{\pi r^2}{2} - r^2$$
$$\text{shaded} = \frac{\pi r^2}{2} - \left(\frac{\pi r^2}{2} - r^2\right) = r^2$$

10.

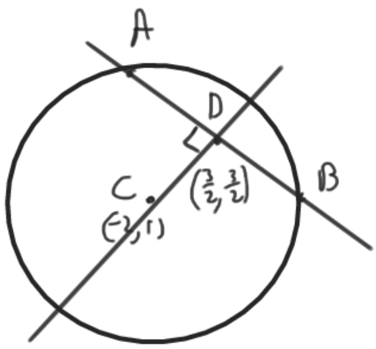
The circle $x^2 + y^2 + 4x - 2y - 20 = 0$ has centre C and passes through points A and B.

(a) State the coordinates of C. [1]

$$(x+2)^2 - 2^2 + (y-1)^2 - 1 = 20$$
$$(x+2)^2 + (y-1)^2 = 5^2$$
$$\mathrm{C} = (-2, 1)$$

It is given that the midpoint, D, of AB has coordinates $(1\frac{1}{2}, 1\frac{1}{2})$.

(b) Find the equation of AB, giving your answer in the form $y = mx + c$. [4]



$$\text{gradient of CD} = \frac{\frac{3}{2} - 1}{\frac{3}{2} + 2} = \frac{1}{7}$$
$$\text{gradient of AB} = -7$$
$$y - \frac{3}{2} = -7\left(x - \frac{3}{2}\right)$$
$$y = -7x + 12$$

(c) Find, by calculation, the x-coordinates of A and B. [3]

$$(x+2)^2 + (-7x + 12 - 1)^2 = 25$$
$$x^2 + 4x + 4 + 49x^2 - 154x + 121 = 25$$
$$50x^2 - 150x + 100 = 0$$
$$x^2 - 3x + 2 = 0$$
$$(x-2)(x-1) = 0$$
$$x = 1, 2$$

11.

The function f is defined, for $x \in \mathbb{R}$, by $f : x \mapsto x^2 + ax + b$, where a and b are constants.

(a) It is given that $a = 6$ and $b = -8$.

Find the range of f. [3]

$$\begin{aligned} f(x) &= x^2 + 6x - 8 \\ &= (x+3)^2 - 17 \end{aligned}$$
range: $y \geqslant -17$

(b) It is given instead that $a = 5$ and that the roots of the equation $f(x) = 0$ are k and $-2k$, where k is a constant.

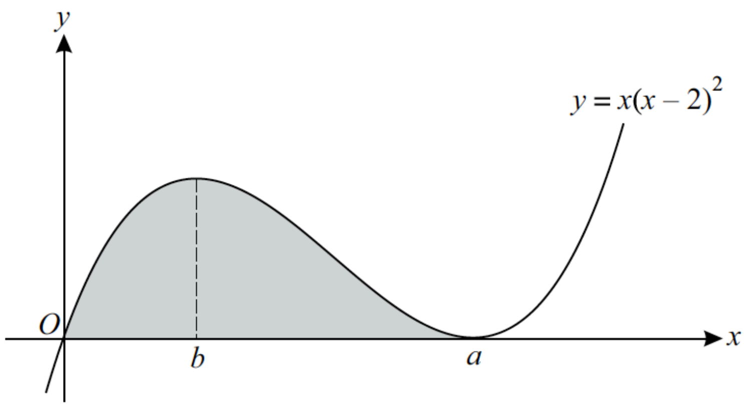
Find the values of b and k. [3]

$$\begin{aligned} x^2 + 5x + b &= 0 \\ (x-k)(x+2k) &= 0 \\ x^2 + kx - 2k^2 &= 0 \\ k &= 5 \\ b &= -2(25) = -50 \end{aligned}$$

(c) Show that if the equation $f(x + a) = a$ has no real roots then $a^2 < 4(b - a)$. [3]

$$f(x+a) = (x+a)^2 + a(x+a) + b = a$$
$$\begin{aligned} x^2 + 2ax + a^2 + ax + a^2 + b &= a \\ x^2 + 3ax + 2a^2 + b - a &= 0 \\ (3a)^2 - 4\left(2a^2 + b - a\right) &< 0 \\ a^2 &< 4(b-a) \end{aligned}$$

12.



The diagram shows the curve with equation $y = x(x-2)^2$. The minimum point on the curve has coordinates $(a, 0)$ and the x-coordinate of the maximum point is b, where a and b are constants.

(a) State the value of a. [1]

$a = 2$

(b) Calculate the value of b. [4]

$$\begin{aligned} y &= x^3 - 4x^2 + 4x \\ \frac{dy}{dx} &= 3x^2 - 8x + 4 \\ 0 &= (x-2)(3x-2) \\ b &= \frac{2}{3} \end{aligned}$$

(c) Find the area of the shaded region. [4]

$$\begin{aligned} \text{area} &= \int_0^2 (x^3 - 4x^2 + 4x)\,dx \\ &= \left[\frac{x^4}{4} - \frac{4}{3}x^3 + 2x^2\right]_0^2 \\ &= \frac{16}{4} - \frac{4}{3} \times 8 + 2 \times 4 \\ &= \frac{4}{3}\text{ units}^2 \end{aligned}$$

(d) The gradient, $\frac{dy}{dx}$, of the curve has a minimum value m.

Calculate the value of m. [4]

$$\begin{aligned} \frac{dy}{dx} &= 3x^2 - 8x + 4 \\ \frac{d^2y}{dx^2} &= 6x - 8 = 0 \\ x &= \frac{4}{3} \end{aligned}$$

$$f'\left(\frac{4}{3}\right) = 3\left(\frac{4}{3}\right)^2 - 8\left(\frac{4}{3}\right) + 4 = -\frac{4}{3} = m$$