Tab 1

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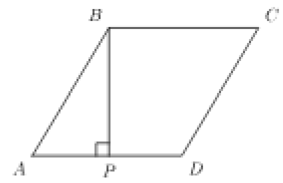
AI HOMEWORK

WRITTEN BY NYC SPECIALIZED HS ALUMNI #2 SCHOOL IN USA

AMC 2022B

1.
Define $x \diamond y$ to be $|x - y|$ for all real numbers x and y. What is the value of $(1 \diamond (2 \diamond 3)) - ((1 \diamond 2) \diamond 3)$?
A. -2
B. -1
C. 0
D. 1
E. 2

2.
Define $x \diamond y$ to be $|x - y|$ for all real numbers x and y. What is the value of $(1 \diamond (2 \diamond 3)) - ((1 \diamond 2) \diamond 3)$?
A. -2
B. -1
C. 0
D. 1
E. 2



3.
How many three-digit positive integers have an odd number of even digits?
A. 150
B. 250
C. 350
D. 450
E. 550

4. A donkey suffers an attack of hiccups and the first hiccup happens at 4:00 one afternoon. Suppose that the donkey hiccups regularly every 5 seconds. At what time does the donkey's 700th hiccup occur?
A. 15 seconds after 4:58
B. 20 seconds after 4:58
C. 25 seconds after 4:58
D. 30 seconds after 4:58
E. 35 seconds after 4:58

5. What is the value of ((1 + 1/3)(1 + 1/5)(1 + 1/7)) / ((1 - 1/(3^2))(1 - 1/(5^2))(1 - 1/(7^2)))?
A. 3/2
B. 2
C. 15/4
D. 4
E. 105/8

6. How many of the first ten numbers of the sequence 121, 11211, 1112111, ... are prime numbers?

A. 0

B. 1

C. 2

D. 3

E. 4

7. For how many values of the constant k will the polynomial x^2 + kx + 36 have two distinct integer roots?
A. 6
B. 8
C. 9
D. 14
E. 16

8. Consider the following 100 sets of 10 elements each:
{1, 2, 3, ..., 10},
{11, 12, 13, ..., 20},
{21, 22, 23, ..., 30},
...
{991, 992, 993, ..., 1000}.
How many of these sets contain exactly two multiples of 7?
A. 40
B. 42
C. 43
D. 49
E. 50

9. The sum 1/(2!) + 2/(3!) + 3/(4!) + ... + 2021/(2022!) can be expressed as a - 1/(b!), where a and b are positive integers. What is a + b?
A. 2020
B. 2021
C. 2022
D. 2023
E. 2024

10. Camila writes down five positive integers. The unique mode of these integers is 2 greater than their median, and the median is 2 greater than their arithmetic mean. What is the least possible value for the mode?
A. 5
B. 7
C. 9
D. 11
E. 13

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11. All the high schools in a large school district are involved in a fundraiser selling T-shirts. Which of the choices below is logically equivalent to the statement "No school bigger than Euclid HS sold more T-shirts than Euclid HS"?
A. All schools smaller than Euclid HS sold fewer T-shirts than Euclid HS.
B. No school that sold more T-shirts than Euclid HS is bigger than Euclid HS.
C. All schools bigger than Euclid HS sold fewer T-shirts than Euclid HS.
D. All schools that sold fewer T-shirts than Euclid HS are smaller than Euclid HS.
E. All schools smaller than Euclid HS sold more T-shirts than Euclid HS.

12. A pair of fair 6-sided dice is rolled n times. What is the least value of n such that the probability that the sum of the numbers face up on a roll equals 7 at least once is greater than 1/2?
A. 2
B. 3
C. 4
D. 5
E. 6

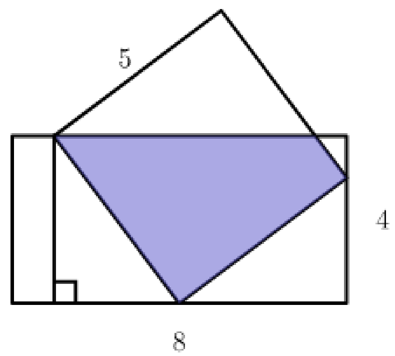
13. The positive difference between a pair of primes is equal to 2, and the positive difference between the cubes of the two primes is 31106. What is the sum of the digits of the least prime that is greater than those two primes?
A. 8
B. 10
C. 11
D. 13
E. 16

14. Suppose that S is a subset of {1, 2, 3, ..., 25} such that the sum of any two (not necessarily distinct) elements of S is never an element of S. What is the maximum number of elements S may contain?
A. 12
B. 13
C. 14
D. 15
E. 16

15. Let S_n be the sum of the first n term of an arithmetic sequence that has a common difference of 2. The quotient S_{3n} / S_n does not depend on n. What is S_{20}?
A. 340
B. 360
C. 380
D. 400
E. 420

16. The diagram below shows a rectangle with side lengths 4 and 8 and a square with side length 5. Three vertices of the square lie on three different sides of the rectangle, as shown. What is the area of the region inside both the square and the rectangle?

A. 15 1/8
B. 15 3/8
C. 15 1/2
D. 15 5/8
E. 15 7/8



17. One of the following numbers is not divisible by any prime number less than 10. Which is it?
A. $2^{606} - 1$
B. $2^{606} + 1$
C. $2^{607} - 1$
D. $2^{607} + 1$
E. $2^{607} + 3^{607}$

18. Consider systems of three linear equations with unknowns x, y, and z, where each of the coefficients is either 0 or 1 and the system has a solution other than x = y = z = 0. How many such systems of equations are there? (The equations in a system need not be distinct, and two systems containing the same equations in a different order are considered different.)
A. 302
B. 338
C. 340
D. 343
E. 344

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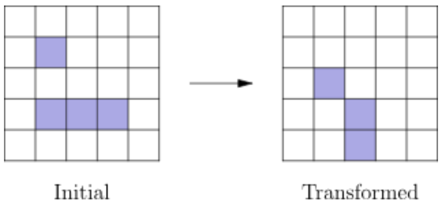
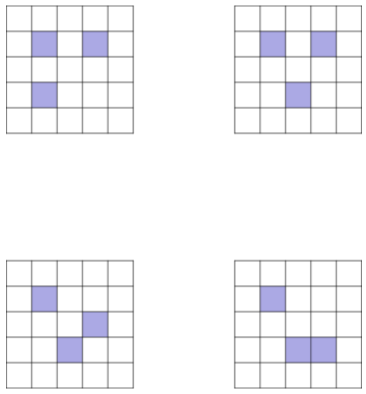
19. Each square in a 5 x 5 grid is either filled or empty, and has up to eight adjacent neighboring squares, where neighboring squares share either a side or a corner. The grid is transformed by rules concerning filled neighbors. Suppose the 5 x 5 grid has a border of empty squares surrounding a 3 x 3 subgrid. How many initial configurations will lead to a transformed grid consisting of a single filled square in the center after a single transformation? (Rotations and reflections of the same configuration are considered different.)

A. 14
B. 18
C. 22
D. 26
E. 30



20. Let ABCD be a rhombus with angle ADC = 46 degrees. Let E be the midpoint of CD, and let F be the point on BE such that AF is perpendicular to BE. What is the degree measure of angle BFC?

A. 110
B. 111
C. 112
D. 113
E. 114



21. Let P(x) be a polynomial with rational coefficients such that when P(x) is divided by the polynomial x^2 + x + 1, the remainder is x + 2, and when P(x) is divided by the polynomial x^2 + 1, the remainder is 2x + 1. There is a unique polynomial of least degree with these two properties. What is the sum of the squares of the coefficients of that polynomial?
A. 10
B. 13
C. 19
D. 20
E. 23

22. Let S be the set of circles in the coordinate plane that are tangent to each of the three circles with equations x^2 + y^2 = 4, x^2 + y^2 = 64, and (x-5)^2 + y^2 = 3. What is the sum of the areas of all circles in S?
A. 48pi
B. 68pi
C. 96pi
D. 102pi
E. 136pi

23. Ant Amelia starts on the number line at 0 and crawls in the following manner. For n = 1, 2, 3, Amelia chooses a time duration t_n and an increment x_n independently and uniformly at random from the interval (0,1). During the nth step of the process, Amelia moves x_n units in the positive direction, using up t_n minutes. If the total elapsed time has exceeded 1 minute during the nth step, she stops at the end of that step; otherwise, she continues with the next step, taking at most 3 steps in all. What is the probability that Amelia's position when she stops will be greater than 1?
A. 1/3
B. 1/2
C. 2/3
D. 3/4
E. 5/6

24. Consider functions f that satisfy |f(x) - f(y)| <= (1/2)|x - y| for all real numbers x and y. Of all such functions that also satisfy the equation f(300) = f(900), what is the greatest possible value of f(f(800)) - f(f(400))?
A. 25
B. 50
C. 100
D. 150
E. 200

25. Let x_0, x_1, x_2, ... be a sequence of numbers, where each x_k is either 0 or 1. For each positive integer n, define S_n = sum from k=0 to n-1 of (x_k * 2^k). Suppose 7S_n ≡ 1 (mod 2^n) for all n >= 1. What is the value of the sum x_2019 + 2x_2020 + 4x_2021 + 8x_2022? A. 6 B. 7 C. 12 D. 14 E. 15	Answer Key: 1. A 2. D 3. D 4. A 5. B 6. A 7. B 8. B 9. D 10. D 11. B 12. C 13. E 14. B 15. D 16. D 17. C 18. B 19. C 20. D 21. E 22. E 23. C 24. B 25. A

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AMC 2022A

1.
What is the value of 3 + 1/(3 + 1/(3 + 1/3))?

$$3 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{3}}} ?$$

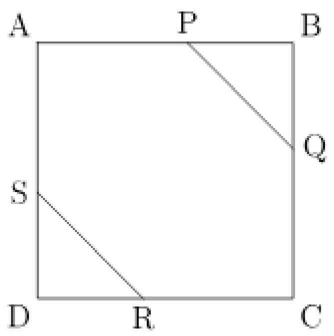
A. 31/10
B. 49/15
C. 33/10
D. 109/33
E. 15/4

2.
Mike cycled 15 laps in 57 minutes. Assume he cycled at a constant speed throughout. Approximately how many laps did he complete in the first 27 minutes?
A. 5
B. 7
C. 9
D. 11
E. 13

3.
The sum of three numbers is 96. The first number is 6 times the third number, and the third number is 40 less than the second number. What is the absolute value of the difference between the first and second numbers?
A. 1
B. 2
C. 3
D. 4
E. 5

4. In some countries, automobile fuel efficiency is measured in liters per 100 kilometers while other countries use miles per gallon. Suppose that 1 kilometer equals m miles, and 1 gallon equals l liters. Which of the following gives the fuel efficiency in liters per 100 kilometers for a car that gets x miles per gallon?
A. x/(100*l*m)
B. (x*l*m)/100
C. (l*m)/(100*x)
D. (100x)/(lm)
E. (100*l*m)/x

5. Square ABCD has side length 1. Points P, Q, R, and S each lie on a side of ABCD such that APQCRS is an equilateral convex hexagon with side length s. What is s?
A. sqrt(2)/3
B. 1/2
C. 2 - sqrt(2)
D. 1 - sqrt(2)/4
E. 2/3



6. Which expression is equal to |a - 2 - sqrt((a-1)^2)| for a < 0?

A. 3 - 2a

B. 1 - a

C. 1

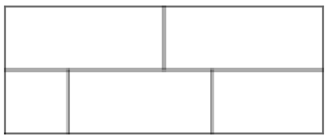
D. a + 1

E. 3

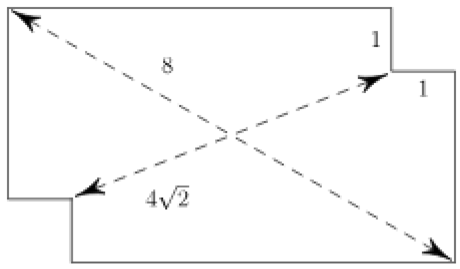
7. The least common multiple of a positive integer n and 18 is 180, and the greatest common divisor of n and 45 is 15. What is the sum of the digits of n?
A. 3
B. 6
C. 8
D. 9
E. 12

8. A data set consists of 6 (not distinct) positive integers: 1, 7, 5, 2, 5, and X. The average (arithmetic mean) of the 6 numbers equals a value in the data set. What is the sum of all positive values of X?
A. 10
B. 26
C. 32
D. 36
E. 40

9. A rectangle is partitioned into 5 regions as shown. Each region is to be painted a solid color - red, orange, yellow, blue, or green - so that regions that touch are painted different colors, and colors can be used more than once. How many different colorings are possible?
A. 120
B. 270
C. 360
D. 540
E. 720



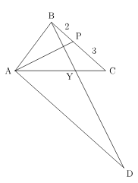
10. Daniel finds a rectangular index card and measures its diagonal to be 8 centimeters. Daniel then cuts out equal squares of side 1 cm at two opposite corners of the index card and measures the distance between the two closest vertices of these squares to be 4sqrt(2) centimeters, as shown below. What is the area of the original index card?
A. 14
B. 10sqrt(2)
C. 16
D. 12*sqrt(2)
E. 18



11. Ted mistakenly wrote 2^m * (1/4096) as 2 * (1/4096)^m. What is the sum of all real numbers m for which these two expressions have the same value?
A. 5
B. 6
C. 7
D. 8
E. 9

12. On Halloween 31 children walked into the principal's office asking for candy. They can be classified into three types: Some always lie; some always tell the truth; and some alternately lie and tell the truth. The principal asked everyone the same three questions in this order: "Are you a truth-teller?"; "Are you an alternater?"; "Are you a liar?". The principal gave candy to 22, 15, and 9 children for their "yes" answers to the respective questions. How many pieces of candy in all did the principal give to the children who always tell the truth?
A. 7
B. 12
C. 21
D. 27
E. 31

13. Let triangle ABC be a scalene triangle. Point P lies on BC so that AP bisects angle BAC. The line through B perpendicular to AP intersects the line through A parallel to BC at point D. Suppose BP = 2 and PC = 3. What is AD?
A. 8
B. 9
C. 10
D. 11
E. 12



14. How many ways are there to split the integers 1 through 14 into 7 pairs such that in each pair, the greater number is at least 2 times the lesser number?
A. 108
B. 120
C. 126
D. 132
E. 144

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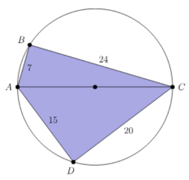
15. Quadrilateral ABCD with side lengths AB = 7, BC = 24, CD = 20, DA = 15 is inscribed in a circle. The area interior to the circle but exterior to the quadrilateral can be written in the form (a*pi - b)/c, where a, b, and c are positive integers such that a and c have no common prime factor. What is a + b + c?

A. 260
B. 855
C. 1235
D. 1565
E. 1997



16. The roots of the polynomial 10x^3 - 39x^2 + 29x - 6 are the height, length, and width of a rectangular box (right rectangular prism). A new rectangular box is formed by lengthening each edge of the original box by 2 units. What is the volume of the new box?
A. 24/5
B. 42/5
C. 81/5
D. 30
E. 48

17. How many three-digit positive integers abc are there whose nonzero digits a, b, and c satisfy 0.abc (repeating) = (1/3)(0.a + 0.b + 0.c)? (The bar indicates repetition.)
A. 9
B. 10
C. 11
D. 13
E. 14

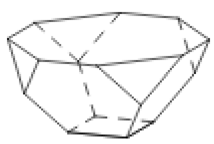
18. Let T_k be the transformation of the coordinate plane that first rotates the plane k degrees counterclockwise around the origin and then reflects the plane across the y-axis. What is the least positive integer n such that performing the sequence of transformations T_1, T_2, T_3, ..., T_n returns the point (1, 0) back to itself?
A. 359
B. 360
C. 719
D. 720
E. 721

19. Define L_n as the least common multiple of all the integers from 1 to n inclusive. There is a unique integer h such that 1/1 + 1/2 + 1/3 ... + 1/17 = h / L_17. What is the remainder when h is divided by 17?
A. 1
B. 3
C. 5
D. 7
E. 9

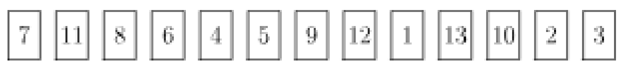
20.A four-term sequence is formed by adding each term of a four-term arithmetic sequence of positive integers to the corresponding term of a four-term geometric sequence of positive integers. The first three terms of the resulting four-term sequence are 57, 60, and 91. What is the fourth term of this sequence?

A. 190
B. 194
C. 198
D. 202
E. 206

21.A bowl is formed by attaching four regular hexagons of side 1 to a square of side 1. The edges of the adjacent hexagons coincide. What is the area of the octagon obtained by joining the top eight vertices of the four hexagons, situated on the rim of the bowl?
A. 6
B. 7
C. 5 + 2*sqrt(2)
D. 8
E. 9



22. Suppose that 13 cards numbered 1, 2, 3, ..., 13 are arranged in a row. The task is to pick them up in numerically increasing order, working repeatedly from left to right. For how many of the 13! possible orderings of the cards will the 13 cards be picked up in exactly two passes?
A. 4082
B. 4095
C. 4096
D. 8178
E. 8191



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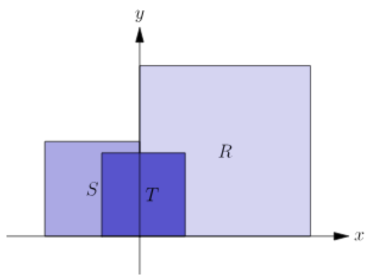
23. Isosceles trapezoid ABCD has parallel sides AD and BC, with BC < AD and AB = CD. There is a point P in the plane such that PA = 1, PB = 2, PC = 3, and PD = 4. What is BC/AD?
A. 1/4
B. 1/3
C. 1/2
D. 2/3
E. 3/4

24. How many strings of length 5 formed from the digits 0, 1, 2, 3, 4 are there such that for each j in {1, 2, 3, 4}, at least j of the digits are less than j?
A. 500
B. 625
C. 1089
D. 1199
E. 1296

25. Let R, S, and T be squares that have vertices at lattice points in the coordinate plane, together with their interiors. The bottom edge of each square is on the x-axis. The left edge of R and the right edge of S are on the y-axis, and R contains 9/4 as many lattice points as does S. The top two vertices of T are in R union S, and T contains 1/4 of the lattice points contained in R union S. The fraction of lattice points in S that are in S intersect T is 27 times the fraction of lattice points in R that are in R intersect T. What is the minimum possible value of the edge length of R plus the edge length of S plus the edge length of T?
A. 336
B. 337
C. 338
D. 339
E. 340



Answer Key:
1. D
2. B
3. E
4. E
5. C
6. A
7. B
8. D
9. D
10. E
11. C
12. A
13. C
14. E
15. D
16. D
17. D
18. A
19. C
20. E
21. B
22. D
23. B
24. E
25. B

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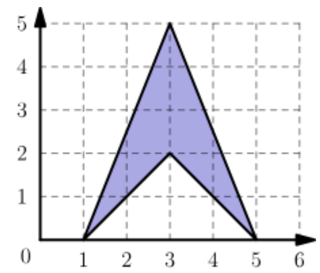
AMC 2021D

1. What is the value of $1234 + 2341 + 3412 + 4123$?
A. 10,000
B. 10,010
C. 10,110
D. 11,000
E. 11,110

2. What is the area of the shaded figure shown below?
A. 4
B. 6
C. 8
D. 10
E. 12



3. The expression $(2021/2020) - (2020/2021)$ is equal to the fraction p/q in which p and q are positive integers whose greatest common divisor is 1. What is p?
A. 1
B. 9
C. 2020
D. 2021
E. 4041

4. At noon on a certain day, Minneapolis is N degrees warmer than St. Louis. At 4:00 the temperature in Minneapolis has fallen by 5 degrees while the temperature in St. Louis has risen by 3 degrees, at which time the temperatures in the two cities differ by 2 degrees. What is the product of all possible values of N?
A. 10
B. 30
C. 60
D. 100
E. 120

5. Let $n = 8^{2022}$. Which of the following is equal to $n/4$?
A. 4^{1010}
B. 2^{2022}
C. 8^{2018}
D. 4^{3031}
E. 4^{3032}

6. The least positive integer with exactly 2021 distinct positive divisors can be written in the form $m * 6^k$, where m and k are integers and 6 is not a divisor of m. What is $m + k$?

A. 47

B. 58

C. 59

D. 88

E. 90

7. Call a fraction a/b, not necessarily in the simplest form, 'special' if a and b are positive integers whose sum is 15. How many distinct integers can be written as the sum of two, not necessarily different, special fractions?
A. 9
B. 10
C. 11
D. 12
E. 13

8. The greatest prime number that is a divisor of 16,384 is 2 because $16{,}384 = 2^{14}$. What is the sum of the digits of the greatest prime number that is a divisor of 16,383?
A. 3
B. 7
C. 10
D. 16
E. 22

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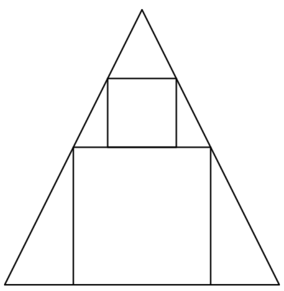
9. The knights in a certain kingdom come in two colors. 2/7 of them are red, and the rest are blue. Furthermore, 1/6 of the knights are magical, and the fraction of red knights who are magical is 2 times the fraction of blue knights who are magical. What fraction of red knights are magical?
A. 2/9
B. 3/13
C. 7/27
D. 2/7
E. 1/3

10. Forty slips of paper numbered 1 to 40 are placed in a hat. Alice and Bob each draw one number from the hat without replacement, keeping their numbers hidden from each other. Alice says, "I can't tell who has the larger number." Then Bob says, "I know who has the larger number." Alice says, "You do? Is your number prime?" Bob replies, "Yes." Alice says, "In that case, if I multiply your number by 100 and add my number, the result is a perfect square." What is the sum of the two numbers drawn from the hat?
A. 27
B. 37
C. 47
D. 57
E. 67

11. A regular hexagon of side length 1 is inscribed in a circle. Each minor arc of the circle determined by a side of the hexagon is reflected over that side. What is the area of the region bounded by these 6 reflected arcs?
A. (5*sqrt(3))/2 - pi
B. 3*sqrt(3) - pi
C. 4*sqrt(3) - (3*pi)/2
D. pi - 3/2
E. pi + (sqrt(3))/2

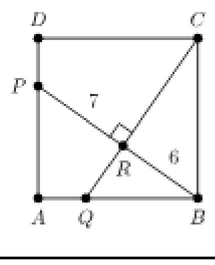
12. Which of the following conditions is sufficient to guarantee that integers x, y, and z satisfy the equation x(x-y) + y(y-z) + z(z-x) = 1?
A. x > y and y = z
B. x = y - 1 and y = z - 1
C. x = z + 1 and y = x + 1
D. x = z and y - 1 = x
E. x + y + z = 1

13. A square with side length 3 is inscribed in an isosceles triangle with one side of the square along the base of the triangle. A square with side length 2 has two vertices on the other square and the other two on sides of the triangle, as shown. What is the area of the triangle?
A. 19 1/4
B. 20 1/4
C. 21 3/4
D. 22 1/2
E. 23 3/4



14. Una rolls 6 standard 6-sided dice simultaneously and calculates the product of the 6 numbers obtained. What is the probability that the product is divisible by 4?
A. 3/4
B. 57/64
C. 59/64
D. 187/192
E. 63/64

15. In square ABCD, points P and Q lie on AD and AB, respectively. Segments BP and CQ intersect at right angles at R, with BR = 6 and PR = 7. What is the area of the square?
A. 85
B. 93
C. 100
D. 117
E. 125



16. Five balls are arranged around a circle. Chris chooses two adjacent balls at random and interchanges them. Then Silva does the same, with her choice of adjacent balls to interchange being independent of Chris's. What is the expected number of balls that occupy their original positions after these two successive transpositions?
A. 1.6
B. 1.8
C. 2.0
D. 2.2
E. 2.4

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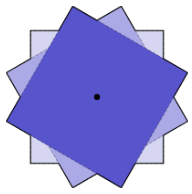
17. Distinct lines ℓ and m lie in the xy-plane. They intersect at the origin. Point P(-1, 4) is reflected about line ℓ to point P', and then P' is reflected about line m to point P''. The equation of line ℓ is 5x - y = 0, and the coordinates of P'' are (4, 1). What is the equation of line m?
A. 5x + 2y = 0
B. 3x + 2y = 0
C. x - 3y = 0
D. 2x - 3y = 0
E. 5x - 3y = 0

18. Three identical square sheets of paper each with side length 6 are stacked on top of each other. The middle sheet is rotated clockwise 30 degrees about its center and the top sheet is rotated clockwise 60 degrees about its center, resulting in the 24-sided polygon shown in the figure below. The area of this polygon can be expressed in the form a - b*sqrt(c), where a, b, and c are positive integers, and c is not divisible by the square of any prime. What is a + b + c?
A. 75
B. 93
C. 96
D. 129
E. 147



19. Let N be the positive integer 7777…777, a 313-digit number where each digit is a 7. Let f(r) be the leading digit of the r-th root of N. What is f(2) + f(3) + f(4) + f(5) + f(6)?
A. 8
B. 9
C. 11
D. 22
E. 29

20. In a particular game, each of 4 players rolls a standard 6-sided die. The winner is the player who rolls the highest number. If there is a tie for the highest roll, those involved in the tie will roll again and this process will continue until one player wins. Hugo is one of the players in this game. What is the probability that Hugo's first roll was a 5, given that he won the game?
A. 61/216
B. 367/1296
C. 41/144
D. 185/648
E. 11/36

21. Regular polygons with 5, 6, 7, and 8 sides are inscribed in the same circle. No two of the polygons share a vertex, and no three of their sides intersect at a common point. At how many points inside the circle do two of their sides intersect?
A. 52
B. 56
C. 60
D. 64
E. 68

22. For each integer n ≥ 2, let S_n be the sum of all products j*k, where j and k are integers and 1 ≤ j < k ≤ n. What is the sum of the 10 least values of n such that S_n is divisible by 3?
A. 196
B. 197
C. 198
D. 199
E. 200

23. Each of the 5 sides and the 5 diagonals of a regular pentagon are randomly and independently drawn as solid or dashed with equal probability. What is the probability that there will be a triangle whose vertices are among the vertices of the pentagon such that all of its sides have the same stroke type?
A. 2/3
B. 105/128
C. 125/128
D. 253/256
E. 1

24. A cube is constructed from 4 white unit cubes and 4 blue unit cubes. How many different ways are there to construct the 2x2x2 cube using these smaller cubes? (Two constructions are considered the same if one can be rotated to match the other.)
A. 7
B. 8
C. 9
D. 10
E. 11

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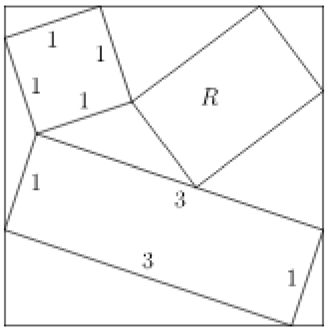
25. A rectangle with side lengths 1 and 3, a square with side length 1, and a rectangle R are inscribed inside a larger square as shown. The sum of all possible values for the area of R can be written in the form m/n, where m and n are relatively prime positive integers. What is m + n? A. 14 B. 23 C. 46 D. 59 E. 67 	Answer Key: 1. E 2. B 3. E 4. C 5. E 6. B 7. C 8. C 9. C 10. A 11. B 12. D 13. B 14. C 15. D 16. D 17. D 18. E 19. A 20. C 21. E 22. B 23. D 24. A 25. E

AMC 2021C

1. What is the value of (2112 - 2021)^2 / 169?
A. 7
B. 21
C. 49
D. 64
E. 91

2. Menkara has a 4 x 6 index card. If she shortens the length of one side of this card by 1 inch, the card would have area 18 square inches. What would the area of the card be in square inches if instead she shortens the length of the other side by 1 inch?
A. 16
B. 17
C. 18
D. 19
E. 20

3. What is the maximum number of balls of clay of radius 2 that can completely fit inside a cube of side length 6 assuming the balls can be reshaped but not compressed before they are packed in the cube?
A. 3
B. 4
C. 5
D. 6
E. 7

4. Mr. Lopez has a choice of two routes to get to work. Route A is 6 miles long, and his average speed along this route is 30 miles per hour. Route B is 5 miles long, and his average speed along this route is 40 miles per hour, except for a 1/2-mile stretch in a school zone where his average speed is 20 miles per hour. By how many minutes is Route B quicker than Route A?
A. 2 3/4
B. 3 3/4
C. 4 1/2
D. 5 1/2
E. 6 3/4

5. The six-digit number 2 0 2 1 0 A is prime for only one digit A. What is A?
A. 1
B. 3
C. 5
D. 7
E. 9

6. Elmer the emu takes 44 equal strides to walk between consecutive telephone poles on a rural road. Oscar the ostrich can cover the same distance in 12 equal leaps. The telephone poles are evenly spaced, and the 41st pole along this road is exactly one mile (5280 feet) from the first pole. How much longer, in feet, is Oscar's leap than Elmer's stride?

A. 6

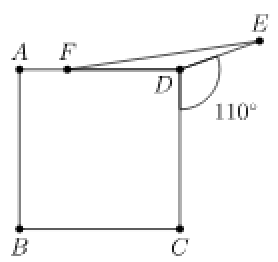
B. 8

C. 10

D. 11

E. 15

7. As shown in the figure below, point E lies on the opposite half-plane determined by line CD from point A so that angle CDE = 110 degrees. Point F lies on AD so that DE = DF, and ABCD is a square. What is the degree measure of angle AFE?
A. 160
B. 164
C. 166
D. 170
E. 174



8. A two-digit positive integer is said to be cuddly if it is equal to the sum of its nonzero tens digit and the square of its units digit. How many two-digit positive integers are cuddly?
A. 0
B. 1
C. 2
D. 3
E. 4

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9. When a certain unfair die is rolled, an even number is 3 times as likely to appear as an odd number. The die is rolled twice. What is the probability that the sum of the numbers rolled is even?
A. 3/8
B. 4/9
C. 5/9
D. 9/16
E. 5/8

10. A school has 100 students and 5 teachers. In the first period, each student is taking one class, and each teacher is teaching one class. The enrollments in the classes are 50, 20, 20, 5, and 5. Let t be the average value obtained if a teacher is picked at random and the number of students in their class is noted. Let s be the average value obtained if a student was picked at random and the number of students in their class, including the student, is noted. What is $t - s$?
A. -18.5
B. -13.5
C. 0
D. 13.5
E. 18.5

11. Emily sees a ship traveling at a constant speed along a straight section of a river. She walks parallel to the riverbank at a uniform rate faster than the ship. She counts 210 equal steps walking from the back of the ship to the front. Walking in the opposite direction, she counts 42 steps of the same size from the front of the ship to the back. In terms of Emily's equal steps, what is the length of the ship?
A. 70
B. 84
C. 98
D. 105
E. 126

12. The base-nine representation of the number N is 27,006,000,052 base nine. What is the remainder when N is divided by 5?
A. 0
B. 1
C. 2
D. 3
E. 4

13. Each of 6 balls is randomly and independently painted either black or white with equal probability. What is the probability that every ball is different in color from more than half of the other 5 balls?
A. 1/64
B. 1/6
C. 1/4
D. 5/16
E. 1/2

14. How many ordered pairs (x, y) of real numbers satisfy the following system of equations?
$x^2 + 3y = 9$
$(|x| + |y| - 4)^2 = 1$
A. 1
B. 2
C. 3
D. 5
E. 7

15. Isosceles triangle ABC has $AB = AC = 3\sqrt{6}$, and a circle with radius $5\sqrt{2}$ is tangent to line AB at B and to line AC at C. What is the area of the circle that passes through vertices A, B, and C?
A. 24π
B. 25π
C. 26π
D. 27π
E. 28π

16. The graph of $f(x) = |\text{floor}(x)| - |\text{floor}(1-x)|$ is symmetric about which of the following? (Here floor(x) is the greatest integer not exceeding x.)
A. the y-axis
B. the line $x = 1$
C. the origin
D. the point (1/2, 0)
E. the point (1, 0)

17. An architect is building a structure that will place vertical pillars at the vertices of regular hexagon ABCDEF, which is lying horizontally on the ground. The six pillars will hold up a flat solar panel that will not be parallel to the ground. The heights of pillars at A, B, and C are 12, 9, and 10 meters, respectively. What is the height, in meters, of the pillar at E?
A. 9
B. $6\sqrt{3}$
C. $8\sqrt{3}$
D. 17
E. $12\sqrt{3}$

18. A farmer's rectangular field is partitioned into a 2 by 2 grid of 4 rectangular sections as shown in the figure. In each section the farmer will plant one crop: corn, wheat, soybeans, or potatoes. The farmer does not want to grow corn and wheat in any two sections that share a border, and the farmer does not want to grow soybeans and potatoes in any two sections that share a border. Given these restrictions, in how many ways can the farmer choose crops to plant in each of the four sections of the field?
A. 12
B. 64
C. 84
D. 90
E. 144

19. A disk of radius 1 rolls all the way around the inside of a square of side length $s > 4$ and sweeps out a region of area A. A second disk of radius 1 rolls all the way around the outside of the same square and sweeps out a region of area 2A. The value of s can be written as $a + (b\pi)/c$, where a, b, and c are positive integers and b and c are relatively prime. What is $a + b + c$?
A. 10
B. 11
C. 12
D. 13
E. 14

20. How many ordered pairs of positive integers (b, c) exist where both $x^2 + bx + c = 0$ and $x^2 + cx + b = 0$ do not have distinct, real solutions?
A. 4
B. 6
C. 8
D. 10
E. 12

21. Each of 20 balls is tossed independently and at random into one of the 5 bins. Let p be the probability that some bin ends up with 3 balls, another with 5 balls, and the other three with 4 balls each. Let q be the probability that every bin ends up with 4 balls. What is p/q?
A. 1
B. 4
C. 8
D. 12
E. 16

22. Inside a right circular cone with base radius 5 and height 12 are three congruent spheres with radius r. Each sphere is tangent to the other two spheres and also tangent to the base and side of the cone. What is r?
A. 3/2
B. $(90 - 40\sqrt{3})/11$
C. 2
D. $(144 - 25\sqrt{3})/44$
E. 5/2

23. For each positive integer n, let $f_1(n)$ be twice the number of positive integer divisors of n, and for $j \geq 2$, let $f_j(n) = f_1(f_{j-1}(n))$. For how many values of $n \leq 50$ is $f_{50}(n) = 12$?
A. 7
B. 8
C. 9
D. 10
E. 11

24. Each of the 12 edges of a cube is labeled 0 or 1. Two labelings are considered different even if one can be obtained from the other by a sequence of one or more rotations and/or reflections. For how many such labelings is the sum of the labels on the edges of each of the 6 faces of the cube equal to 2?
A. 8
B. 10
C. 12
D. 16
E. 20

25. A quadratic polynomial with real coefficients and leading coefficient 1 is called disrespectful if the equation p(p(x)) = 0 is satisfied by exactly three real numbers. Among all the disrespectful quadratic polynomials, there is a unique such polynomial p~(x) for which the sum of the roots is maximized. What is p~(1)? A. 5/16 B. 1/2 C. 5/8 D. 1 E. 9/8	Answer Key: 1. C 2. E 3. D 4. B 5. E 6. B 7. D 8. B 9. E 10. B 11. A 12. D 13. D 14. D 15. C 16. D 17. D 18. C 19. A 20. B 21. E 22. B 23. D 24. E 25. A

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AMC 2021B

1. How many integer values of x satisfy |x| < 3π?
A. 9
B. 10
C. 18
D. 19
E. 20

2. What is the value of sqrt((3 - 2*sqrt(3))^2) + sqrt((3 + 2*sqrt(3))^2)?
A. 0
B. 4*sqrt(3) - 6
C. 6
D. 4*sqrt(3)
E. 4*sqrt(3) + 6

3. In an after-school program for juniors and seniors, there is a debate team with an equal number of students from each class on the team. Among the 28 students in the program, 25% of the juniors and 10% of the seniors are on the debate team. How many juniors are in the program?
A. 5
B. 6
C. 8
D. 11
E. 20

4. At a math contest, 57 students are wearing blue shirts, and another 75 students are wearing yellow shirts. The 132 students are assigned into 66 pairs. In exactly 23 of these pairs, both students are wearing blue shirts. In how many pairs are both students wearing yellow shirts?
A. 23
B. 32
C. 37
D. 41
E. 64

5. The ages of Jonie's four cousins are distinct single-digit positive integers. Two of the cousins' ages multiplied together give 24, while the other two multiply to 30. What is the sum of the ages of Jonie's four cousins?
A. 21
B. 22
C. 23
D. 24
E. 25

6. Ms. Blackwell gives an exam to two classes. The mean of the scores of the students in the morning class is 84, and the afternoon class's mean score is 70. The ratio of the number of students in the morning class to the number of students in the afternoon class is 3:4. What is the mean of the scores of all the students?

A. 74

B. 75

C. 76

D. 77

E. 78

7. In a plane, four circles with radii 1, 3, 5, and 7 are tangent to line ℓ at the same point A, but they may be on either side of ℓ. Region S consists of all the points that lie inside exactly one of the four circles. What is the maximum possible area of region S?
A. 24π
B. 32π
C. 64π
D. 65π
E. 84π

8. Mr. Zhou places all the integers from 1 to 225 into a 15 by 15 grid. He places 1 in the middle square (eighth row and eighth column) and places other numbers one by one clockwise, as shown in part in the diagram below. What is the sum of the greatest number and the least number that appear in the second row from the top?
A. 367
B. 368
C. 369
D. 379
E. 380

⋯	⋯	⋯	⋯	⋯	⋯	⋯
⋯	21	22	23	24	25	⋯
⋯	20	7	8	9	10	⋯
⋯	19	6	1	2	11	⋯
⋯	18	5	4	3	12	⋯
⋯	17	16	15	14	13	⋯
⋯	⋯	⋯	⋯	⋯	⋯	⋯

9. The point P(a, b) in the xy-plane is first rotated counterclockwise by 90 degrees around the point (1, 5) and then reflected about the line y = -x. The image of P after these two transformations is at (-6, 3). What is b - a? A. 1 B. 3 C. 5 D. 7 E. 9	10. An inverted cone with base radius 12 cm and height 18 cm is full of water. The water is poured into a tall cylinder whose horizontal base has radius of 24 cm. What is the height in centimeters of the water in the cylinder? A. 1.5 B. 3 C. 4 D. 4.5 E. 6
11. Grandma has just finished baking a large rectangular pan of brownies. She is planning to make rectangular pieces of equal size and shape, with straight cuts parallel to the sides of the pan. Each cut must be made entirely across the pan. Grandma wants to make the same number of interior pieces as pieces along the perimeter of the pan. What is the greatest possible number of brownies she can produce? A. 24 B. 30 C. 48 D. 60 E. 64	12. Let N = 34 * 34 * 63 * 270. What is the ratio of the sum of the odd divisors of N to the sum of the even divisors of N? A. 1:16 B. 1:15 C. 1:14 D. 1:8 E. 1:3
13. Let n be a positive integer and d be a digit such that the value of the numeral 32d in base n equals 263, and the value of the numeral 324 in base n equals the value of the numeral 11d1 in base six. What is n + d? A. 10 B. 11 C. 13 D. 15 E. 16	14. Three equally spaced parallel lines intersect a circle, creating three chords of lengths 38, 38, and 34. What is the distance between two adjacent parallel lines? A. 5 1/2 B. 6 C. 6 1/2 D. 7 E. 7 1/2
15. The real number x satisfies the equation x + 1/x = sqrt(5). What is the value of x^11 - 7x^7 + x^3? A. -1 B. 0 C. 1 D. 2 E. sqrt(5)	16. Call a positive integer an uphill integer if every digit is strictly greater than the previous digit. For example, 1357, 89, and 5 are all uphill integers, but 32, 1240, and 466 are not. How many uphill integers are divisible by 15? A. 4 B. 5 C. 6 D. 7 E. 8
17. Ravon, Oscar, Aditi, Tyrone, and Kim play a card game. Each person is given 2 cards out of a set of 10 cards numbered 1, 2, 3, …, 10. The score of a player is the sum of the numbers of their cards. The scores of the players are as follows: Ravon--11, Oscar--4, Aditi--7, Tyrone--16, Kim--17. Which of the following statements is true? A. Ravon was given card 3. B. Aditi was given card 3. C. Ravon was given card 4. D. Aditi was given card 4. E. Tyrone was given card 7.	18. A fair 6-sided die is repeatedly rolled until an odd number appears. What is the probability that every even number appears at least once before the first occurrence of an odd number? A. 1/120 B. 1/32 C. 1/20 D. 3/20 E. 1/6

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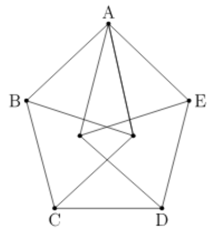
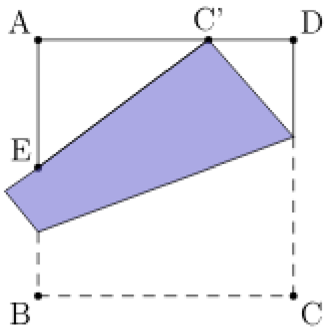
19. Suppose that S is a finite set of positive integers. If the greatest integer in S is removed from S, then the average value (arithmetic mean) of the integers remaining is 32. If the least integer in S is also removed, then the average value of the integers remaining is 35. If the greatest integer is then returned to the set, the average value of the integers rises to 40. The greatest integer in the original set S is 72 greater than the least integer in S. What is the average value of all the integers in the set S?
A. 36.2
B. 36.4
C. 36.6
D. 36.8
E. 37

20. The figure is constructed from 11 line segments, each of which has length 2. The area of pentagon ABCDE can be written as sqrt(m) + sqrt(n), where m and n are positive integers. What is m + n?
A. 20
B. 21
C. 22
D. 23
E. 24

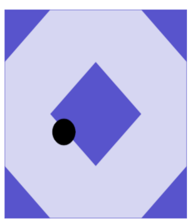


21. A square piece of paper has side length 1 and vertices A, B, C, and D in that order. As shown in the figure, the paper is folded so that vertex C meets edge AD at point C′, and edge BC intersects edge AB at point E.
Suppose that C′D = 1/3. What is the perimeter of triangle AEC′?
A. 2
B. 1 + (2*sqrt(3))/3
C. 13/6
D. 1 + (3*sqrt(3))/4
E. 7/3



22. Ang, Ben, and Jasmin each have 5 blocks, colored red, blue, yellow, white, and green; and there are 5 empty boxes. Each of the people randomly and independently of the other two people places one of their blocks into each box. The probability that at least one box receives 3 blocks all of the same color is m/n, where m and n are relatively prime positive integers. What is m + n?
A. 47
B. 94
C. 227
D. 471
E. 542

23. A square with side length 8 is colored a dark purple, except for 4 bright isosceles right triangular regions with legs of length 2 in each corner of the square and a bright diamond with side length 2*sqrt(2) in the center of the square, as shown in the diagram. A circular coin with diameter 1 is dropped onto the square and lands in a random location where the coin is completely contained within the square. The probability that the coin will cover part of the darker region of the square can be written as 1/196 * (a + b*sqrt(2) + π), where a and b are positive integers. What is a + b?
A. 64
B. 66
C. 68
D. 70
E. 72



24. Arjun and Beth play a game in which they take turns removing one brick or two adjacent bricks from one "wall" among a set of several walls of bricks, with gaps possibly creating new walls. The walls are one brick tall. Arjun plays first, and the player who removes the last brick wins. For which starting configuration is there a strategy that guarantees a win for Beth?
A. (6, 1, 1)
B. (6, 2, 1)
C. (6, 2, 2)
D. (6, 3, 1)
E. (6, 3, 2)

25. Let S be the set of lattice points in the coordinate plane, both of whose coordinates are integers between 1 and 30, inclusive. Exactly 300 points in S lie on or below a line with equation y = mx. The possible values of m lie in an interval of length a/b, where a and b are relatively prime positive integers. What is a + b? A. 31 B. 47 C. 62 D. 72 E. 85	Answer Key: 1. D 2. D 3. C 4. B 5. B 6. C 7. D 8. A 9. D 10. A 11. D 12. C 13. B 14. B 15. B 16. C 17. C 18. C 19. D 20. D 21. A 22. D 23. C 24. B 25. E

AMC 2021A

1. What is the value of (2^2 - 2) - (3^2 - 3) + (4^2 - 4)? A. 1 B. 2 C. 5 D. 8 E. 12	2. Portia's high school has 3 times as many students as Lara's high school. The two high schools have a total of 2600 students. How many students does Portia's high school have? A. 600 B. 650 C. 1950 D. 2000 E. 2050
3. The sum of two natural numbers is 17,402. One of the two numbers is divisible by 10. If the units digit of that number is erased, the other number is obtained. What is the difference of these two numbers? A. 10,272 B. 11,700 C. 13,362 D. 14,238 E. 15,426	4. A cart rolls down a hill, travelling 5 inches the first second and accelerating so that during each successive 1-second time interval, it travels 7 inches more than during the previous 1-second interval. The cart takes 30 seconds to reach the bottom of the hill. How far, in inches, does it travel? A. 215 B. 360 C. 2992 D. 3195 E. 3242
5. The quiz scores of a class with k > 12 students have a mean of 8. The mean of a collection of 12 of these quiz scores is 14. What is the mean of the remaining quiz scores in terms of k? A. (14 - 8)/(k - 12) B. (8k - 168)/(k - 12) C. 14/12 - 8/k D. (14(k - 12))/k^2 E. (14(k - 12))/(8k)	6. Chantal and Jean start hiking from a trailhead toward a fire tower. Jean is wearing a heavy backpack and walks slower. Chantal starts walking at 4 miles per hour. Halfway to the tower, the trail becomes really steep, and Chantal slows down to 2 miles per hour. After reaching the tower, she immediately turns around and descends the steep part of the trail at 3 miles per hour. She meets Jean at the halfway point. What was Jean's average speed, in miles per hour, until they meet? A. 12/13 B. 1 C. 13/12 D. 24/13 E. 2
7. Tom has a collection of 13 snakes, 4 of which are purple and 5 of which are happy. He observes that: • all of his happy snakes can add, • none of his purple snakes can subtract, and • all of his snakes that can't subtract also can't add. Which of these conclusions can be drawn about Tom's snakes? A. Purple snakes can add. B. Purple snakes are happy. C. Snakes that can add are purple. D. Happy snakes are not purple. E. Happy snakes can't subtract.	8. When a student multiplied the number 66 by the repeating decimal 1.abab... = 1.ab (repeating), he did not notice the notation and just multiplied 66 times 1.ab. Later he found that his answer is 0.5 less than the correct answer. What is the 2-digit number ab? A. 15 B. 30 C. 45 D. 60 E. 75

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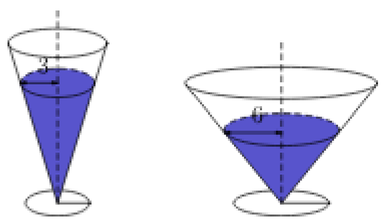
9. What is the least possible value of $(xy - 1)^2 + (x + y)^2$ for real numbers x and y?
A. 0
B. 1/4
C. 1/2
D. 1
E. 2

10. Which of the following is equivalent to $(2+3)(2^2+3^2)(2^4+3^4)(2^8+3^8)(2^{16}+3^{16})(2^{32}+3^{32})(2^{64}+3^{64})$?
A. $3^{127} + 2^{127}$
B. $3^{127} + 2^{127} + 2 \cdot 3^{63} + 3 \cdot 2^{63}$
C. $3^{128} - 2^{128}$
D. $3^{128} + 2^{128}$
E. 5^{127}

11. For which of the following integers b is the base-b number $2021_b - 221_b$ not divisible by 3?
A. 3
B. 4
C. 6
D. 7
E. 8

12. Two right circular cones with vertices facing down as shown in the figure below contain the same amount of liquid. The radii of the tops of the liquid surfaces are 3 cm and 6 cm. Into each cone is dropped a spherical marble of radius 1 cm, which sinks to the bottom and is completely submerged without spilling any liquid. What is the ratio of the rise of the liquid level in the narrow cone to the rise of the liquid level in the wide cone?
A. 1:1
B. 47:43
C. 2:1
D. 40:13
E. 4:1



13. What is the volume of tetrahedron $ABCD$ with edge lengths $AB = 2$, $AC = 3$, $AD = 4$, $BC = \sqrt{13}$, $BD = 2\sqrt{5}$, and $CD = 5$?
A. 3
B. $2\sqrt{3}$
C. 4
D. $3\sqrt{3}$
E. 6

14. All the roots of the polynomial $z^6 - 10z^5 + Az^4 + Bz^3 + Cz^2 + Dz + 16$ are positive integers, possibly repeated. What is the value of B?
A. -88
B. -80
C. -64
D. -41
E. -40

15. Values for A, B, C, and D are to be selected from {1, 2, 3, 4, 5, 6} without replacement (i.e. no two letters have the same value). How many ways are there to make such choices so that the two curves $y = Ax^2 + B$ and $y = Cx^2 + D$ intersect? (The order in which the curves are listed does not matter.)
A. 30
B. 60
C. 90
D. 180
E. 360

16. In the following list of numbers, the integer n appears n times in the list for $1 \le n \le 200$:
1, 2, 2, 3, 3, 3, 4, 4, 4, 4, ..., 200, 200, ..., 200
What is the median of the numbers in this list?
A. 100.5
B. 134
C. 142
D. 150.5
E. 167

17. Trapezoid $ABCD$ has $AB \parallel CD$, $BC = CD = 43$, and $AD \perp BD$. Let O be the intersection of the diagonals AC and BD, and let P be the midpoint of BD. Given that $OP = 11$, the length of AD can be written in the form $m\sqrt{n}$, where m and n are positive integers and n is not divisible by the square of any prime. What is $m + n$?
A. 65
B. 132
C. 157
D. 194
E. 215

18. Let f be a function defined on the set of positive rational numbers with the property that $f(a \cdot b) = f(a) + f(b)$ for all positive rational numbers a and b. Suppose that f also has the property that $f(p) = p$ for every prime number p. For which of the following numbers x is $f(x) < 0$?
A. 17/32
B. 11/16
C. 7/9
D. 7/6
E. 25/11

19. The area of the region bounded by the graph of $x^2 + y^2 = 3\|x - y\| + 3\|x + y\|$ is $m + n\pi$, where m and n are integers. What is m + n? A. 18 B. 27 C. 36 D. 45 E. 54	20. In how many ways can the sequence 1, 2, 3, 4, 5 be rearranged so that no three consecutive terms are increasing and no three consecutive terms are decreasing? A. 10 B. 18 C. 24 D. 32 E. 44
21. Let ABCDEF be an equiangular hexagon. The lines AB, CD, and EF determine a triangle with area 192sqrt(3), and the lines BC, DE, and FA determine a triangle with area 324sqrt(3). The perimeter of hexagon ABCDEF can be expressed as m + n*sqrt(p), where m, n, and p are positive integers and p is not divisible by the square of any prime. What is m + n + p? A. 47 B. 52 C. 55 D. 58 E. 63	22. Hiram's algebra notes are 50 pages long and are printed on 25 sheets of paper; the first sheet contains pages 1 and 2, the second sheet contains pages 3 and 4, and so on. One day he leaves his notes on the table before leaving for lunch, and his roommate decides to borrow some pages from the middle of the notes. When Hiram comes back, he discovers that his roommate has taken a consecutive set of sheets from the notes and that the average (mean) of the page numbers on all remaining sheets is exactly 19. How many sheets were borrowed? A. 10 B. 13 C. 15 D. 17 E. 20
23. Frieda the frog begins a sequence of hops on a 3×3 grid of squares, moving one square on each hop and choosing at random the direction of each hop-up, down, left, or right. She does not hop diagonally. When the direction of a hop would take Frieda off the grid, she "wraps around" and jumps to the opposite edge. Suppose Frieda starts from the center square, makes at most four hops at random, and stops hopping if she lands on a corner square. What is the probability that she reaches a corner square on one of the four hops? A. 9/16 B. 5/8 C. 3/4 D. 25/32 E. 13/16	24. The interior of a quadrilateral is bounded by the graphs of (x + ay)^2 = 4a^2 and (ax - y)^2 = a^2, where a is a positive real number. What is the area of this region in terms of a, valid for all a > 0? A. (8a^2)/((a+1)^2) B. (4a)/(a+1) C. (8a)/(a+1) D. (8a^2)/(a^2+1) E. (8a)/(a^2+1)

25. How many ways are there to place 3 indistinguishable red chips, 3 indistinguishable blue chips, and 3 indistinguishable green chips in the squares of a 3×3 grid so that no two chips of the same color are directly adjacent to each other, either vertically or horizontally? A. 12 B. 18 C. 24 D. 30 E. 36	Answer Key: 1. D 2. C 3. D 4. D 5. B 6. A 7. D 8. E 9. D 10. C 11. E 12. E 13. C 14. A 15. C 16. C 17. D 18. E 19. E 20. D 21. C 22. B 23. D 24. D 25. E

AMC 2020B

1. What is the value of $1 - (-2) - 3 - (-4) - 5 - (-6)$? A. -20 B. -3 C. 3 D. 5 E. 21	2. Carl has 5 cubes each of side length 1, and Kate has 5 cubes each of side length 2. What is the total volume of the 10 cubes? A. 24 B. 25 C. 28 D. 40 E. 45
3. The ratio of w to x is 4:3, the ratio of y to z is 3:2, and the ratio of z to x is 1:6. What is the ratio of w to y? A. 4:3 B. 3:2 C. 8:3 D. 4:1 E. 16:3	4. The acute angles of a right triangle are $a°$ and $b°$, where $a > b$ and both a and b are prime numbers. What is the least possible value of b? A. 2 B. 3 C. 5 D. 7 E. 11
5. How many distinguishable arrangements are there of 1 brown tile, 1 purple tile, 2 green tiles, and 3 yellow tiles in a row? A. 210 B. 420 C. 630 D. 840 E. 1050	6. Megan's odometer shows 15951. 2 hours later, it shows the next palindrome. What is her average speed? A. 50 B. 55 C. 60 D. 65 E. 70
7. How many positive even multiples of 3 less than 2020 are perfect squares? A. 7 B. 8 C. 9 D. 10 E. 12	8. Points P and Q lie in a plane with PQ = 8. How many locations for point R make $\triangle PQR$ a right triangle of area 12? A. 2 B. 4 C. 6 D. 8 E. 12

9. How many integer solutions (x, y) satisfy $x^{2020} + y^2 = 2y$?

A. 1

B. 2

C. 3

D. 4

E. infinitely many

10. A three-quarter sector of a circle of radius 4 inches forms the lateral surface of a cone. What is the volume of the cone?

A. $3\pi/5$

B. $4\pi/3$

C. $3\pi/7$

D. $6\pi/3$

E. $6\pi/7$

11. Harold and Betty each select 5 books from 10. Probability exactly 2 are the same?

A. 1/8

B. 5/36

C. 14/45

D. 25/63

E. 1/2

12. Decimal representation of $1 / 2^{2020}$: How many zeros after the decimal point?

A. 23

B. 24

C. 25

D. 26

E. 27

13. Andy the Ant starts at $(-20,20)$ facing east, moves 1, turns 90° left, then moves 2, turns 90° left, etc. Location at 2020th left turn?

A. $(-1030, -994)$

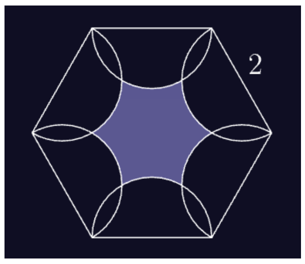
B. $(-1030, -990)$

C. $(-1026, -994)$

D. $(-1026, -990)$

E. $(-1022, -994)$

14. Six semicircles lie inside a regular hexagon of side 2. Area inside hexagon but outside semicircles?



A. $6\sqrt{3} - 3\pi$

B. $9\sqrt{3}/2 - 2\pi$

C. $3\sqrt{3}/2 - \pi$

D. $3\sqrt{3} - \pi$

E. $9\sqrt{3}/2 - \pi$

15. Steve writes digits 1,2,3,4,5 repeatedly to form 10,000-digit list. He removes every 3rd, then 4th, then 5th digit. Sum of digits in positions 2019, 2020, 2021?

A. 7

B. 9

C. 10

D. 11

E. 12

16. Game on $[0,n]$. Bela chooses first. Players must pick numbers >1 unit from previous numbers. Who wins with optimal strategy?

A. Bela always wins

B. Jenn always wins

C. Bela wins iff n is odd

D. Jenn wins iff n is odd

E. Bela wins iff n > 8

AI HOMEWORK

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17. 10 people stand in circle. Each knows 2 neighbors + 1 opposite. How many ways to split into 5 pairs where members know each other?

A. 11

B. 12

C. 13

D. 14

E. 15

18. Urn with 1 red and 1 blue. Repeat drawing and adding same color 4 times. Probability 3 of each color?

A. 1/6

B. 1/5

C. 1/4

D. 1/3

E. 1/2

19. Number of 10-card hands from 52 cards: 158A00A4AA0. Find A.

A. 2

B. 3

C. 4

D. 6

E. 7

20. Box with edges 1,3,4. Volume of $S(r) = a r^3 + b r^2 + c r + d$. Find $b \cdot c / a \cdot d$.

A. 6

B. 19

C. 24

D. 26

E. 38

21. In square ABCD, points E and H lie on AB and DA so that AE = AH. Points F and G lie on BC and CD, and I and J lie on EH such that FI ⊥ EH and GJ ⊥ EH. Each polygon has area 1. What is FI^2?

A. 7/3

B. $8 - 4\sqrt{2}$

C. $1 + \sqrt{2}$

D. $7/4\sqrt{2}$

E. 2^2

22. What is the remainder when 2^202 + 202 is divided by 2^101 + 2^51 + 1?

A. 100

B. 101

C. 200

D. 201

E. 202

23. A cylindrical tank has radius 3 meters and height 5 meters. Water is poured into the tank at a rate of 9 cubic meters per minute. How long, in minutes, will it take to fill the tank completely?

A. 5

B. 15

C. 20

D. 25

E. 30

24. If f(x) = 2x + 1 and g(x) = x^2, what is (g \circ f)(3)?

A. 49

B. 25

C. 36

D. 13

E. 12

25. Let D(n) denote the number of ways of writing the positive integer n as a product $n = f_1 * f_2 * \ldots * f_k$, where $k \geq 1$, the f_i are integers strictly greater than 1, and the order in which the factors are listed matters (that is, two representations that differ only in the order of the factors are counted as distinct). For example, the number 6 can be written as 6, 2 * 3, and 3 * 2, so D(6) = 3. What is D(96)? A. 112 B. 128 C. 144 D. 172 E. 184	Answer Key: 1. D 2. E 3. E 4. D 5. B 6. B 7. A 8. D 9. D 10. C 11. D 12. D 13. B 14. D 15. D 16. A 17. C 18. B 19. A 20. B 21. B 22. D 23. C 24. C 25. A

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AMC 2020A

1. What value of x satisfies $x - 3/4 = 5/12 - 1/3$?

A. $-2/3$

B. $7/36$

C. $7/12$

D. $2/3$

E. $5/6$

2. The numbers 3, 5, 7, a, and b have an average of 15. What is the average of a and b?

A. 0

B. 15

C. 30

D. 45

E. 60

3. Assuming $a \neq 3$, $b \neq 4$, and $c \neq 5$, what is the value in simplest form of the expression:

$(5-c)/(a-3) * (3-a)/(b-4) * (4-b)/(c-5)$

A. -1

B. 1

C. $abc/60$

D. $1/(abc) - 60$

E. $1/60 - abc$

4. A driver travels for 2 hours at 60 miles per hour. Her car gets 30 miles per gallon of gasoline. She is paid \$0.50 per mile, and her only expense is gasoline at \$2.00 per gallon. What is her net rate of pay, in dollars per hour, after this expense?

A. 20

B. 22

C. 24

D. 25

E. 26

5. What is the sum of all real numbers x for which $|x^2 - 12x + 34| = 2$?

A. 12

B. 15

C. 18

D. 21

E. 25

6. How many 4-digit positive integers with only even digits are divisible by 5?

A. 80

B. 100

C. 125

D. 200

E. 500

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7. The 25 integers from -10 to 14 can be arranged to form a 5-by-5 square such that the sum of each row, column, and main diagonals are equal. What is this common sum?

A. 2

B. 5

C. 10

D. 25

E. 50

8. What is the value of $1 + 2 + 3 - 4 + 5 + 6 + 7 - 8 + \ldots + 197 + 198 + 199 - 200$?

A. 9,800

B. 9,900

C. 10,000

D. 10,100

E. 10,200

9. A bench holds either 7 adults or 11 children. If N benches are connected end to end, an equal number of adults and children seated together will occupy all space. What is the least positive integer N?

A. 9

B. 18

C. 27

D. 36

E. 77

10. Seven cubes with volumes 1, 8, 27, 64, 125, 216, 343 are stacked from largest to smallest to form a tower. What is the total surface area of the tower?

A. 644

B. 658

C. 664

D. 720

E. 749

11. What is the median of the list of 4040 numbers: $1, 2, \ldots, 2020, 1^2, 2^2, \ldots, 2020^2$?

A. 1974.5

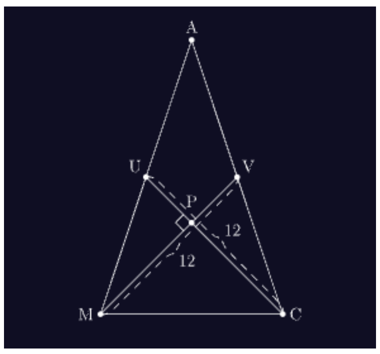
B. 1975.5

C. 1976.5

D. 1977.5

E. 1978.5

12. Triangle AMC is isosceles with AM = AC. Medians MV and CU are perpendicular and MV = CU = 12. What is the area of triangle AMC?



A. 48

B. 72

C. 96

D. 144

E. 192

13. A frog starts at (1,2) and jumps length 1 along axes randomly until reaching a square with vertices (0,0), (0,4), (4,4), (4,0). What is the probability the frog ends on a vertical side?

A. 1/2

B. 5/8

C. 2/3

D. 3/4

E. 7/8

14. Real numbers x and y satisfy $x + y = 4$ and $x*y = -2$. What is the value of $x + y^2/x^3 + x^2/y^3 + y$?

A. 360

B. 400

C. 420

D. 440

E. 480

15. A positive integer divisor of 12! is chosen randomly. The probability that it is a perfect square is m/n, with m and n coprime. What is m + n?

A. 3

B. 5

C. 12

D. 18

E. 23

16. A point is randomly chosen inside a square with vertices (0,0), (2020,0), (2020,2020), (0,2020). Probability that it is within d units of a lattice point is 1/2. Find d to the nearest tenth.

A. 0.3

B. 0.4

C. 0.5

D. 0.6

E. 0.7

17. Define $P(x) = (x-1^2)(x-2^2)\ldots(x-100^2)$. How many integers n satisfy $P(n) \le 0$?

A. 4900

B. 4950

C. 5000

D. 5050

E. 5100

18. Let (a,b,c,d) be integers in {0,1,2,3}. For how many quadruples is $ad - bc$ odd?

A. 48

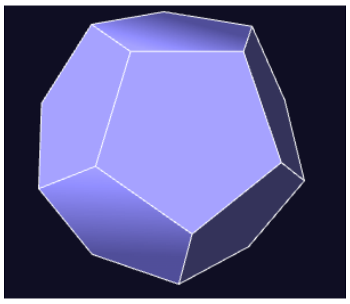
B. 64

C. 96

D. 128

E. 192

19. A regular dodecahedron floats in space with two horizontal faces. How many ways are there to move from the top face to the bottom face via adjacent faces visiting each face at most once?



A. 125

B. 250

C. 405

D. 640

E. 810

20. Quadrilateral ABCD satisfies $\angle ABC = \angle ACD = 90°$, $AC = 20$, $CD = 30$, and diagonals AC and BD intersect at E with $AE = 5$. What is the area of ABCD?

A. 330

B. 340

C. 350

D. 360

E. 370

21. There exists a unique strictly increasing sequence of nonnegative integers $a_1 < a_2 < \ldots < a_k$ such that $2^{17} + 1/2^{289} + 1 = 2^{a_1} + 2^{a_2} + \ldots + 2^{a_k}$. What is k?

A. 117

B. 136

C. 137

D. 273

E. 306

22. For how many positive integers $n \leq 1000$ is $\lfloor 998/n \rfloor + \lfloor 999/n \rfloor + \lfloor 1000/n \rfloor$ not divisible by 3?

A. 22

B. 23

C. 24

D. 25

E. 26

23. Let T be the triangle with vertices (0,0), (4,0), (0,3). How many of the 125 sequences of three plane transformations (rotations of 90°, 180°, 270°, reflections across x-axis or y-axis) return T to its original position?

A. 12

B. 15

C. 17

D. 20

E. 25

24. Let n be the least positive integer greater than 1000 such that $\gcd(63, n+120) = 21$ and $\gcd(n+63, 120) = 60$. What is the sum of the digits of n?

A. 12

B. 15

C. 18

D. 21

E. 24

25. Jason rolls three fair dice. He rerolls a subset of dice to maximize chances of winning a sum of 7. What is the probability he rerolls exactly two dice? A. 7/36 B. 5/24 C. 2/9 D. 17/22 E. 1/4	Answer Key: 1. E 2. C 3. A 4. E 5. C 6. B 7. C 8. B 9. B 10. B 11. C 12. C 13. B 14. D 15. E 16. B 17. E 18. C 19. E 20. D 21. C 22. A 23. A 24. C 25. A

AMC 2019B

1. Alicia had two containers. The first was 5/6 full of water and the second was empty. She poured all the water from the first container into the second container, at which point the second container was 3/4 full of water. What is the ratio of the volume of the first container to the volume of the second container? A. 5/8 B. 4/5 C. 7/8 D. 9/10 E. 11/12	2. Consider the statement, "If n is not prime, then n − 2 is prime." Which of the following values of n is a counterexample to this statement? A. 11 B. 15 C. 19 D. 21 E. 27
3. In a high school with 500 students, 40% of the seniors play a musical instrument, while 30% of the non-seniors do not play a musical instrument. In all, 46.8% of the students do not play a musical instrument. How many non-seniors play a musical instrument? A. 66 B. 154 C. 186 D. 220 E. 266	4. All lines with equation ax + by = c such that a, b, c form an arithmetic progression pass through a common point. What are the coordinates of that point? A. (-1, 2) B. (0, 1) C. (1, -2) D. (1, 0) E. (1, 2)
5. Triangle ABC lies in the first quadrant. Points A, B, and C are reflected across the line y = x to points A', B', and C', respectively. Assume that none of the vertices lie on the line y = x. Which statement is not always true? A. Triangle A'B'C' lies in the first quadrant. B. Triangles ABC and A'B'C' have the same area. C. The slope of line AA' is -1. D. The slopes of lines AA' and CC' are the same. E. Lines AB and A'B' are perpendicular.	6. A positive integer n satisfies the equation (n+1)! + (n+2)! = n! * 440. What is the sum of the digits of n? A. 3 B. 8 C. 10 D. 11 E. 12

AI HOMEWORK

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7. Each piece of candy costs a whole number of cents. Casper has enough money to buy either 12 pieces of red candy, 14 pieces of green candy, 15 pieces of blue candy, or n pieces of purple candy. A piece of purple candy costs 20 cents. What is the smallest possible value of n?

A. 18

B. 21

C. 24

D. 25

E. 28

8. A square has four equilateral triangles on its sides, each of side length 2, with their third vertices meeting at the center. The region inside the square but outside the triangles is shaded. What is the area of the shaded region?

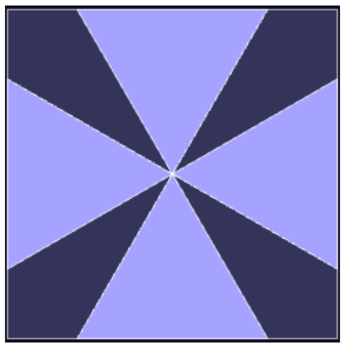
A. 4

B. $12 - 4\sqrt{3}$

C. $3\sqrt{3}$

D. $4\sqrt{3}$

E. $16 - 4\sqrt{3}$



9. The function f(x) = floor(|x|) − |floor(x)| for all real x. What is the range of f?

A. {-1, 0}

B. The set of nonpositive integers

C. {-1, 0, 1}

D. {0}

E. The set of nonnegative integers

10. In a plane, points A and B are 10 units apart. How many points C exist such that the perimeter of triangle ABC is 50 units and the area is 100 square units?

A. 0

B. 2

C. 4

D. 8

E. Infinitely many

11. Two jars have the same number of marbles, each marble blue or green. Jar 1 has a 9:1 blue-to-green ratio; Jar 2 has 8:1. There are 95 green marbles in total. How many more blue marbles are in Jar 1 than Jar 2?

A. 5

B. 10

C. 25

D. 45

E. 50

12. What is the greatest possible sum of the digits in the base-seven representation of a positive integer less than 2019?

A. 11

B. 14

C. 22

D. 23

E. 27

13. What is the sum of all real numbers x for which the median of 4, 6, 8, 17, x equals the mean of these five numbers?

A. -5

B. 0

C. 5

D. 15/4

E. 35/4

14. The base-ten representation of 19! is partially given as 121,T5,100,40M,832,H00. What is T + M + H?

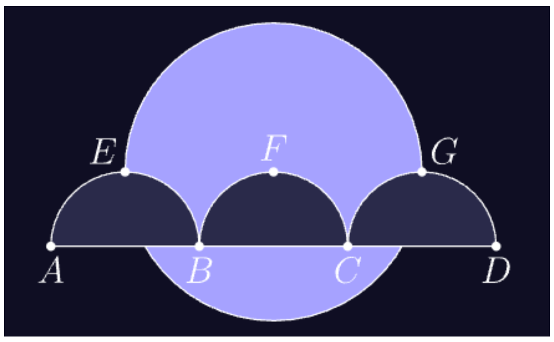
A. 3

B. 8

C. 12

D. 14

E. 17

15. Right triangles T1 and T2 have areas 1 and 2, respectively. A side of T1 is congruent to a side of T2, and a different side of T1 is congruent to a different side of T2. What is the square of the product of the lengths of the other sides? A. 28/3 B. 10 C. 32/3 D. 34/3 E. 12	16. In triangle ABC (right angle at C), points D on AB and E on BC satisfy AC = CD, DE = EB, and AC:DE = 4:3. What is AD:DB? A. 2:3 B. 2:5 C. 1:1 D. 3:5 E. 3:2
17. A red and green ball are independently tossed into positive-integer-numbered bins with probability 2^-k for bin k. What is the probability that the red ball lands in a higher-numbered bin than the green ball? A. 1/4 B. 2/7 C. 1/3 D. 3/8 E. 3/7	18. Henry walks 3/4 of the way to his gym 2 km from home, then 3/4 back toward home, then 3/4 toward the gym, and repeats. He eventually oscillates between points A and B. What is |A − B|? A. 2/3 B. 1 C. 6/5 D. 1/4 E. 1/2
19. Let S be the set of positive divisors of 100,000. How many numbers are the product of two distinct elements of S? A. 98 B. 100 C. 117 D. 119 E. 121	20. Line segment AD is trisected by points B and C (AB=BC=CD=2). Semicircles of radius 1 with diameters on AD are tangent to line EG. A circle of radius 2 is centered at F. The area inside the circle but outside semicircles is a/b * π − c + d. What is a + b + c + d? A. 13 B. 14 C. 15 D. 16 E. 17 

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21. Debra flips a fair coin until she gets either two heads or two tails in a row. What is the probability she gets two heads but sees a second tail before a second head?

A. 1/36

B. 1/24

C. 1/18

D. 1/12

E. 1/6

22. Raashan, Sylvia, and Ted each start with $1. Every 15 seconds, players with money give $1 to a randomly chosen other player. What is the probability that after 2019 rounds each has $1?

A. 1/7

B. 1/4

C. 1/3

D. 1/2

E. 2/3

23. Points A = (6, 13) and B = (12, 11) lie on circle ω in the plane. Suppose that the tangent lines to ω at A and B intersect at a point on the x-axis. What is the area of ω?

A. (83π)/8

B. (21π)/2

C. (85π)/8

D. (43π)/4

E. (87π)/8

24. Define a sequence recursively by $x_0 = 5$ and

$x_{n+1} = (x_n^2 + 5x_n + 4) / (x_n + 6)$

for all nonnegative integers n. Let m be the least positive integer such that

$x_m \leq 4 + 1/(2^{20})$.

In which of the following intervals does m lie?

A. [9, 26]

B. [27, 80]

C. [81, 242]

D. [243, 728]

E. [729, ∞)

25. How many sequences of 0s and 1s of length 19 are there that begin and end with a 0, contain no two consecutive 0s, and contain no three consecutive 1s? A. 55 B. 60 C. 65 D. 70 E. 75	Answer Key: 1. D 2. E 3. B 4. A 5. E 6. C 7. B 8. B 9. A 10. A 11. A 12. C 13. A 14. C 15. A 16. A 17. C 18. C 19. C 20. E 21. B 22. B 23. C 24. C 25. C

AI HOMEWORK

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AMC 2019A

1. What is the value of

$$2^{\left(0^{\left(1^{9}\right)}\right)} + \left(\left(2^{0}\right)^{1}\right)^{9} ?$$

A. 0

B. 1

C. 2

D. 3

E. 4

2. What is the hundreds digit of (20! – 15!) ?

A. 0

B. 1

C. 2

D. 4

E. 5

3. Ana and Bonita were born on the same date in different years, n years apart. Last year Ana was 5 times as old as Bonita. This year Ana's age is the square of Bonita's age. What is n?

A. 3

B. 5

C. 9

D. 12

E. 15

4. A box contains 28 red balls, 20 green balls, 19 yellow balls, 13 blue balls, 11 white balls, and 9 black balls. What is the minimum number of balls that must be drawn to guarantee at least 15 of one color?

A. 75

B. 76

C. 79

D. 84

E. 91

5. What is the greatest number of consecutive integers whose sum is 45?

A. 9

B. 25

C. 45

D. 90

E. 120

6. For how many of the following quadrilaterals does there exist a point equidistant from all four vertices?

• a square

• a rectangle (not a square)

• a rhombus (not a square)

• a parallelogram (not rectangle/rhombus)

• an isosceles trapezoid (not parallelogram)

A. 1

B. 2

C. 3

D. 4

E. 5

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7. Two lines with slopes 1/2 and 2 intersect at (2, 2). What is the area of the triangle enclosed by these lines and $x + y = 10$?

A. 4

B. $4\sqrt{2}$

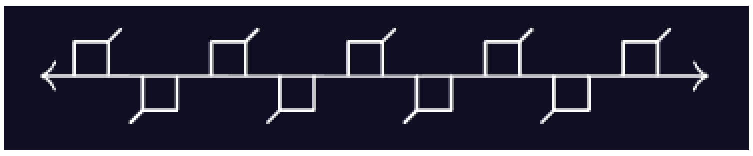
C. 6

D. 8

E. $6\sqrt{2}$

8. The figure below shows line ℓ with a regular, infinite, recurring pattern of squares and line segments.

How many of the following four kinds of rigid motion transformations of the plane in which this figure is drawn, other than the identity transformation, will transform this figure into itself?



A. Some rotation around a point of line ℓ

B. Some translation in the direction parallel to line ℓ

C. The reflection across line ℓ

D. Some reflection across a line perpendicular to line ℓ

E. None of the above

9. What is the greatest three-digit positive integer n for which the sum of the first n positive integers is not a divisor of the product of the first n positive integers?

A. 995

B. 996

C. 997

D. 998

E. 999

10. A rectangular floor that is 10 feet wide and 17 feet long is tiled with 170 one-foot square tiles. A bug walks from one corner to the opposite corner in a straight line. Including the first and last tiles, how many tiles does the bug visit?

A. 17

B. 25

C. 26

D. 27

E. 28

11. How many positive integer divisors of 2019 are perfect squares or perfect cubes (or both)?

A. 32

B. 36

C. 37

D. 39

E. 41

12. Melanie computes the mean (μ), the median (M), and the modes of the 365 values that are the dates in the months of 2019. Thus, her data consist of 12 1s, 12 2s, …, 12 28s, 11 29s, 11 30s, and 7 31s. Let d be the median of the modes. Which of the following statements is true?

A. $\mu < d < M$

B. $M < d < \mu$

C. $d = M = \mu$

D. $d < M < \mu$

E. $d < \mu < M$

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13. Let △ABC be an isosceles triangle with BC = AC and ∠ACB = 40°. Construct the circle with diameter BC, and let D and E be the other intersection points of the circle with AC and AB respectively. Let F be the intersection of diagonals of quadrilateral BCDE. What is ∠BFC?

A. 90°

B. 100°

C. 105°

D. 110°

E. 120°

14. For a set of four distinct lines in a plane, there are exactly N distinct points that lie on two or more of the lines. What is the sum of all possible values of N?

A. 14

B. 16

C. 18

D. 19

E. 21

15. A sequence is defined by $a_1 = 1$, $a_2 = 7/3$, and $a_n = (2a_{n-2} - a_{n-1})/(a_{n-2} \times a_{n-1})$ for $n \geq 3$. Then $a_{2019} = p/q$, where p and q are coprime. Find p + q.

A. 2020

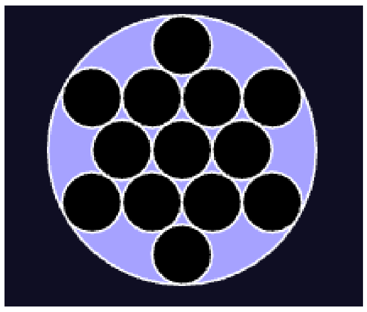
B. 4039

C. 6057

D. 6061

E. 8078

16. The figure below shows 13 circles of radius 1 within a larger circle. All the intersections occur at points of tangency. What is the area of the region, shaded in the figure, inside the larger circle but outside all the circles of radius 1?



A. 4π/3

B. 7π

C. π(3 + 2)

D. 10π(3 − 1)

E. π(3 + 6)

17. A child builds towers using identically shaped cubes of different colors. How many different towers with a height of 8 cubes can the child build with 2 red cubes, 3 blue cubes, and 4 green cubes? (One cube will be left out.)

A. 24

B. 288

C. 312

D. 1,260

E. 40,320

18. For some positive integer k, the repeating base-k representation of 7/51 is 0.23 repeating. What is k?

A. 13

B. 14

C. 15

D. 16

E. 17

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19. What is the least possible value of $(x + 1)(x + 2)(x + 3)(x + 4) + 2019$, where x is a real number? A. 2017 B. 2018 C. 2019 D. 2020 E. 2021	20. The numbers 1,2,…,9 are randomly placed into a 3×3 grid. What is the probability that the sum of numbers in each row and column is odd? A. 1/21 B. 1/14 C. 5/63 D. 2/21 E. 1/7
21. A sphere with center O has radius 6. A triangle with sides 15, 15, 24 is tangent to the sphere. What is the distance from O to the plane of the triangle? A. $2\sqrt{3}$ B. 4 C. $3\sqrt{2}$ D. $2\sqrt{5}$ E. 5	22. Real numbers x and y are chosen as follows: • Flip a coin. If heads, flip again: number is 0 if heads, 1 if tails. • If tails, choose uniformly from [0,1]. What is the probability that $\lvert x - y\rvert > 1/2$? A. 1/3 B. 7/16 C. 1/2 D. 9/16 E. 2/3
23. Travis plays a counting game with the Thompson triplets. What is the 2019th number said by Tadd? A. 5743 B. 5885 C. 5979 D. 6001 E. 6011	24. Let p, q, r be distinct roots of $x^3 - 22x^2 + 80x - 67$. If $1/(s - p) + 1/(s - q) + 1/(s - r) = 1/(x^3 - 22x^2 + 80x - 67)$, what is $1/A + 1/B + 1/C$? A. 243 B. 244 C. 245 D. 246 E. 247

25. For how many integers n between 1 and 50 is $(n^2 - 1)! / (n!)^n$ an integer? A. 31 B. 32 C. 33 D. 34 E. 35	Answer Key: 1. C 2. A 3. D 4. B 5. D 6. C 7. C 8. C 9. B 10. C 11. C 12. E 13. D 14. D 15. E 16. A 17. D 18. D 19. B 20. B 21. D 22. B 23. C 24. B 25. D

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AMC 2018A

1. What is the value of ((2+1)-1+1)-1+1)-1+1? A. 5/8 B. 11/7 C. 8/5 D. 18/11 E. 15/8	2. Liliane has 50% more soda than Jacqueline, and Alice has 25% more soda than Jacqueline. What is the relationship between the amounts of soda that Liliane and Alice have? A. Liliane has 20% more soda than Alice. B. Liliane has 25% more soda than Alice. C. Liliane has 45% more soda than Alice. D. Liliane has 75% more soda than Alice. E. Liliane has 100% more soda than Alice.
3. A unit of blood expires after 10! = 10\cdot9\cdot8\cdots1 seconds. Yasin donates a unit at noon of January 1. On what day does it expire? A. January 2 B. January 12 C. January 22 D. February 11 E. February 12	4. How many ways can a student schedule 3 mathematics courses (algebra, geometry, number theory) in a 6-period day if no two courses are consecutive? A. 3 B. 6 C. 12 D. 18 E. 24
5. Alice: “≥6 miles away.” Bob: “≤5 miles away.” Charlie: “≤4 miles away.” None are true. Let d be the distance. Which interval represents all possible d? A. (0,4) B. (4,5) C. (4,6) D. (5,6) E. (5,∞)	6. Sangho’s video score = 90, 65% likes. How many votes cast? A. 200 B. 300 C. 400 D. 500 E. 600

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7.

7. For how many (not necessarily positive) integer values of n is the following value an integer?

$$4000 \cdot \left(\frac{2}{5}\right)^n$$

A. 3

B. 4

C. 6

D. 8

E. 9

8. Joe has 23 coins (5c, 10c, 25c). 10c coins = 5c coins + 3. Total value = 320c. How many more 25c than 5c coins?

A. 0

B. 1

C. 2

D. 3

E. 4

9. Triangles similar to isosceles ABC, 7 smallest triangles area = 1, △ABC area = 40. Area of trapezoid DBCE?

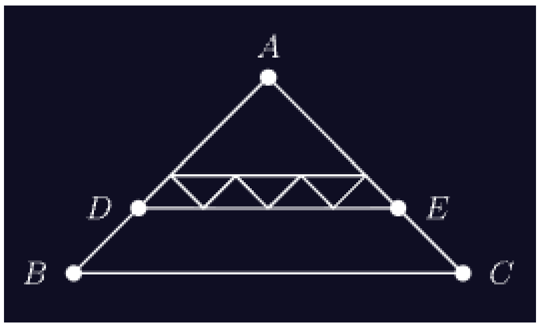
A. 16

B. 18

C. 20

D. 22

E. 24



10.

10. Suppose that real number x satisfies

$$\sqrt{49-x^2}-\sqrt{25-x^2}=3.$$

What is the value of

$$\sqrt{49-x^2}+\sqrt{25-x^2}?$$

A. 8

B. 33/8

C. 9

D. 2√10 + 4

E. 12

11. 7 dice rolled, probability sum = 10 → n/6^7. Find n.

A. 42

B. 49

C. 56

D. 63

E. 84

12. How many ordered pairs (x,y) satisfy \{x+3y=3, |x|-|y|=1\}?

A. 1

B. 2

C. 3

D. 4

E. 8

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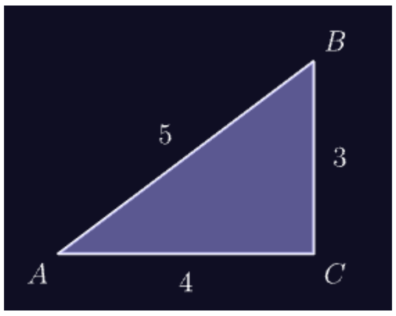
13. Triangle sides 3,4,5 folded so A falls on B. Length of crease?



A. 1+1/2

B. 3

C. 7/4

D. 15/8

E. 2

14. What is the greatest integer less than or equal to {3^{100}+2^{100}} / {3^{96}+2^{96}}?

A. 80

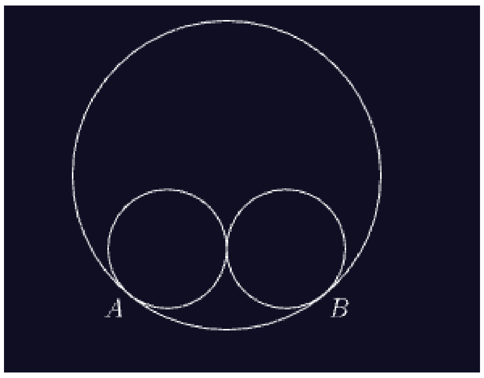
B. 81

C. 96

D. 97

E. 625

15. Two circles of radius 5 are externally tangent and internally tangent to a circle of radius 13 at points A and B. Distance AB = m/n, with m,n relatively prime. Find m+n.



A. 21

B. 29

C. 58

D. 69

E. 93

16. Right triangle ABC, legs AB=20, BC=21. How many line segments with integer length can be drawn from B to a point on hypotenuse AC?

A. 5

B. 8

C. 12

D. 13

E. 15

17. S is a set of 6 integers from {1,…,12}, such that if a<b in S, then b is not a multiple of a. Least possible element?

A. 2

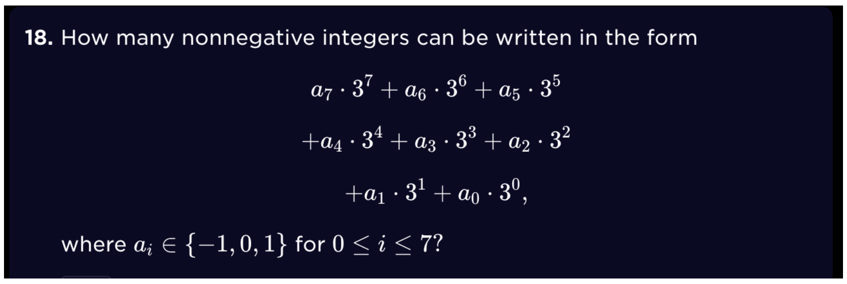
B. 3

C. 4

D. 5

E. 7

18. How many nonnegative integers can be written as ?



18. How many nonnegative integers can be written in the form

$$a_7 \cdot 3^7 + a_6 \cdot 3^6 + a_5 \cdot 3^5$$
$$+a_4 \cdot 3^4 + a_3 \cdot 3^3 + a_2 \cdot 3^2$$
$$+a_1 \cdot 3^1 + a_0 \cdot 3^0,$$

where $a_i \in \{-1, 0, 1\}$ for $0 \le i \le 7$?

A. 512

B. 729

C. 1094

D. 3281

E. 59,048

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19. Randomly select m ∈ {11,13,15,17,19} and n ∈ {1999,…,2018}. Probability that m^n has units digit 1?

A. 1/5

B. 1/4

C. 3/10

D. 7/20

E. 2/5

20. 7×7 scanning code, symmetric under rotations/reflections. At least one black and one white square. Total number of symmetric codes?

A. 510

B. 1022

C. 8190

D. 8192

E. 65,534

21. Find values of a for which x^2+y^2=a^2 and y=x^2-a intersect at exactly 3 points.

A. a=1/4

B. 1/4 < a < 1/2

C. a>1/4

D. a=1/2

E. a>1/2

22. Positive integers a,b,c,d with gcd(a,b)=24, gcd(b,c)=36, gcd(c,d)=54, 70 < gcd(d,a) < 100. Which must divide a?

A. 5

B. 7

C. 11

D. 13

E. 17

23. Right triangle, legs 3,4, unplanted square in right angle corner, shortest distance to hypotenuse = 2. Fraction of field planted?

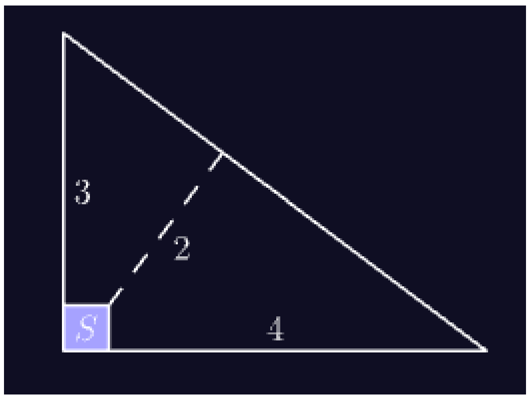
A. 25/27

B. 26/27

C. 73/75

D. 145/147

E. 74/75



24. Triangle ABC, AB=50, AC=10, area=120. D,E midpoints of AB,AC. Angle bisector of ∠BAC intersects DE and BC at F,G. Area of FDBG?

A. 60

B. 65

C. 70

D. 75

E. 80

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25. Positive integer n, nonzero digits a,b,c. A_n = n-digit with all a, B_n = n-digit with all b, C_n = 2n-digit with all c. Find max a+b+c such that C_n - B_n = A_n^2 has ≥2 values of n. A. 12 B. 14 C. 16 D. 18 E. 20	Answer Key: 1. B 2. A 3. E 4. E 5. D 6. B 7. E 8. C 9. E 10. A 11. E 12. C 13. D 14. A 15. D 16. D 17. C 18. D 19. E 20. B 21. E 22. D 23. D 24. D 25. D

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AMC 2017B

1. Mary thought of a positive two-digit number. She multiplied it by 3 and added 11. Then she switched the digits of the result, obtaining a number between 71 and 75, inclusive. What was Mary's number? A. 11 B. 12 C. 13 D. 14 E. 15	2. Sofia ran 5 laps around the 400-meter track at her school. For each lap, she ran the first 100 meters at an average speed of 4 meters per second and the remaining 300 meters at an average speed of 5 meters per second. How much time did Sofia take running the 5 laps? A. 5 minutes and 35 seconds B. 6 minutes and 40 seconds C. 7 minutes and 5 seconds D. 7 minutes and 25 seconds E. 8 minutes and 10 seconds
3. Real numbers x, y, and z satisfy the inequalities $0 < x < 1$, $-1 < y < 0$, and $1 < z < 2$. Which of the following numbers is necessarily positive? A. $y + x/2$ B. $y + xz$ C. $y + y/2$ D. $y + 2y/2$ E. $y + z$	4. Supposed that x and y are nonzero real numbers such that $(3x + y)/(x - 3y) = -2$. What is the value of $(x + 3y)/(3x - y)$? A. -3 B. -1 C. 1 D. 2 E. 3
5. Camilla had twice as many blueberry jelly beans as cherry jelly beans. After eating 10 pieces of each kind, she now has three times as many blueberry jelly beans as cherry jelly beans. How many blueberry jelly beans did she originally have? A. 10 B. 20 C. 30 D. 40 E. 50	6. What is the largest number of solid 2 in × 2 in × 1 in blocks that can fit in a 3 in × 2 in × 3 in box? A. 3 B. 4 C. 5 D. 6 E. 7

7. Samia set off on her bicycle to visit her friend, traveling at an average speed of 17 kilometers per hour. When she had gone half the distance to her friend's house, a tire went flat, and she walked the rest of the way at 5 kilometers per hour. In all, it took her 44 minutes to reach her friend's house. In kilometers rounded to the nearest tenth, how far did Samia walk?

A. 2.0

B. 2.2

C. 2.8

D. 3.4

E. 4.4

8. Points A(11,9) and B(2,−3) are vertices of △ABC with AB = AC. The altitude from A meets the opposite side at D(−1,3). What are the coordinates of point C?

A. (−8,9)

B. (−4,8)

C. (−4,9)

D. (−2,3)

E. (−1,0)

9. A radio program has a quiz consisting of 3 multiple-choice questions, each with 3 choices. A contestant wins if he or she gets 2 or more of the questions right. The contestant answers randomly to each question. What is the probability of winning?

A. 1/27

B. 1/9

C. 2/9

D. 7/27

E. 1/2

10. The lines with equations $ax - 2y = c$ and $2x + by = -c$ are perpendicular and intersect at $(1, -5)$. What is c?

A. −13

B. −8

C. 2

D. 8

E. 13

11. At Typico High School, 60% of the students like dancing, and the rest dislike it. Of those who like dancing, 80% say that they like it, and the rest say that they dislike it. Of those who dislike dancing, 90% say that they dislike it, and the rest say that they like it. What fraction of students who say they dislike dancing actually like it?

A. 10%

B. 12%

C. 20%

D. 25%

E. 33 1/3%

12. Elmer's new car gives 50% better fuel efficiency, measured in kilometers per liter, than his old car. However, his new car uses diesel fuel, which is 20% more expensive per liter than the gasoline his old car used. By what percent will Elmer save money if he uses his new car instead of his old car for a long trip?

A. 20%

B. 26 2/3%

C. 27 7/9%

D. 33 1/3%

E. 66 2/3%

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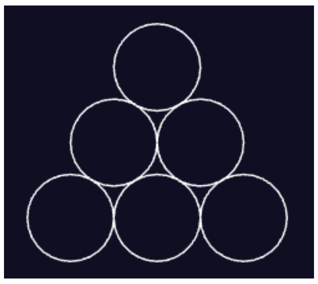
13. There are 20 students participating in an after-school program offering classes in yoga, bridge, and painting. Each student must take at least one of these three classes, but may take two or all three. There are 10 students taking yoga, 13 taking bridge, and 9 taking painting. There are 9 students taking at least two classes. How many students are taking all three classes? A. 1 B. 2 C. 3 D. 4 E. 5	14. An integer N is selected at random in the range $1 \le N \le 2020$. What is the probability that the remainder when N^16 is divided by 5 is 1? A. 1/5 B. 2/5 C. 3/5 D. 4/5 E. 1
15. Rectangle ABCD has AB = 3 and BC = 4. Point E is the foot of the perpendicular from B to diagonal AC. What is the area of △AED? A. 42/25 B. 28/15 C. 2 D. 54/25 E. 54/25	16. How many of the base-ten numerals for positive integers ≤ 2017 contain the digit 0? A. 469 B. 471 C. 475 D. 478 E. 481
17. A positive integer is monotonous if it is one-digit or digits form a strictly increasing or decreasing sequence. How many monotonous positive integers are there? A. 1024 B. 1524 C. 1533 D. 1536 E. 2048	18. In a figure with 6 disks: 3 blue, 2 red, 1 green. Rotations/reflections considered identical. How many different paintings? A. 6 B. 8 C. 9 D. 12 E. 15 

19. Equilateral △ABC extended at each side: BB' = 3AB, CC' = 3BC, AA' = 3CA. Find the ratio of areas △A'B'C' to △ABC. A. 9:1 B. 16:1 C. 25:1 D. 36:1 E. 37:1	20. 21! has over 60,000 divisors. Probability a randomly chosen divisor is odd? A. 1/21 B. 1/19 C. 1/18 D. 1/2 E. 11/21
21. △ABC: AB = 6, AC = 8, BC = 10. D midpoint of BC. Sum of inradii of △ADB and △ADC? A. 5 B. 11/4 C. 2 D. 17/6 E. 3	22. Circle radius 2, diameter AB extended to D (BD=3). ED = 5 perpendicular to AD. Segment AE intersects circle at C. Area △ABC? A. 120/37 B. 140/39 C. 145/39 D. 140/37 E. 120/31
23. N = 123456789101112…4344. Remainder when N divided by 45? A. 1 B. 4 C. 9 D. 18 E. 44	24. Vertices of equilateral triangle lie on hyperbola xy=1, and a vertex is centroid. Square of area? A. 48 B. 60 C. 108 D. 120 E. 169

25. Isabella took 7 math tests, all integers 91–100, averages integer after each test. 7th test = 95. Score on 6th test? A. 92 B. 94 C. 96 D. 98 E. 100	Answer Key: 1. B 2. C 3. E 4. D 5. D 6. B 7. C 8. C 9. D 10. E 11. D 12. A 13. C 14. D 15. E 16. A 17. B 18. D 19. E 20. B 21. D 22. D 23. C 24. C 25. E

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AMC 2017A

1. What is the value of 2(2(2(2(2(2+1)+1)+1)+1)+1)+1?

A. 70

B. 97

C. 127

D. 159

E. 729

2. Pablo buys popsicles for his friends. The store sells single popsicles for $1 each, 3-popsicle boxes for $2 each, and 5-popsicle boxes for $3. What is the greatest number of popsicles that Pablo can buy with $8?

A. 8

B. 11

C. 12

D. 13

E. 15

3. Tamara has three rows of two 6-feet by 2-feet flower beds in her garden. The beds are separated and surrounded by 1-foot-wide walkways. What is the total area of the walkways, in square feet?

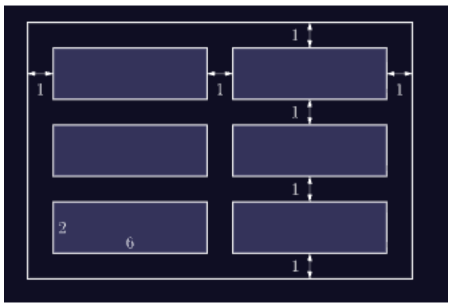
A. 72

B. 78

C. 90

D. 120

E. 150



4. Mia is helping her mom pick up 30 toys. Mia's mom puts 3 toys into the toy box every 30 seconds, but Mia takes 2 toys out immediately after those 30 seconds. How much time, in minutes, will it take to put all 30 toys into the box for the first time?

A. 13.5

B. 14

C. 14.5

D. 15

E. 15.5

5. The sum of two nonzero real numbers is 4 times their product. What is the sum of the reciprocals of the two numbers?

A. 1

B. 2

C. 4

D. 8

E. 12

6. Ms. Carroll promised that anyone who got all the multiple-choice questions right would receive an A. Which statement necessarily follows logically?

A. If Lewis did not receive an A, then he got all of the multiple-choice questions wrong.

B. If Lewis did not receive an A, then he got at least one of the multiple-choice questions wrong.

C. If Lewis got at least one question wrong, then he did not receive an A.

D. If Lewis received an A, then he got all of the multiple-choice questions right.

E. If Lewis received an A, then he got at least one of the multiple-choice questions right.

7. Jerry and Silvia want to go from the southwest corner to the northeast corner of a square field. Jerry walks east then north; Silvia walks straight northeast. How much shorter is Silvia's trip compared to Jerry's? A. 30% B. 40% C. 50% D. 60% E. 70%	8. At a gathering of 30 people, 20 know each other and 10 know no one. People who know each other hug; people who do not know each other shake hands. How many handshakes occur? A. 240 B. 245 C. 290 D. 480 E. 490
9. Minnie rides at 20 kph on flat, 30 kph downhill, and 5 kph uphill. Penny rides at 30 kph flat, 40 kph downhill, 10 kph uphill. The route is 10 km uphill, 15 km downhill, and 20 km flat. How many more minutes does it take Minnie than Penny? A. 45 B. 60 C. 65 D. 90 E. 95	10. Joy has 30 rods, one of each length from 1 cm to 30 cm. She places rods of lengths 3, 7, and 15 cm. How many remaining rods can she choose as a fourth rod to form a quadrilateral with positive area? A. 16 B. 17 C. 18 D. 19 E. 20
11. The region within 3 units of line segment AB has volume 216π. What is the length AB? A. 6 B. 12 C. 18 D. 20 E. 24	12. Let S be points (x, y) such that two of the three quantities 3, $x+2$, and $y-4$ are equal, and the third is no greater than the common value. Which describes S? A. a single point B. two intersecting lines C. three lines whose pairwise intersections are three distinct points D. a triangle E. three rays with a common endpoint

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13. Define a sequence recursively: $F_0 = 0$, $F_1 = 1$, and F_n = remainder of $(F_{n-1} + F_{n-2}) \div 3$ for $n \geq 2$. Sequence starts 0,1,1,2,0,2,… What is $F_{2017} + F_{2018} + F_{2019} + F_{2020} + F_{2021} + F_{2022} + F_{2023} + F_{2024}$?

A. 6

B. 7

C. 8

D. 9

E. 10

14. Every week Roger pays for a movie ticket and a soda out of his allowance. Last week, Roger's allowance was A dollars. The cost of his movie ticket was 20% of the difference between A and the cost of his soda, while the cost of his soda was 5% of the difference between A and the cost of his movie ticket. To the nearest whole percent, what fraction of A did Roger pay for his movie ticket and soda?

A. 9%

B. 19%

C. 22%

D. 23%

E. 25%

15. Chloe chooses a real number uniformly at random from [0, 2017]. Independently, Laurent chooses a real number uniformly at random from [0, 4034]. What is the probability that Laurent's number is greater than Chloe's number?

A. 1/2

B. 2/3

C. 3/4

D. 5/6

E. 7/8

16. There are 10 horses named Horse 1, Horse 2, …, Horse 10. Horse k runs one lap in k minutes. At time 0 all horses are together. The least time $S > 0$ when all 10 horses meet is $S = 2520$. Let $T > 0$ be the least time such that at least 5 horses are again at the starting point. What is the sum of the digits of T?

A. 2

B. 3

C. 4

D. 5

E. 6

17. Distinct points P, Q, R, S lie on the circle $x^2 + y^2 = 25$ with integer coordinates. Distances PQ and RS are irrational. What is the greatest possible value of RS/PQ?

A. 3

B. 5

C. 3/5

D. 7

E. 5/2

18. Amelia has a coin that lands heads with probability 1/3, Blaine with probability 2/5. They alternate tossing, Amelia first. Probability Amelia wins = p/q. What is $q - p$?

A. 1

B. 2

C. 3

D. 4

E. 5

19. Alice refuses to sit next to Bob or Carla. Derek refuses to sit next to Eric. How many ways can 5 people sit in a row under these conditions? A. 12 B. 16 C. 28 D. 32 E. 40	20. Let S(n) be the sum of digits of n. If S(n) = 1274, which could be S(n + 1)? A. 1 B. 3 C. 12 D. 1239 E. 1265
21. A square with side x is inscribed in a 3-4-5 right triangle at the right-angle vertex. Another square with side y is inscribed with a side on the hypotenuse. What is x/y? A. 12/13 B. 35/37 C. 1 D. 37/35 E. 13/12	22. Sides AB and AC of equilateral triangle ABC are tangent to a circle at B and C. What fraction of the area of ABC lies outside the circle? A. $4/3\ \pi - 1/3$ B. $3/2 - \pi/8$ C. 1/2 D. $3 - 2\sqrt{3}\ \pi/9$ E. $4/3 - 4\pi/27$
23. How many triangles with positive area have vertices at (i, j), $i, j \in \{1, 2, 3, 4, 5\}$? A. 2128 B. 2148 C. 2160 D. 2200 E. 2300	24. Polynomial $g(x) = x^3 + ax^2 + x + 10$ has three distinct roots, each also a root of $f(x) = x^4 + x^3 + bx^2 + 100x + c$. What is f(1)? A. −9009 B. −8008 C. −7007 D. −6006 E. −5005

25. How many integers 100–999 have a permutation divisible by 11? A. 226 B. 243 C. 270 D. 469 E. 486	Answer Key: 1. C 2. D 3. B 4. B 5. C 6. B 7. A 8. B 9. C 10. B 11. D 12. E 13. D 14. D 15. C 16. B 17. D 18. D 19. C 20. D 21. D 22. E 23. B 24. C 25. A

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AMC 2016B

1. What is the value of $(2a^{-1} + a^{-1}/2)/a$ when $a = 1/2$? A) 1 B) 2 C) 5/2 D) 10 E) 20	2. If $n\heartsuit m = n^3 / m^2$, what is $(2\heartsuit 4) / (4\heartsuit 2)$? A) 1/4 B) 1/2 C) 1 D) 2 E) 4
3. Let $x = -2016$. What is the value of $\lvert \lvert x\rvert - x \rvert - \lvert x\rvert - x$? A) −2016 B) 0 C) 2016 D) 4032 E) 6048	4. Zoey read 15 books, one at a time. The first book took her 1 day to read, the second took 2 days, the third 3 days, and so on, with each book taking 1 more day than the previous book. Zoey finished the first book on a Monday, and the second on a Wednesday. On what day of the week did she finish her 15th book? A) Sunday B) Monday C) Wednesday D) Friday E) Saturday
5. The mean age of Amanda's 4 cousins is 8, and their median age is 5. What is the sum of the ages of Amanda's youngest and oldest cousins? A) 13 B) 16 C) 19 D) 22 E) 25	6. Laura added two three-digit positive integers. All six digits in these numbers are different. Laura's sum is a three-digit number S. What is the smallest possible value for the sum of the digits of S? A) 1 B) 4 C) 5 D) 15 E) 21
7. The ratio of the measures of two acute angles is 5:4, and the complement of one of these two angles is twice as large as the complement of the other. What is the sum of the degree measures of the two angles? A) 75 B) 90 C) 135 D) 150 E) 270	8. What is the tens digit of $2015^{2016} - 2017$? A) 0 B) 1 C) 3 D) 5 E) 8

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9. Each of 21 teams in a tournament plays every other team once. Every team wins 10 games and loses 10 games, with no ties. How many sets of three teams are there in which each team beats one of the others and loses to the third? A) 105 B) 210 C) 315 D) 385 E) 420	10. Positive integers a and b satisfy $a^4b = b^4a$. What is the smallest possible value of a + b greater than 1? A) 5 B) 9 C) 10 D) 17 E) 18
11. A regular octagon has side length 1. What is the distance between opposite sides? A) $\sqrt{2}$ B) $1 + \sqrt{2}$ C) 2 D) $2\sqrt{2}$ E) 4	12. How many positive integers less than 1000 are not divisible by 2, 3, or 5? A) 133 B) 200 C) 267 D) 333 E) 400
13. Find the remainder when $2025^4 + 4$ is divided by 25. A) 0 B) 4 C) 5 D) 9 E) 21	14. How many distinct prime factors does 2025 have? A) 1 B) 2 C) 3 D) 4 E) 5
15. What is the sum of all positive integers less than 100 that are multiples of 3 or 5 but not both? A) 2318 B) 2418 C) 2518 D) 2618 E) 2718	16. How many positive divisors of 3600 are perfect squares? A) 6 B) 8 C) 9 D) 10 E) 12
17. What is the smallest positive integer n such that n! is divisible by 10^6? A) 10 B) 15 C) 25 D) 30 E) 50	18. What is the units digit of 7^{2016}? A) 1 B) 3 C) 5 D) 7 E) 9

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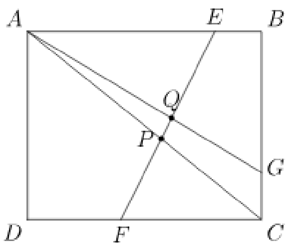
19. What is the remainder when $1^2 + 2^2 + 3^2 + \ldots + 100^2$ is divided by 8?



A) 0
B) 1
C) 2
D) 3
E) 4

20. How many three-digit numbers have the property that their digits, from left to right, form an increasing arithmetic sequence?

A) 6
B) 7
C) 8
D) 9
E) 10

21. A fair six-sided die is rolled three times. What is the probability that the three numbers shown can be the side lengths of a triangle?

A) 1/2
B) 5/9
C) 2/3
D) 7/9
E) 8/9

22. How many zeros are at the end of 100!?

A) 22
B) 23
C) 24
D) 25
E) 26

23. In regular hexagon ABCDEF, points W, X, Y, and Z are chosen on sides BC, CD, EF, and FA respectively, so lines AB, ZW, YX, and ED are parallel and equally spaced.
What is the ratio of the area of hexagon WCXYFZ to the area of hexagon ABCDEF?

A) 1/3
B) 10/27
C) 11/27
D) 4/9
E) 13/27

24. How many four-digit integers abcd, with $a \neq 0$, have the property that the three two-digit integers ab, bc, and cd form an increasing arithmetic sequence?

(Example: 4692 works since $46 < 69 < 92$.)

A) 9
B) 15
C) 16
D) 17
E) 20

25. Let
$f(x) = \Sigma$ (from $k=2$ to 10) of $[\lfloor k \cdot x \rfloor - k\lfloor x \rfloor]$,
where $\lfloor r \rfloor$ denotes the greatest integer less than or equal to r.
How many distinct values does $f(x)$ assume for $x \geq 0$?

A) 32
B) 36
C) 45
D) 46
E) Infinitely many

Answer Key:

1. D
2. B
3. D
4. B
5. D
6. B
7. C
8. A
9. C
10. D
11. A
12. C
13. E
14. D
15. B
16. C
17. A
18. C
19. D
20. B
21. C
22. D
23. C
24. D
25. A

AMC 2016A

1. What is the value of $(11! - 10!) / 9!$? A) 99 B) 100 C) 110 D) 121 E) 132	2. For what value of x does $10^x \times 100^{(2x)} = 1000^5$? A) 1 B) 2 C) 3 D) 4 E) 5
3. For every dollar Ben spent on bagels, David spent 25 cents less. Ben paid $12.50 more than David. How much did they spend in the bagel store together? A) $37.50 B) $50.00 C) $87.50 D) $90.00 E) $92.50	4. The remainder can be defined for all real numbers x and y with $y \neq 0$ by $\text{rem}(x, y) = x - y\lfloor x/y \rfloor$, where $\lfloor x/y \rfloor$ denotes the greatest integer less than or equal to x/y. What is the value of rem(3/8, −2/5)? A) −3/8 B) −1/40 C) 0 D) 3/8 E) 31/40
5. A rectangular box has integer side lengths in the ratio 1:3:4. Which of the following could be the volume of the box? A) 48 B) 56 C) 64 D) 96 E) 144	6. Ximena lists the whole numbers 1 through 30 once. Emilio copies Ximena's numbers, replacing each occurrence of the digit 2 by the digit 1. Ximena adds her numbers and Emilio adds his numbers. How much larger is Ximena's sum than Emilio's? A) 13 B) 26 C) 102 D) 103 E) 110
7. The mean, median, and mode of the 7 data values 60, 100, x, 40, 50, 200, 90 are all equal to x. What is the value of x? A) 50 B) 60 C) 75 D) 90 E) 100	8. Trickster Rabbit agrees with Foolish Fox to double Fox's money every time Fox crosses the bridge by Rabbit's house, as long as Fox pays 40 coins in toll to Rabbit after each crossing. The payment is made after the doubling. Fox crosses the bridge three times and ends up with 0 coins. How many coins did Fox have at the beginning? A) 20 B) 30 C) 35 D) 40 E) 45

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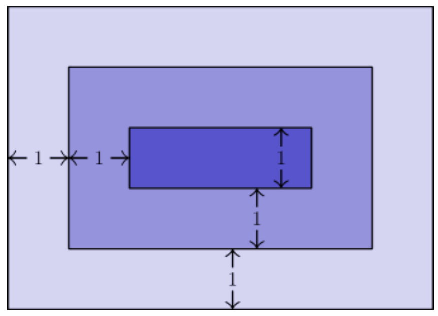
9. A triangular array of 2016 coins has 1 coin in the first row, 2 coins in the second row, 3 coins in the third row, and so on up to N coins in the Nth row.
What is the sum of the digits of N?

A) 6
B) 7
C) 8
D) 9
E) 10

10. A rug is made with three different colors. The areas of the three regions form an arithmetic progression. The inner rectangle is 1 foot wide, and each of the two outer regions are 1 foot wide on all sides.
What is the length in feet of the inner rectangle?

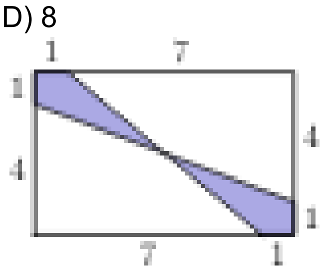


A) 1
B) 2
C) 4
D) 6
E) 8

11. Find the area of the shaded region.

A) 4 3/5
B) 5 5/4
C) 6 ½

D) 8



12. Three distinct integers are selected at random between 1 and 2016, inclusive. Which statement about the probability p that the product is odd is correct?

A) $p < 1/8$
B) $p = 1/8$
C) $1/8 < p < 1/3$
D) $p = 1/3$
E) $p > 1/3$

13. Five friends sat in a row of 5 seats (1–5). Ada left to get popcorn. Upon return: Bea moved 2 seats right, Ceci moved 1 seat left, Dee and Edie switched seats, leaving an end seat for Ada.
Which seat had Ada been sitting in?

A) 1
B) 2
C) 3
D) 4
E) 5

14. How many ways are there to write 2016 as the sum of twos and threes, ignoring order?

A) 236
B) 336
C) 337
D) 403
E) 672

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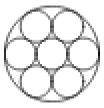
15. Seven cookies of radius 1 inch are cut from a circular dough. Neighboring cookies are tangent, and all except the center cookie are tangent to the dough edge. Leftover scrap is reshaped into another cookie of the same thickness.
What is the radius of the scrap cookie?



A) 2
B) 1.5
C) π
D) 2π
E) π

16. A triangle with vertices A(0,2), B(−3,2), C(−3,0) is reflected about the x-axis, then rotated 90° counterclockwise about the origin.
Which transformation returns the image to △ABC?

A) Rotate 90° counterclockwise
B) Rotate 90° clockwise
C) Reflect about x-axis
D) Reflect about y=x
E) Reflect about y-axis

17. Let N be a positive multiple of 5. One red ball and N green balls are arranged in a line in random order. Let P(N) be the probability that at least 3/5 of the green balls are on the same side of the red ball. Observe that P(5) = 1 and that P(N) approaches 4/5 as N grows large. What is the sum of the digits of the least value of N such that P(N) < 321/400?

A) 12
B) 14
C) 16
D) 18
E) 20

18. Each vertex of a cube is labeled with integers 1 through 8, each used once, so that the sum of the four numbers on each face is the same. Rotations of the cube are considered the same. How many different arrangements are possible?

A) 1
B) 3
C) 6
D) 12
E) 24

19. In rectangle ABCD, AB = 6 and BC = 3. Point E between B and C, and F between E and C such that BE = EF = FC. Segments AE and AF intersect BD at P and Q, respectively. The ratio BP:PQ:QD can be written as r:s:t where gcd(r,s,t) = 1. What is r+s+t?

A) 7
B) 9
C) 12
D) 15
E) 20

20. For some N, when (a+b+c+d+1)^N is expanded and like terms combined, the resulting expression contains exactly 1001 terms that include all four variables a, b, c, d, each to some positive power. What is N?

A) 9
B) 14
C) 16
D) 17
E) 19

21. Circles with centers P, Q, R and radii 1, 2, 3 lie on the same side of line l and are tangent to l at P', Q', R', with Q' between P' and R'. Circle Q is externally tangent to the other two circles. What is the area of triangle △PQR?

A) 0
B) 2√3
C) 1
D) 6−2√2
E) 3√2

22. For some positive integer n, the number 110n^3 has 110 positive integer divisors. How many positive integer divisors does 81n^4 have?

A) 110
B) 191
C) 261
D) 325
E) 425

23. A binary operation ♢ satisfies a ♢ (b ♢ c) = (a ♢ b) · c and a ♢ a = 1 for all nonzero real numbers. Solve 2016 ♢ (6 ♢ x) = 100. Express x as p/q in lowest terms. What is p+q?

A) 109
B) 201
C) 301
D) 3049
E) 33,601

24. A quadrilateral is inscribed in a circle of radius 200√2. Three sides have length 200. What is the length of the fourth side?

A) 200
B) 200√2
C) 200√3
D) 300√2
E) 500

25. How many ordered triples (x,y,z) of positive integers satisfy lcm(x,y)=72, lcm(x,z)=600, lcm(y,z)=900?

A) 15
B) 16
C) 24
D) 27
E) 64

Answer Key:

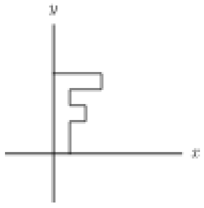
1. B
2. C
3. C
4. B
5. D
6. D
7. C
8. C
9. D
10. B
11. D
12. A
13. B
14. C
15. A
16. D
17. A
18. C
19. E
20. B
21. D
22. D
23. A
24. E
25. A

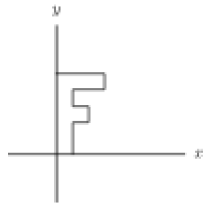
AMC 2015B

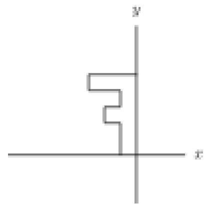
1. What is the value of $2 - (-2)^{(-2)}$? A. -2 B. 1/16 C. 7/4 D. 9/4 E. 6	2. Marie does three equally time-consuming tasks in a row without taking breaks. She begins the first task at 1:00 PM and finishes the second task at 2:40 PM. When does she finish the third task? A. 3:10 PM B. 3:30 PM C. 4:00 PM D. 4:10 PM E. 4:30 PM
3. Kaashish has written down one integer two times and another integer three times. The sum of the five numbers is 100, and one of the numbers is 28. What is the other number? A. 8 B. 11 C. 14 D. 15 E. 18	4. Four siblings ordered an extra large pizza. Alex ate 1/5, Beth 1/3, and Cyril 1/4 of the pizza. Dan got the leftovers. What is the sequence of the siblings in decreasing order of the part of pizza they consumed? A. Alex, Beth, Cyril, Dan B. Beth, Cyril, Alex, Dan C. Beth, Cyril, Dan, Alex D. Beth, Dan, Cyril, Alex E. Dan, Beth, Cyril, Alex
5. David, Hikmet, Jack, Marta, Rand, and Todd were in a 12-person race with 6 other people. Rand finished 6 places ahead of Hikmet. Marta finished 1 place behind Jack. David finished 2 places behind Hikmet. Jack finished 2 places behind Todd. Todd finished 1 place behind Rand. Marta finished in 6th place. Who finished in 8th place? A. David B. Hikmet C. Jack D. Rand E. Todd	6. Mahdi practices exactly one sport each day of the week. He runs three days a week but never on two consecutive days. On Monday he plays basketball and two days later golf. He swims and plays tennis, but he never plays tennis the day after running or swimming. Which day of the week does Mahdi swim? A. Sunday B. Tuesday C. Thursday D. Friday E. Saturday

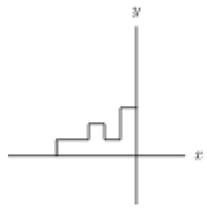
7. Consider the operation "minus the reciprocal of," defined by $a \diamond b = a - 1/b$. What is $((1 \diamond 2) \diamond 3) - (1 \diamond (2 \diamond 3))$?
A. −7/30
B. −1/6
C. 0
D. 1/6
E. 7/30

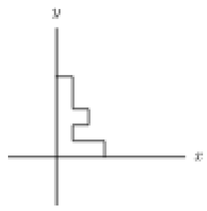
8. The letter F is rotated 90° clockwise around the origin, then reflected in the y-axis, and then rotated a half turn around the origin. What is the final image?

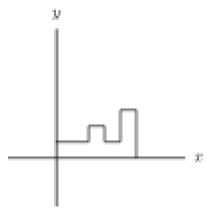


A. 

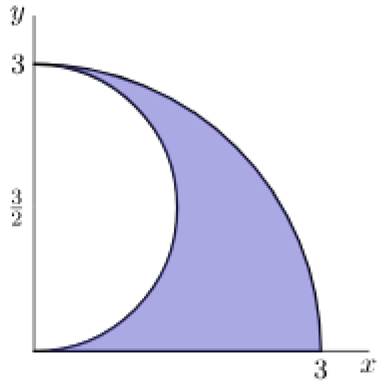
B. 

C. 

D. 

E. 

9. The shaded region below is called a shark's fin falcata, a figure studied by Leonardo da Vinci. It is bounded by the portion of the circle of radius 3 and center (0,0) that lies in the first quadrant, the portion of the circle with radius 3/2 and center (0,3/2) that lies in the first quadrant, and the line segment from (0,0) to (3,0). What is the area of the shark's fin falcata?



A. 4π/5
B. 9π/8
C. 4π/3
D. 7π/5
E. 3π/2

10. What are the sign and units digit of the product of all the odd negative integers strictly greater than −2015?
A. Negative ending with 1
B. Positive ending with 1
C. Negative ending with 5
D. Positive ending with 5
E. Negative ending with 0

11. Among the positive integers less than 100, each of whose digits is a prime number, one is selected at random. What is the probability that the selected number is prime?
A. 8/99
B. 2/5
C. 9/20
D. 1/2
E. 9/16

12. For how many integers x is the point (x,−x) inside or on the circle of radius 10 centered at (5,5)?
A. 11
B. 12
C. 13
D. 14
E. 15

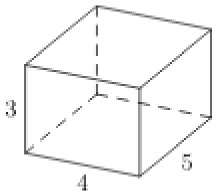
13. The line 12x + 5y = 60 forms a triangle with the coordinate axes. What is the sum of the lengths of the altitudes of this triangle?
A. 20
B. 360/17
C. 107/5
D. 43/2
E. 281/13

14. Let a, b, and c be three distinct one-digit numbers. What is the maximum value of the sum of the roots of the equation (x−a)(x−b) + (x−b)(x−c) = 0?
A. 15
B. 15.5
C. 16
D. 16.5
E. 17

15. The town of Hamlet has 3 people for each horse, 4 sheep for each cow, and 3 ducks for each person. Which of the following could not possibly be the total number of people, horses, sheep, cows, and ducks in Hamlet?
A. 41
B. 47
C. 59
D. 61
E. 66

16. Al, Bill, and Cal will each randomly be assigned a whole number from 1 to 10, inclusive, with no two of them getting the same number. What is the probability that Al's number will be a whole number multiple of Bill's and Bill's number will be a whole number multiple of Cal's?
A. 9/1000
B. 1/90
C. 1/80
D. 1/72
E. 2/121

17. The centers of the faces of the right rectangular prism shown below are joined to create an octahedron. What is the volume of this octahedron?



A. 75/12
B. 10
C. 12
D. 10/2
E. 15

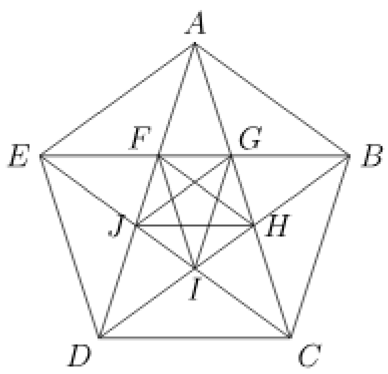
18. Johann has 64 fair coins. He flips all the coins. Any coin that lands on tails is tossed again. Coins that land on tails on the second toss are tossed a third time. What is the expected number of coins that are now heads?
A. 32
B. 40
C. 48
D. 56
E. 64

19. In △ABC, ∠C = 90° and AB = 12. Squares ABXY and ACWZ are constructed outside of the triangle. The points X, Y, Z, and W lie on a circle. What is the perimeter of the triangle?
A. $12 + 9\sqrt{3}$
B. $18 + 6\sqrt{3}$
C. $12 + 12\sqrt{2}$
D. 30
E. 32

20. Erin the ant starts at a given corner of a cube and crawls along exactly 7 edges in such a way that she visits every corner exactly once and then finds that she is unable to return along an edge to her starting point. How many paths are there meeting these conditions?
A. 6
B. 9
C. 12
D. 18
E. 24

21. Cozy the Cat and Dash the Dog are going up a staircase with a certain number of steps. Cozy jumps two steps at a time; Dash jumps five steps at a time. Dash takes 19 fewer jumps than Cozy to reach the top. Let s denote the sum of all possible numbers of steps in the staircase. What is the sum of the digits of s?
A. 9
B. 11
C. 12
D. 13
E. 15

22. In the figure shown, ABCDE is a regular pentagon and AG = 1. What is FG + JH + CD?



A. 3
B. $12 - 4\sqrt{5}$
C. $5 + 2\sqrt{5}$
D. 1 + 5
E. 11 + 11/5

23. Let n be a positive integer greater than 4 such that the decimal representation of n! ends in k zeros and the decimal representation of (2n)! ends in 3k zeros. Let s denote the sum of the four least possible values of n. What is the sum of the digits of s?
A. 7
B. 8
C. 9
D. 10
E. 11

24. Aaron the ant walks on the coordinate plane according to the following rules. He starts at the origin (0,0) facing east and walks one unit to (1,0). For n = 1,2,3,..., right after arriving at pn, if Aaron can turn 90° left and walk one unit to an unvisited point pn+1, he does that. Otherwise, he walks straight ahead. This creates a counterclockwise spiral. What is p2015?
A. (−22, −13)
B. (−13, −22)
C. (−13, 22)
D. (13, −22)
E. (22, −13)

25. A rectangular box measures a × b × c, where a, b, and c are integers and $1 \le a \le b \le c$. The volume and the surface area of the box are numerically equal. How many ordered triples (a, b, c) are possible? A. 4 B. 10 C. 12 D. 21 E. 26	Answer Key: 1. C 2. B 3. A 4. C 5. B 6. E 7. A 8. E 9. B 10. C 11. B 12. A 13. E 14. D 15. B 16. C 17. B 18. D 19. C 20. A 21. D 22. D 23. B 24. D 25. B

AI HOMEWORK

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AMC 2015A

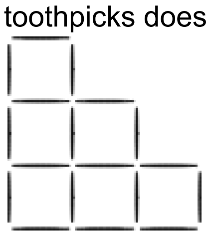
1. What is the value of $(2^0 - 1 + 5^2 - 0) - 1 \times 5$?

A. −125
B. −120
C. 5
D. 24
E. 25

2. A box contains a collection of triangular and square tiles. There are 25 tiles in the box, containing 84 edges total. How many square tiles are there in the box?

A. 3
B. 5
C. 7
D. 9
E. 11

3. Ann made a 3-step staircase using 18 toothpicks. How many toothpicks does she need to add to complete a 5-step staircase?



A. 9
B. 18
C. 20
D. 22
E. 24

4. Pablo, Sofia, and Mia got some candy eggs at a party. Pablo had three times as many eggs as Sofia, and Sofia had twice as many eggs as Mia. Pablo decides to give some of his eggs to Sofia and Mia so that all three will have the same number of eggs. What fraction of his eggs should Pablo give to Sofia?

A. 1/12
B. ⅙
C. ¼
D. ⅓
E. 1/2

5. Mr. Patrick teaches math to 15 students. The average grade for the class except Payton's was 80. After grading Payton's test, the average became 81. What was Payton's score?

A. 81
B. 85
C. 91
D. 94
E. 95

6. The sum of two positive numbers is 5 times their difference. What is the ratio of the larger number to the smaller number?

A. 5/4
B. 3/2
C. 9/5
D. 2
E. 2/5

7. How many terms are in the arithmetic sequence 13, 16, 19, …, 70, 73?

A. 20
B. 21
C. 24
D. 60
E. 61

8. Two years ago Pete was three times as old as his cousin Claire. Two years before that, Pete was four times as old as Claire. In how many years will the ratio of their ages be 2:1?

A. 2
B. 4
C. 5
D. 6
E. 8

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9. Two right circular cylinders have the same volume. The radius of the second cylinder is 10% more than the radius of the first. What is the relationship between the heights of the two cylinders?

A. The second height is 10% less than the first.
B. The first height is 10% more than the second.
C. The second height is 21% less than the first.
D. The first height is 21% more than the second.
E. The second height is 80% of the first.

10. How many rearrangements of abcd are there in which no two adjacent letters are also adjacent letters in the alphabet?

A. 0
B. 1
C. 2
D. 3
E. 4

11. The ratio of the length to the width of a rectangle is 4:3. If the rectangle has diagonal of length d, then the area may be expressed as $k \cdot d^2$ for some constant k. What is k?

A. 2/7
B. 3/7
C. 12/25
D. 16/25
E. 3/4

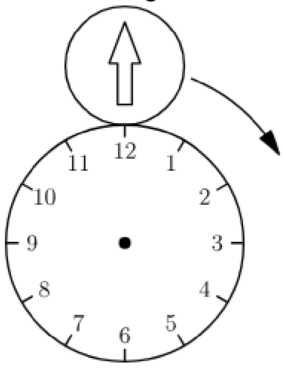
12. Points (π, a) and (π, b) are distinct points on the graph of $y^2 + x^4 = 2x^2y + 1$. What is $|a - b|$?

A. 1
B. $\pi/2$
C. 2
D. $1 + \pi$

13. Claudia has 12 coins, each of which is a 5-cent coin or a 10-cent coin. There are exactly 17 different values that can be obtained as combinations of one or more of her coins. How many 10-cent coins does Claudia have?

A. 3
B. 4
C. 5
D. 6
E. 7

14. A circular disk with radius 10 cm is externally tangent to a clock face of radius 20 cm at 12 o'clock. The disk has an arrow painted on it, initially pointing upward. Let the disk roll clockwise around the clock face. At what point on the clock face will the disk be tangent when the arrow is next pointing upward?



A. 2 o'clock
B. 3 o'clock
C. 4 o'clock
D. 6 o'clock
E. 8 o'clock

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15. Consider the set of all fractions x/y, where x and y are relatively prime positive integers. How many fractions have the property that if both numerator and denominator are increased by 1, the value of the fraction increases by 10%?

A. 0
B. 1
C. 2
D. 3
E. infinitely many

16. If $y + 4 = (x - 2)^2$, $x + 4 = (y - 2)^2$, and $x \neq y$, what is the value of $x^2 + y^2$?

A. 10
B. 15
C. 20
D. 25
E. 30

17. A line that passes through the origin intersects both the line $x = 1$ and the line $y = 1 + \sqrt{3}x$. The three lines create an equilateral triangle. What is the perimeter of the triangle?

A. $2\sqrt{6}$
B. $2 + 2\sqrt{3}$
C. 6
D. $3 + 2\sqrt{3}$
E. $6 + 3\sqrt{3}$

18. Hexadecimal (base-16) numbers are written using numeric digits 0 through 9 and letters A through F to represent 10 through 15. Among the first 1000 positive integers, there are n whose hexadecimal representation contains only numeric digits. What is the sum of the digits of n?

A. 17
B. 18
C. 19
D. 20
E. 21

19. The isosceles right triangle ABC has a right angle at C and area 12.5. The rays trisecting $\angle ACB$ intersect AB at D and E. What is the area of $\triangle CDE$?

A. $5\sqrt{2} / 3$
B. $50/3 - 75/4$
C. $15\sqrt{3} / 8$
D. $50 - 25\sqrt{3} / 2$
E. $25 / 6$

20. A rectangle with positive integer side lengths in cm has area A cm² and perimeter P cm. Which of the following numbers cannot equal A + P?

A. 100
B. 102
C. 104
D. 106
E. 108

21. Tetrahedron ABCD has AB = 5, AC = 3, BC = 4, BD = 4, AD = 3, and CD = $5\sqrt{12} / 2$. What is the volume of the tetrahedron?

A. $3\sqrt{2}$
B. $2\sqrt{5}$
C. $24 / 5$
D. $3\sqrt{3}$
E. $24\sqrt{5} / 2$

22. Eight people are sitting around a circular table, each holding a fair coin. All eight people flip their coins and those who flip heads stand while those who flip tails remain seated. What is the probability that no two adjacent people will stand?

A. $47 / 256$
B. $3 / 16$
C. $49 / 256$
D. $25 / 128$
E. $51 / 256$

23. The zeroes of the function $f(x) = x^2 - a x + 2a$ are integers. What is the sum of the possible values of a?

A. 7
B. 8
C. 16
D. 17
E. 18

24. For some positive integers p, there is a quadrilateral ABCD with positive integer side lengths, perimeter p, right angles at B and C, AB = 2, and CD = AD. How many different values of $p < 2015$ are possible?

A. 30
B. 31
C. 61
D. 62
E. 63

25. Let S be a square of side length 1. Two points are chosen independently at random on the sides of S. The probability that the straight-line distance between the points is at least $\sqrt{2} / 2$ is $a - b\pi / c$, where a, b, and c are positive integers with gcd(a, b, c) = 1. What is a + b + c?

A. 59
B. 60
C. 61
D. 62
E. 63

Answer Key:

1. C
2. D
3. D
4. B
5. E
6. B
7. B
8. B
9. D
10. C
11. C
12. C
13. C
14. C
15. B
16. B
17. D
18. E
19. D
20. B
21. C
22. A
23. C
24. B
25. A

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AMC 2014B

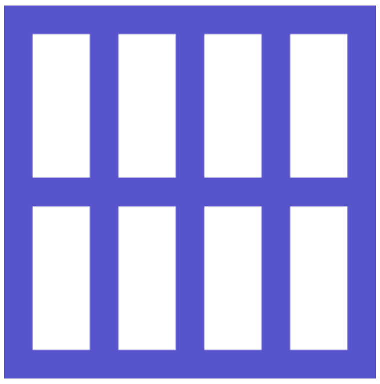
1. Leah has 13 coins, all of which are pennies and nickels. If she had one more nickel than she has now, then she would have the same number of pennies and nickels. In cents, how much are Leah's coins worth?
   A. 33
   B. 35
   C. 37
   D. 39
   E. 41

2. What is 2^3 + 2^3 ÷ 2^(-3) + 2^(-3)?
   A. 16
   B. 24
   C. 32
   D. 48
   E. 64

3. Randy drove the first third of his trip on a gravel road, the next 20 miles on pavement, and the remaining one-fifth on a dirt road. In miles, how long was Randy's trip?
   A. 30
   B. 400/11
   C. 75/2
   D. 40
   E. 300/7

4. Susie pays for 4 muffins and 3 bananas. Calvin spends twice as much paying for 2 muffins and 16 bananas. A muffin is how many times as expensive as a banana?
   A. 3/2
   B. 5/3
   C. 7/4
   D. 2
   E. 13/4

5. Doug constructs a square window using 8 equal-size panes of glass, as shown. The ratio of the height to width for each pane is 5:2, and the borders around and between the panes are 2 inches wide. In inches, what is the side length of the square window?



   A. 26
   B. 28
   C. 30
   D. 32
   E. 34

6. Orvin went to the store with just enough money to buy 30 balloons. When he arrived, he discovered that the store had a special sale on balloons: buy 1 balloon at the regular price and get a second at 1/3 off the regular price. What is the greatest number of balloons Orvin could buy?
   A. 33
   B. 34
   C. 36
   D. 38
   E. 39

7. Suppose A > B > 0 and A is x% greater than B. What is x?
   A. 100((A-B)/B)
   B. 100((A+B)/B)
   C. 100((A+B)/A)
   D. 100((A-B)/A)
   E. 100(A/B)

8. A truck travels b/6 feet every t seconds. There are 3 feet in a yard. How many yards does the truck travel in 3 minutes?
   A. b*1080/t
   B. 30b/t
   C. 30t/b
   D. 10b/t
   E. 10t/b

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9. For real numbers w and z,

(1/w + 1/z) / (1/w - 1/z) = 2014

What is (w - z) / (w + z)?
   A. -2014
   B. -1/2014
   C. 1/2014
   D. 1
   E. 2014

10. In the addition shown below, A, B, C, and D are distinct digits. How many different values are possible for D?

ABBCB + BCADA = DBDDD

   A. 2
   B. 4
   C. 7
   D. 8
   E. 9

11. For the consumer, a single discount of n% is more advantageous than any of the following discounts:
 (1) Two successive 15% discounts
 (2) Three successive 10% discounts
 (3) A 25% discount followed by a 5% discount

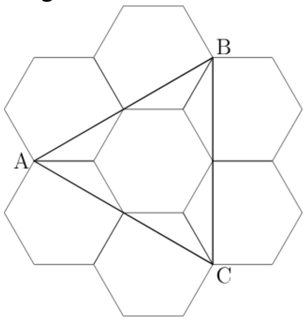
What is the smallest possible positive integer value of n?

   A. 27
   B. 28
   C. 29
   D. 31
   E. 33

12. The largest divisor of 2,014,000,000 is itself. What is its fifth-largest divisor?

   A. 125,875,000
   B. 201,400,000
   C. 251,750,000
   D. 402,800,000
   E. 503,500,000

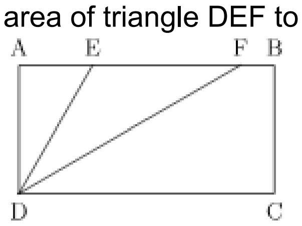
13. Six regular hexagons surround a regular hexagon of side length 1 as shown. What is the area of triangle ABC?



   A. $2\sqrt{3}$
   B. $3\sqrt{3}$
   C. $1 + \sqrt{3}/2$
   D. $2 + 2\sqrt{3}$
   E. $3 + 2\sqrt{3}$

14. Danica drove her new car on a trip for a whole number of hours, averaging 55 miles per hour. At the beginning of the trip, abc miles were displayed on the odometer, where $a \geq 1$ and $a + b + c \leq 7$. At the end of the trip, the odometer showed cba miles. What is $a^2 + b^2 + c^2$?
   A. 26
   B. 27
   C. 36
   D. 37
   E. 41

15. In rectangle ABCD, DC = 2 × CB and points E and F lie on AB so that ED and FD trisect angle ADC. What is the ratio of the area of triangle DEF to the area of rectangle ABCD?



   A. 3/6
   B. 6/8
   C. 3/16
   D. ⅓
   E. 2/4

16. Four fair six-sided dice are rolled. What is the probability that at least three of the four dice show the same value?
   A. 1/36
   B. 7/72
   C. 1/9
   D. 5/36
   E. 1/6

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17. What is the greatest power of 2 that is a factor of $10^{1002} - 4^{501}$?
    A. 2^{1002}
    B. 2^{1003}
    C. 2^{1004}
    D. 2^{1005}
    E. 2^{1006}

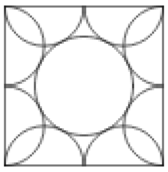
18. A list of 11 positive integers has a mean of 10, a median of 9, and a unique mode of 8. What is the largest possible value of an integer in the list?
    A. 24
    B. 30
    C. 31
    D. 33
    E. 35

19. Two concentric circles have radii 1 and 2. Two points on the outer circle are chosen independently and uniformly at random. What is the probability that the chord joining the two points intersects the inner circle?
    A. $\frac{1}{6}$
    B. $\frac{1}{4}$
    C. $2-\sqrt{2}/2$
    D. $\frac{1}{3}$
    E. $1/2$

20. For how many integers x is the number $x^4 - 51x^2 + 50$ negative?
    A. 8
    B. 10
    C. 12
    D. 14
    E. 16

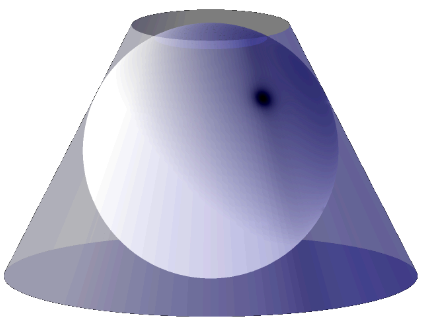
21. Trapezoid ABCD has parallel sides AB of length 33 and CD of length 21. The other two sides are of lengths 10 and 14. The angles A and B are acute. What is the length of the shorter diagonal of ABCD?
    A. $10\sqrt{6}$
    B. 25
    C. $8\sqrt{10}$
    D. $18\sqrt{2}$
    E. 26

22. Eight semicircles line the inside of a square with side length 2 as shown. What is the radius of the circle tangent to all of these semicircles?



    A. $1 + \sqrt{2}/4$
    B. $5 - \sqrt{2}/2$
    C. $3 + \frac{1}{4}$
    D. $2\sqrt{3}/5$
    E. $5\sqrt{3}/5$

23. A sphere is inscribed in a truncated right circular cone as shown. The volume of the truncated cone is twice that of the sphere. What is the ratio of the radius of the bottom base of the truncated cone to the radius of the top base of the truncated cone?



    A. $3/2$
    B. $1 + \sqrt{5}/2$
    C. $3/3$
    D. $2/2$
    E. $3 + \sqrt{5}/2$

24. The numbers 1, 2, 3, 4, 5 are to be arranged in a circle. An arrangement is bad if it is not true that for every n from 1 to 15 one can find a subset of the numbers that appear consecutively on the circle that sum to n. Arrangements that differ only by a rotation or a reflection are considered the same. How many different bad arrangements are there?
    A. 1
    B. 2
    C. 3
    D. 4
    E. 5

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25. In a small pond there are eleven lily pads in a row labeled 0 through 10. A frog is sitting on pad 1. When the frog is on pad N, 0 < N < 10, it will jump to pad N−1 with probability N/10 and to pad N+1 with probability 1 − N/10. Each jump is independent of the previous jumps.

If the frog reaches pad 0 it will be eaten by a patiently waiting snake. If the frog reaches pad 10 it will exit the pond, never to return. What is the probability that the frog will escape without being eaten by the snake?

A. 32/79
B. 161/384
C. 63/146
D. 7/16
E. 1/2
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Answer Key:

1. C
2. E
3. E
4. B
5. A
6. C
7. A
8. E
9. A
10. C
11. C
12. C
13. B
14. D
15. A
16. B
17. D
18. E
19. D
20. C
21. B
22. B
23. E
24. B
25. C
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AMC 2014A

1. What is $10 * (1/2 + 1/5 + 1/10) - 1$? A. 3 B. 8 C. 25 D. 170 E. 1703	2. Roy's cat eats 1/3 of a can of cat food every morning and 1/4 of a can every evening. Before feeding his cat on Monday morning, Roy opened a box containing 6 cans of cat food. On what day of the week did the cat finish eating all the cat food in the box? A. Tuesday B. Wednesday C. Thursday D. Friday E. Saturday
3. Bridget bakes 48 loaves of bread for her bakery. She sells half of them in the morning for $2.50 each. In the afternoon she sells two thirds of what she has left, and because they are not fresh, she charges only half price. In the late afternoon she sells the remaining loaves at a dollar each. Each loaf costs $0.75 for her to make. In dollars, what is her profit for the day? A. 24 B. 36 C. 44 D. 48 E. 52	4. Walking down Jane Street, Ralph passed four houses in a row, each painted a different color. He passed the orange house before the red house, and he passed the blue house before the yellow house. The blue house was not next to the yellow house. How many orderings of the colored houses are possible? A. 2 B. 3 C. 4 D. 5 E. 6
5. On an algebra quiz, 10% of the students scored 70 points, 35% scored 80 points, 30% scored 90 points, and the rest scored 100 points. What is the difference between the mean and median score of the students' scores on this quiz? A. 1 B. 2 C. 3 D. 4 E. 5	6. Suppose that a cows give b gallons of milk in c days. At this rate, how many gallons of milk will d cows give in e days? A. $b * d * e / (a * c)$ B. $a * c / (b * d * e)$ C. $a * b * d * e / c$ D. $a * b * c * d * e / ?$ E. $d * e / (a * b * c)$
7. Nonzero real numbers x, y, a, and b satisfy $x < a$ and $y < b$. How many of the following inequalities must be true? (I) $x + y < a + b$ (II) $x - y < a - b$ (III) $x * y < a * b$ (IV) $x / y < a / b$ A. 0 B. 1 C. 2 D. 3 E. 4	8. Which of the following numbers is a perfect square? A. $2 * 14! / 15!$ B. $2 * 15! / 16!$ C. $2 * 16! / 17!$ D. $2 * 17! / 18!$ E. $2 * 18! / 19!$
9. The two legs of a right triangle, which are altitudes, have lengths 2 and 6. How long is the third altitude of the triangle? A. 1 B. 2 C. 3 D. 4 E. 5	10. Five positive consecutive integers starting with a have average b. What is the average of 5 consecutive integers that start with b? A. $a + 3$ B. $a + 4$ C. $a + 5$ D. $a + 6$ E. $a + 7$

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11. A customer who intends to purchase an appliance has three coupons, only one of which may be used:
 Coupon 1: 10% off the listed price if the listed price is at least \$50
 Coupon 2: \$20 off the listed price if the listed price is at least \$100
 Coupon 3: 18% off the amount by which the listed price exceeds \$100

For which of the following listed prices will coupon 1 offer a greater price reduction than either coupon 2 or coupon 3?

A. \$179.99
B. \$199.95
C. \$219.95
D. \$239.95
E. \$259.95

12. A regular hexagon has side length 6. Congruent arcs with radius 3 are drawn with the center at each of the vertices, creating circular sectors. The region inside the hexagon but outside the sectors is shaded. What is the area of the shaded region?
A. $27\sqrt{3} - 9\pi$
B. $27\sqrt{3} - 6\pi$
C. $54\sqrt{3} - 18\pi$
D. $54\sqrt{3} - 12\pi$
E. $108\sqrt{3} - 9\pi$

13. Equilateral triangle ABC has side length 1, and squares ABDE, BCHI, CAFG lie outside the triangle. What is the area of hexagon DEFGHI?
A. $12 + 3\sqrt{3}$
B. 9/2
C. $3 + \sqrt{3}$
D. $6 + 3\sqrt{3}/2$
E. 6

14. The y-intercepts, P and Q, of two perpendicular lines intersecting at the point A(6, 8) have a sum of zero. What is the area of triangle APQ?
A. 45
B. 48
C. 54
D. 60
E. 72

15. David drives from his home to the airport. He drives 35 miles in the first hour, but realizes he will be 1 hour late if he continues at this speed. He increases his speed by 15 miles per hour for the rest of the way and arrives 30 minutes early. How many miles is the airport from his home?
A. 140
B. 175
C. 210
D. 245
E. 280

16. In rectangle ABCD, AB = 1, BC = 2, and points E, F, and G are midpoints of BC, CD, and AD respectively. Point H is the midpoint of GE. What is the area of the shaded region?
A. 1/12
B. 3/18
C. 2/12
D. 3/12
E. 1/6

17. Three fair six-sided dice are rolled. What is the probability that the values shown on two of the dice sum to the value shown on the remaining die?
A. ⅙
B. 13/72
C. 7/36
D. 5/24
E. 2/9

18. A square in the coordinate plane has vertices whose y-coordinates are 0, 1, 4, and 5. What is the area of the square?
A. 16
B. 17
C. 25
D. 26
E. 27

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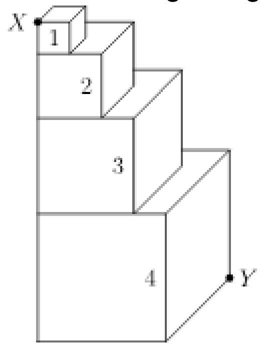
19. Four cubes with edge lengths 1, 2, 3, and 4 are stacked as shown. What is the length of the portion of XY contained in the cube with edge length 3?



A. 3/3
B. 5/5
C. ⅔
D. 2/3/3
E. 4
F. 3/2

20. The product (8)(888…8), where the second factor has k digits, is an integer whose digits have a sum of 1000. What is k?

A. 901
B. 911
C. 919
D. 991
E. 999

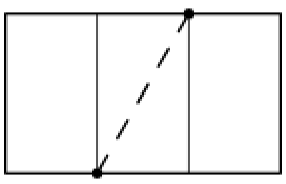
21. Positive integers a and b are such that the graphs of $y = ax + 5$ and $y = 3x + b$ intersect the x-axis at the same point. What is the sum of all possible x-coordinates of these points of intersection?

A. −20
B. −18
C. −15
D. −12
E. −8

22. In the rectangle ABCD, AB = 20 and BC = 10. Let E be a point on CD such that angle CBE = 15 degrees. What is AE?

A. 20/3
B. 10/3
C. 18
D. 11/3
E. 20

23. A rectangular piece of paper whose length is 3 times the width has area A. The paper is divided into three equal sections along the opposite lengths, and a dotted line is drawn from the first divider to the second divider on the opposite side. The paper is folded flat along this dotted line to create a new shape with area B. What is the ratio B/A?



A. ½
B. ⅗
C. ⅔
D. ¾
E. 4/5

24. A sequence of natural numbers is constructed by listing the first 4, then skipping one, listing the next 5, skipping 2, listing 6, skipping 3, and so on. On the nth iteration, listing n+3 and skipping n. The sequence begins 1, 2, 3, 4, 6, 7, 8, 9, 10, 13. What is the 500,000th number in the sequence?

A. 996,506
B. 996,507
C. 996,508
D. 996,509
E. 996,510

25. The number 5867 is between 2^2013 and 2^2014. How many pairs of integers (m, n) are there such that 1 ≤ m ≤ 2012 and 5^n < 2^m < 2^(m+2) < 5^(n+1)? A. 278 B. 279 C. 280 D. 281 E. 282	Answer Key: 1. C 2. C 3. E 4. B 5. C 6. A 7. B 8. D 9. C 10. B 11. C 12. C 13. C 14. D 15. B 16. E 17. D 18. B 19. A 20. D 21. E 22. E 23. C 24. A 25. B

AMC 2013B

1. What is (1+3+5)/(2+4+6) - (2+4+6)/(1+3+5)? A. -1 B. 5/36 C. 7/12 D. 49/20 E. 43/3	2. Mr. Green measures his rectangular garden by walking two of the sides and finds that it is 15 steps by 20 steps. Each of Mr. Green's steps is 2 feet long. Mr. Green expects a half a pound of potatoes per square foot from his garden. How many pounds of potatoes does Mr. Green expect from his garden? A. 600 B. 800 C. 1000 D. 1200 E. 1400
3. On a particular January day, the high temperature in Lincoln, Nebraska, was 16 degrees higher than the low temperature, and the average of the high and low temperatures was 3 degrees. What was the low temperature in Lincoln that day (in degrees)? A. -13 B. -8 C. -5 D. -3 E. 11	4. When counting from 3 to 201, 53 is the 51st number counted. When counting backwards from 201 to 3, 53 is the nth number counted. What is n? A. 146 B. 147 C. 148 D. 149 E. 150
5. Positive integers a and b are each less than 6. What is the smallest possible value for 2 * a - a * b? A. -20 B. -15 C. -10 D. 0 E. 2	6. The average age of 33 fifth-graders is 11. The average age of 55 of their parents is 33. What is the average age of all of these parents and fifth-graders? A. 22 B. 23.25 C. 24.75 D. 26.25 E. 28
7. Six points are equally spaced around a circle of radius 1. Three of these points are the vertices of a triangle that is neither equilateral nor isosceles. What is the area of this triangle? A. sqrt(3)/3 B. sqrt(3)/2 C. 1 D. sqrt(2)/2 E. sqrt(2)	8. Ray's car averages 40 miles per gallon of gasoline, and Tom's car averages 10 miles per gallon of gasoline. Ray and Tom each drive the same number of miles. What is the cars' combined rate of miles per gallon of gasoline? A. 10 B. 16 C. 25 D. 30 E. 40
9. Three positive integers are each greater than 1, have a product of 27000, and are pairwise relatively prime. What is their sum? A. 100 B. 137 C. 156 D. 160 E. 165	10. A basketball team's players were successful on 50% of their two-point shots and 40% of their three-point shots, which resulted in 54 points. They attempted 50% more two-point shots than three-point shots. How many three-point shots did they attempt? A. 10 B. 15 C. 20 D. 25 E. 30

AI HOMEWORK

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11. Real numbers x and y satisfy the equation $x^2 + y^2 = 10x - 6y - 34$. What is $x + y$?

A. 1
B. 2
C. 3
D. 6
1. E. 8

12. Let S be the set of sides and diagonals of a regular pentagon. A pair of elements of S are selected at random without replacement. What is the probability that the two chosen segments have the same length?

A. 2/5
B. 4/9
C. 1/2
D. 5/9
E. 4/5

13. Jo and Blair take turns counting from 1 to one more than the last number said by the other person. Jo starts by saying "1", so Blair follows by saying "1, 2". Jo then says "1, 2, 3", and so on. What is the 53rd number said?

A. 2
B. 3
C. 5
D. 6
E. 8

14. Define $a \clubsuit b = a^2 b - a b^2$. Which of the following describes the set of points (x, y) for which $x \clubsuit y = y \clubsuit x$?

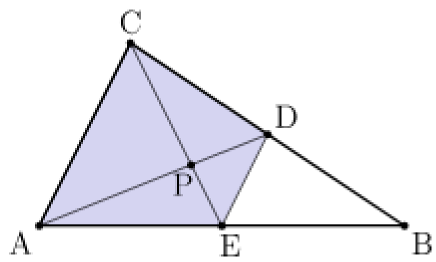
A. A finite set of points
B. One line
C. Two parallel lines
D. Two intersecting lines
E. Three lines

15. A wire is cut into two pieces, one of length a and the other of length b. The piece of length a is bent to form an equilateral triangle, and the piece of length b is bent to form a regular hexagon. The triangle and the hexagon have equal area. What is a/b?

A. 1
B. $\sqrt{6}/2$
C. $\sqrt{3}$
D. 2
E. $3\sqrt{2}/2$

16. In triangle ABC, medians AD and CE intersect at P, PE = 1.5, PD = 2, and DE = 2.5. What is the area of AEDC?

A. 13
B. 13.5
C. 14
D. 14.5
E. 15



17. Alex has 75 red tokens and 75 blue tokens. There is a booth where Alex can give two red tokens and receive in return a silver token and a blue token, and another booth where Alex can give three blue tokens and receive in return a silver token and a red token. Alex continues to exchange tokens until no more exchanges are possible. How many silver tokens will Alex have at the end?

A. 62
B. 82
C. 83
D. 102
E. 103

18. The number 2013 has the property that its units digit is the sum of its other digits, that is 2+0+1=3. How many integers less than 2013 but greater than 1000 have this property?

A. 33
B. 34
C. 45
D. 46
E. 58

19. The real numbers c, b, a form an arithmetic sequence with $a \ge b \ge c \ge 0$. The quadratic $a x^2 + b x + c$ has exactly one root. What is this root?

A. $-7 - 4\sqrt{3}$
B. $-2 - \sqrt{3}$
C. -1
D. $-2 + \sqrt{3}$
E. $-7 + 4\sqrt{3}$

20. The number 2013 is expressed in the form $2013 = (a_1!\, a_2! \ldots a_m!) / (b_1!\, b_2! \ldots b_n!)$, where $a_1 \ge a_2 \ge \ldots \ge a_m$ and $b_1 \ge b_2 \ge \ldots \ge b_n$ are positive integers and $a_1 + b_1$ is as small as possible. What is $|a_1 - b_1|$?

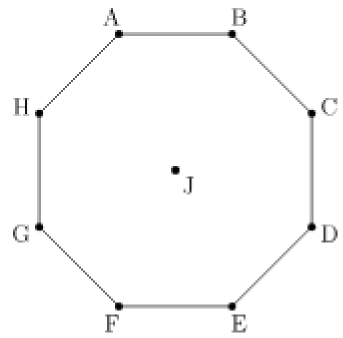
A. 1
B. 2
C. 3
D. 4
E. 5

21. Two non-decreasing sequences of nonnegative integers have different first terms. Each sequence has the property that each term beginning with the third is the sum of the previous two terms, and the seventh term of each sequence is N. What is the smallest possible value of N?

A. 55
B. 89
C. 104
D. 144
E. 273

22. The regular octagon ABCDEFGH has its center at J. Each of the vertices and the center are to be associated with one of the digits 1 through 9, with each digit used once, in such a way that the sums of the numbers on the lines AJE, BJF, CJG, and DJH are all equal. In how many ways can this be done?

A. 384
B. 576
C. 1152
D. 1680
E. 3456



23. In triangle ABC, AB = 13, BC = 14, and CA = 15. Distinct points D, E, and F lie on segments BC, CA, and DE, respectively, such that AD is perpendicular to BC, DE is perpendicular to AC, and AF is perpendicular to BF. The length of segment DF can be written as m/n, where m and n are relatively prime positive integers. What is m + n?
A. 18
B. 21
C. 24
D. 27
E. 30

24. A positive integer n is "nice" if there is a positive integer m with exactly four positive divisors (including 1 and m) such that the sum of the four divisors is equal to n. How many numbers in the set {2010, 2011, 2012, ..., 2019} are nice?

A. 1
B. 2
C. 3
D. 4
E. 5

25. Bernardo chooses a three-digit positive integer N and writes both its base-5 and base-6 representations on a blackboard. Later LeRoy sees the two numbers Bernardo has written. Treating the two numbers as base-10 integers, he adds them to obtain an integer S. For example, if N = 749, Bernardo writes the numbers 10,444 and 3,245, and LeRoy obtains the sum S = 13,689. For how many choices of N are the two rightmost digits of S, in order, the same as those of 2N?

A. 5
B. 10
C. 15
D. 20
E. 25

Answer Key:
1. C
2. A
3. C
4. D
5. B
6. C
7. B
8. B
9. D
10. C
11. B
12. B
13. E
14. E
15. B
16. B
17. E
18. D
19. D
20. B
21. C
22. C
23. B
24. A
25. E

AI HOMEWORK

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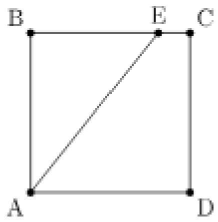
AMC 2013A

1. A taxi ride costs \$1.50 plus \$0.25 per mile traveled. How much does a 5-mile taxi ride cost?
A. \$2.25
B. \$2.50
C. \$2.75
D. \$3.00
E. \$3.75

2. Alice needs 2 1/2 cups of sugar, but her cup holds only 1/4 cup. How many times must she fill the cup?
A. 8
B. 10
C. 12
D. 16
E. 20

3. Square ABCD has side length 10. Point E is on BC, and the area of triangle ABE is 40. What is BE?
A. 4
B. 5
C. 6
D. 7
E. 8



4. A softball team played ten games, scoring 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 runs. They lost by one run in five games. In each other game, they scored twice as many runs as their opponent. How many total runs did their opponents score?
A. 35
B. 40
C. 45
D. 50
E. 55

5. Tom paid \$105, Dorothy \$125, and Sammy \$175 for a shared trip. They split costs evenly. Tom gives Sammy t dollars, and Dorothy gives Sammy d dollars. Find $t - d$.
A. 15
B. 20
C. 25
D. 30
E. 35

6. Joey and his brothers are ages 3, 5, 7, 9, 11, and 13. Two whose ages sum to 16 went to the movies, two younger than 10 played baseball, and Joey stayed home with the 5-year-old. How old is Joey?
A. 3
B. 7
C. 9
D. 11
E. 13

7. A student must choose four courses from English, Algebra, Geometry, History, Art, and Latin. English is required, and at least one math course must be chosen. How many ways?
A. 6
B. 8
C. 9
D. 12
E. 16

8. Evaluate $(2^{2014} + 2^{2012}) / (2^{2014} - 2^{2012})$.
A. -1
B. 1
C. 5/3
D. 2013
E. 2^{4024}

9. Shenille took only 2-point and 3-point shots. She made 30% of 2-pointers and 20% of 3-pointers, with 30 shots total. How many points did she score?
A. 12
B. 18
C. 24
D. 30
E. 36

10. A bouquet has pink roses, red roses, pink carnations, and red carnations. 1/3 of pink flowers are roses, 3/4 of red flowers are carnations, and 6/10 of all flowers are pink. What percent are carnations?
A. 15%
B. 30%
C. 40%
D. 60%
E. 70%

AI HOMEWORK

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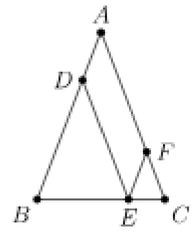
11. A student council must select a two-person welcoming committee and a three-person planning committee from its members. There are exactly 10 ways to pick the welcoming committee. How many ways can the planning committee be chosen? A. 10 B. 12 C. 15 D. 18 E. 25	12. In triangle ABC, AB = AC = 28 and BC = 20. Points D, E, and F are on AB, BC, and AC respectively, with DE // AC and EF // AB. What is the perimeter of parallelogram ADEF? A. 48 B. 52 C. 56 D. 60 E. 72 
13. How many three-digit numbers are not divisible by 5, have first and last digits equal, and digits sum to less than 20? A. 52 B. 60 C. 66 D. 68 E. 70	14. A cube of side 1 is removed from each corner of a cube of side 3. How many edges does the remaining solid have? A. 36 B. 60 C. 72 D. 84 E. 108
15. Triangle sides are 10 and 15. The altitude to the third side equals the average of the altitudes to the given sides. Find the third side. A. 6 B. 8 C. 9 D. 12 E. 18	16. Triangle with vertices (6,5), (8,−3), (9,1) is reflected over x = 8. What is the area of the union of both triangles? A. 9 B. 28/3 C. 10 D. 31/3 E. 32/3
17. Alice visits every 3 days, Beatrix every 4, and Claire every 5. All visited yesterday. In the next 365 days, how many days will exactly two visit? A. 48 B. 54 C. 60 D. 66 E. 72	18. Quadrilateral ABCD has A(0,0), B(1,2), C(3,3), D(4,0). A line through A cuts CD at (p/q, r/s) and halves area. Find p + q + r + s. A. 54 B. 58 C. 62 D. 70 E. 75
19. For how many positive integers b does 2013 base b end with digit 3? A. 6 B. 9 C. 13 D. 16 E. 18	20. A unit square is rotated 45° about its center. Find the area of the region swept by its interior. A. $1 - \sqrt{2}/2 + \pi/4$ B. $1/2 + \pi/4$ C. $2 - \sqrt{2} + \pi/4$ D. $\sqrt{2} + \pi/4$ E. $1 + \sqrt{2}/4 + \pi/8$
21. 12 pirates divide coins. Pirate k takes k/12 of remaining. Find smallest starting number so all get whole coins. How many coins does the 12th pirate get? A. 720 B. 1296 C. 1728 D. 1925 E. 3850	22. Six spheres radius 1 are at hexagon vertices (side 2) and tangent internally to a larger sphere. A smaller eighth sphere is tangent to all six and the large one. Find its radius. A. 1/2 B. 3/2 C. 5/3 D. $\sqrt{3}$ E. 2

23. In triangle ABC, AB = 86, AC = 97. A circle centered at A with radius AB meets BC at B and X. BX and CX are integers. Find BC. A. 11 B. 28 C. 33 D. 61 E. 72	24. Central HS (A,B,C) vs Northern HS (X,Y,Z). Each pair plays twice. Six rounds, 3 games per round. How many different schedules? A. 540 B. 600 C. 720 D. 810 E. 900
25. All 20 diagonals are drawn in a regular octagon. How many distinct interior intersection points occur? A. 49 B. 65 C. 70 D. 96 E. 128	Answer Key: 1. C 2. B 3. E 4. C 5. B 6. D 7. C 8. C 9. B 10. E 11. A 12. C 13. B 14. D 15. D 16. E 17. B 18. B 19. C 20. C 21. D 22. B 23. D 24. E 25. A

AI HOMEWORK

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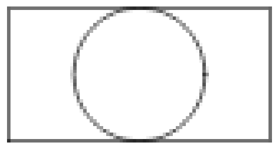
AMC 2012B

1. Each third-grade classroom has 18 students and 2 pet rabbits. How many more students than rabbits are there in all 4 classrooms?
A. 48
B. 56
C. 64
D. 72
E. 80

2. A circle of radius 5 is inscribed in a rectangle. The ratio of the rectangle's length to width is 2:1. What is the area of the rectangle?
A. 50
B. 100
C. 125
D. 150
E. 200



3. The point (1000, 2012) is reflected across y = 2000. What are the coordinates of the reflected point?
A. (998, 2012)
B. (1000, 1988)
C. (1000, 2024)
D. (1000, 4012)
E. (1012, 2012)

4. Ringo has marbles giving remainder 4 mod 6. Paul's marbles give remainder 3 mod 6. Combined, how many marbles are left over when grouped in sixes?
A. 1
B. 2
C. 3
D. 4
E. 5

5. Anna's dinner has 10% tax and 15% tip on pre-tax price. Total = $27.50. What is the meal's base cost?
A. $18
B. $20
C. $21
D. $22
E. $24

6. Xiaoli rounded x up and y down by the same amount, then computed x−y. Which is necessarily true?

A. Estimate > x−y
B. Estimate < x−y
C. Estimate = x−y
D. Estimate = y−x
E. Estimate = 0

7. Chipmunk hides 3 acorns per hole, squirrel hides 4 per hole. They hide same total, but squirrel needs 4 fewer holes. How many acorns did the chipmunk hide?
A. 30
B. 36
C. 42
D. 48
E. 54

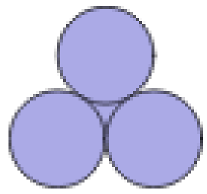
8. Find the sum of all integer solutions to $1 < (x-2)^2 < 25$.
A. 10
B. 12
C. 15
D. 19
E. 25

9. Two integers sum to 26. Adding two more gives 41. Adding two more gives 57. What is the minimum number of odd integers among all six?
A. 1
B. 2
C. 3
D. 4
E. 5

10. How many ordered pairs (M,N) of positive integers satisfy M/6 = 6/N?
A. 6
B. 7
C. 8
D. 9
E. 10

11. A chef picks a dessert each day for a week (Sun–Sat): cake, pie, ice cream, or pudding. Same dessert not two days in a row. Cake required Friday. How many possible menus?
A. 729
B. 972
C. 1024
D. 2187
E. 2304

12. Points A, B, C: B east of A, C north of B. AC = $10\sqrt{2}$, $\angle BAC = 45°$. D is 20 m north of C. AD is between what two integers?
A. 30–31
B. 31–32
C. 32–33
D. 33–34
E. 34–35

13. Clea walks down a non-moving escalator in 60s and a moving one in 24s. How long to ride down standing still? A. 36 B. 40 C. 42 D. 48 E. 52	14. Two equilateral triangles share opposite sides of a square (side $2\sqrt{3}$). Their intersection is a rhombus. Find area of rhombus. A. $\sqrt{3}/2$ B. $\sqrt{3}$ C. $2\sqrt{3}-1$ D. $8\sqrt{3}-12$ E. $4\sqrt{3}/3$
15. In a 6-team round robin, each team plays once per other. Each win = 1. What's the max number of teams tied for most wins? A. 2 B. 3 C. 4 D. 5 E. 6	16. Three circles radius 2 mutually tangent. What is total area of the circles plus the bounded region? A. $10\pi + 4\sqrt{3}$ B. $13\pi - \sqrt{3}$ C. $12\pi + \sqrt{3}$ D. $10\pi + 9$ E. 13π 
17. Disk radius 12 cut into sectors 120° and 240°. Each forms a cone. Ratio of smaller cone volume to larger? A. 1/8 B. 1/4 C. 1/10 D. 5/6 E. 10/5	18. Disease rate 1/500. Test perfect sensitivity, 2% false positive. Find P(has disease | test positive). A. 1/98 B. 1/9 C. 1/11 D. 49/99 E. 98/99
19. In rectangle ABCD, AB = 6, AD = 30, G midpoint of AD, E is 2 units beyond B on AB, F = ED∩BC. Find area of quadrilateral BFDG. A. 133/2 B. 67 C. 135/2 D. 68 E. 137/2	20. Bernardo doubles, Silvia adds 50, stop <1000. Smallest N so Bernardo wins. Sum of digits of N? A. 7 B. 8 C. 9 D. 10 E. 11
21. Four points with pairwise distances a,a,a,a,2a,b. Find ratio b:a. A. $\sqrt{3}$ B. 2 C. $\sqrt{5}$ D. 3 E. π	22. ($a_1, a_2, \ldots, a_{10}$) list of 1–10 s.t. each term ±1 neighbor appears earlier. How many such lists? A. 120 B. 512 C. 1024 D. 181,440 E. 362,880
23. A tetrahedron cut from cube, remaining solid height? A. $\sqrt{3}$ B. $2\sqrt{3}$ C. 1 D. $2\sqrt{3}/3$ E. 2	24. Three girls, four songs. No song liked by all; each pair shares one song liked by both. How many arrangements possible? A. 108 B. 132 C. 671 D. 846 E. 1105

AI HOMEWORK

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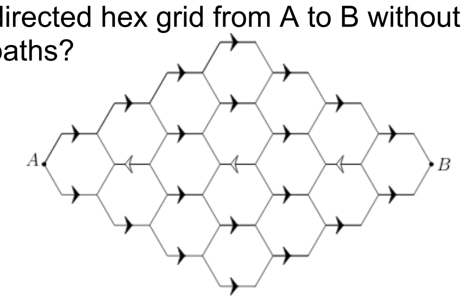
25. Bug travels on directed hex grid from A to B without reusing edges. How many paths?

A. 2112
B. 2304
C. 2368
D. 2384
E. 2400



Answer Key:

1. C
2. E
3. B
4. A
5. D
6. A
7. D
8. B
9. A
10. D
11. A
12. B
13. B
14. D
15. D
16. A
17. C
18. C
19. C
20. A
21. A
22. B
23. D
24. B
25. E

AI HOMEWORK

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AMC 2012A

1. Cagney can frost a cupcake every 20 seconds and Lacey can frost a cupcake every 30 seconds. Working together, how many cupcakes can they frost in 5 minutes? A. 10 B. 15 C. 20 D. 25 E. 30	2. A square with side length 8 is cut in half, creating two congruent rectangles. What are the dimensions of one of these rectangles? A. 2 by 4 B. 2 by 6 C. 2 by 8 D. 4 by 4 E. 4 by 8
3. A bug crawls along a number line, starting at −2. It crawls to −6, then turns around and crawls to 5. How many units does the bug crawl altogether? A. 9 B. 11 C. 13 D. 14 E. 15	4. Let $\angle ABC = 24°$ and $\angle ABD = 20°$. What is the smallest possible degree measure for $\angle CBD$? A. 0 B. 2 C. 4 D. 6 E. 12
5. Last year 100 adult cats, half of whom were female, were brought into the Smallville Animal Shelter. Half of the adult female cats were accompanied by a litter of kittens. The average number of kittens per litter was 4. What was the total number of cats and kittens received by the shelter last year? A. 150 B. 200 C. 250 D. 300 E. 400	5. The product of two positive numbers is 9. The reciprocal of one of these numbers is 4 times the reciprocal of the other number. What is the sum of the two numbers? A. 10/3 B. 20/3 C. 7 D. 15/2 E. 8
7. In a bag of marbles, 3/5 of the marbles are blue and the rest are red. If the number of red marbles is doubled and the number of blue marbles stays the same, what fraction of the marbles will be red? A. 2/5 B. 3/7 C. 4/7 D. 3/5 E. 4/5	8. The sums of three whole numbers taken in pairs are 12, 17, and 19. What is the middle number? A. 4 B. 5 C. 6 D. 7 E. 8
9. A pair of six-sided dice are labeled so that one die has only even numbers (two each of 2, 4, and 6), and the other die has only odd numbers (two each of 1, 3, and 5). The pair of dice is rolled. What is the probability that the sum of the numbers on the tops of the two dice is 7? A. 1/6 B. 1/5 C. 1/4 D. 1/3 E. 1/2	10. Mary divides a circle into 12 sectors. The central angles of these sectors, measured in degrees, are all integers and they form an arithmetic sequence. What is the degree measure of the smallest possible sector angle? A. 5 B. 6 C. 8 D. 10 E. 12

AI HOMEWORK

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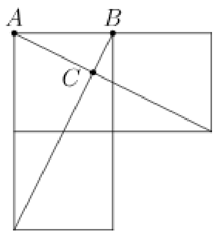
11. Externally tangent circles with centers at points A and B have radii of lengths 5 and 3, respectively. A line externally tangent to both circles intersects ray AB at point C. What is BC?
A. 4
B. 4.8
C. 10.2
D. 12
E. 14.4

12. A year is a leap year if and only if the year number is divisible by 400 (such as 2000) or is divisible by 4 but not 100 (such as 2012). The 200th anniversary of the birth of novelist Charles Dickens was celebrated on February 7, 2012, a Tuesday. On what day of the week was Dickens born?
A. Friday
B. Saturday
C. Sunday
D. Monday
E. Tuesday

13. An iterative average of the numbers 1, 2, 3, 4, and 5 is computed as follows: find the mean of the first two numbers, then find the mean of that with the third, then the mean of that with the fourth, and finally the mean of that with the fifth. What is the difference between the largest and smallest possible values that can be obtained using this procedure?
A. 31/16
B. 2
C. 17/8
D. 3
E. 65/16

14. Chubby makes nonstandard checkerboards that have 31 squares on each side. The checkerboards have a black square in every corner and alternate red and black squares along every row and column. How many black squares are there on such a checkerboard?
A. 480
B. 481
C. 482
D. 483
E. 484

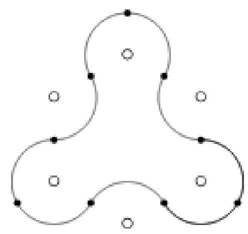
15. Three unit squares and two line segments connecting two pairs of vertices are shown. What is the area of triangle ABC?
A. 1/6
B. 1/5
C. 2/9
D. 1/3
E. 2/4



16. Three runners start running simultaneously from the same point on a 500-meter circular track. They each run clockwise around the course maintaining constant speeds of 4.4, 4.8, and 5.0 meters per second. The runners stop once they are all together again somewhere on the circular course. How many seconds do the runners run?
A. 1000
B. 1250
C. 2500
D. 5000
E. 10000

17. Let a and b be relatively prime positive integers with $a > b > 0$ and $(a^3 - b^3)/(a - b)^3 = 73/3$. What is $a - b$?
A. 1
B. 2
C. 3
D. 4
E. 5

18. The closed curve in the figure is made up of 9 congruent circular arcs each of length $(2\pi/3)$, where each of the centers of the corresponding circles is among the vertices of a regular hexagon of side 2. What is the area enclosed by the curve?
A. $2\pi + 6$
B. $2\pi + 4\sqrt{3}$
C. $3\pi + 4$
D. $2\pi + 3\sqrt{3} + 2$
E. $\pi + 6\sqrt{3}$



AI HOMEWORK

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19. Paula the painter and her two helpers each paint at constant, but different, rates. They always start at 8:00 AM, and all three always take the same amount of time to eat lunch. On Monday the three of them painted 50% of a house, quitting at 4:00 PM. On Tuesday, when Paula wasn't there, the two helpers painted only 24% of the house and quit at 2:12 PM. On Wednesday Paula worked by herself and finished the house by working until 7:12 PM. How long, in minutes, was each day's lunch break? A. 30 B. 36 C. 42 D. 48 E. 60	20. A 3×3 square is partitioned into 9 unit squares. Each unit square is painted either white or black, each color equally likely and independent. The square is rotated 90° clockwise, and every white square that moves into a position formerly occupied by a black square is painted black. What is the probability the grid is now entirely black? A. 49/512 B. 7/64 C. 121/1024 D. 81/512 E. 9/32
21. Let points A = (0,0,0), B = (1,0,0), C = (0,2,0), D = (0,0,3). Points E, F, G, and H are midpoints of BD, AB, AC, and DC, respectively. What is the area of EFGH? A. 2 B. $2\sqrt{3}$ C. $3\sqrt{5}/4$ D. $\sqrt{3}$ E. $2\sqrt{7}/3$	22. The sum of the first m positive odd integers is 212 more than the sum of the first n positive even integers. What is the sum of all possible values of n? A. 255 B. 256 C. 257 D. 258 E. 259
23. Six people each have the same number of internet friends within the group. Some, but not all, are friends with each other, and none have friends outside the group. In how many different ways can this happen? A. 60 B. 170 C. 290 D. 320 E. 660	24. Let a, b, and c be positive integers with $a \geq b \geq c$ satisfying $a^2 - b^2 - c^2 + ab = 2011$ and $a^2 + 3b^2 + 3c^2 - 3ab - 2ac - 2bc = -1997$. What is a? A. 249 B. 250 C. 251 D. 252 E. 253

25. Real numbers x, y, and z are chosen independently and uniformly from [0, n], where n is a positive integer. The probability that no two of x, y, and z are within 1 unit of each other is greater than 1/2. What is the smallest possible value of n? A. 7 B. 8 C. 9 D. 10 E. 11	Answer Key: 1. D 2. E 3. E 4. C 5. B 6. D 7. C 8. D 9. D 10. C 11. D 12. A 13. C 14. B 15. B 16. C 17. C 18. E 19. D 20. A 21. C 22. A 23. B 24. E 25. D

AI HOMEWORK

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AMC 2011B

1. What is (2+4+6)/(1+3+5) − (1+3+5)/(2+4+6)?
A. −1
B. 5/36
C. 7/12
D. 147/60
E. 43/3

2. Josanna’s test scores are 90, 80, 70, 60, and 85. She wants to raise her test average by at least 3 points with her next test. What is the minimum score she needs?
A. 80
B. 82
C. 85
D. 90
E. 95

3. A rectangular tile is reported as 2 inches by 3 inches, meaning each measurement can vary by ±0.5 inches. What is the minimum area of the rectangle?
A. 3.75
B. 4.5
C. 5
D. 6
E. 8.75

4. LeRoy paid A dollars and Bernardo paid B dollars (A < B) for a trip. How many dollars must LeRoy give Bernardo so that they share the costs equally?
A. (A + B)/2
B. (A − B)/2
C. (B − A)/2
D. (B − A)
E. (A + B)

5. Ron reversed the digits of a two-digit number a when multiplying by b. His wrong product was 161. What is the correct product?
A. 116
B. 161
C. 204
D. 214
E. 224

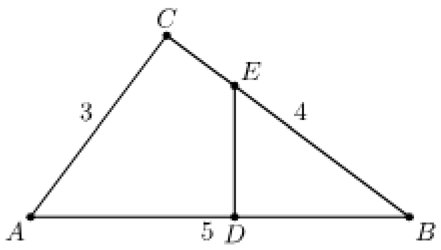
6. Casper ate 1/3 of his candies and gave 2 to his brother. The next day he ate 1/3 of the remainder and gave 4 to his sister. On the third day he ate his final 8 candies. How many candies did he start with?

A. 30
B. 39
C. 48
D. 57
E. 66

7. The sum of two angles of a triangle is (5/6) of a right angle, and one is 30° larger than the other. What is the largest angle?
A. 69°
B. 72°
C. 90°
D. 102°
E. 108°

8. If it is at least 80°F and sunny, then the beach will be crowded. On June 10, the beach was not crowded. What can be concluded?
A. Cooler than 80°F and not sunny
B. Cooler than 80°F or not sunny
C. If ≥80°F, then sunny
D. If <80°F, then sunny
E. If <80°F, then not sunny

9. The area of △EBD is one-third that of △ABC. Segment DE ⊥ AB. What is BD?
A. 4/3
B. 5/4
C. 9/4
D. 4√3/3
E. 5/2



10. For the set {1, 10, 10^2, 10^3, …, 10^{10}}, the ratio of the largest element to the sum of the others is closest to which integer?
A. 1
B. 9
C. 10
D. 11
E. 101

11. There are 52 people in a room. What is the largest n such that “At least n people have birthdays in the same month” is always true?
A. 2
B. 3
C. 4
D. 5
E. 12

12. Keiko walks around a track of width 6 m with straight sides and semicircular ends. It takes her 36 s longer around the outer edge than the inner edge. What is her speed (m/s)?
A. π/3
B. 2π/3
C. π
D. 4π/3
E. 5π/3

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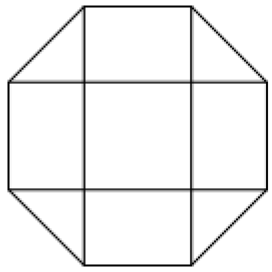
13. Two real numbers are chosen independently from $[-20, 10]$. What is the probability that their product is greater than 0?
A. 1/9
B. 1/3
C. 4/9
D. 5/9
E. 2/3

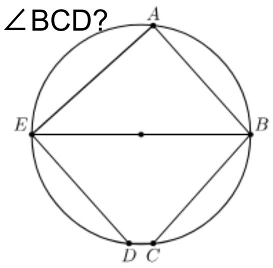
14. A rectangular parking lot has diagonal = 25 m and area = 168 m^2. What is its perimeter?
A. 52
B. 58
C. 62
D. 68
E. 70

15. Let “@” mean averaged with: $a @ b = (a + b)/2$. Which distributive laws hold for all x, y, z?
I. $x @ (y + z) = (x @ y) + (x @ z)$
II. $x + (y @ z) = (x + y) @ (x + z)$
III. $x @ (y @ z) = (x @ y) @ (x @ z)$
A. I only
B. II only
C. III only
D. I and III only
E. II and III only

16. A dart board is a regular octagon divided into regions. A dart lands uniformly on the board. What is the probability it lands in the center square?
A. $2 - \sqrt{2} / 2$
B. 1/4
C. $\sqrt{2} - 1 / 2$
D. $\sqrt{2} / 4$
E. $\sqrt{2} - 2$



17. In a circle, diameter EB ∥ DC and AB ∥ ED. $\angle AEB : \angle ABE = 4 : 5$. What is the measure of $\angle BCD$?
A. 120°
B. 125°
C. 130°
D. 135°
E. 140°



18. Rectangle ABCD has AB = 6, BC = 3. Point M on AB satisfies $\angle AMD = \angle CMD$. What is $\angle AMD$?
A. 15°
B. 30°
C. 45°
D. 60°
E. 75°

19. Find the product of all roots of $5|x| + 8 = x^2 - 16$.
A. −64
B. −24
C. −9
D. 24
E. 576

20. Rhombus ABCD has side = 2 and $\angle B = 120°$. Region R is the set of points inside closer to B than any other vertex. What is area(R)?
A. $\sqrt{3} / 3$
B. $\sqrt{3} / 2$
C. $2\sqrt{3} / 3$
D. $1 + \sqrt{3} / 3$
E. 2

21. Brian writes four integers $w > x > y > z$ with $w + x + y + z = 44$. Their pairwise differences are 1, 3, 4, 5, 6, 9. What is the sum of the possible w values?
A. 16
B. 31
C. 48
D. 62
E. 93

22. A pyramid has a square base (side 1) and equilateral triangle sides. A cube fits inside with one face on the base and the opposite face touching the lateral faces. What is the cube’s volume?
A. $5\sqrt{2} - 7$
B. $7 - 4\sqrt{3}$
C. $2\sqrt{2} / 27$
D. $2\sqrt{9} / 9$
E. $3\sqrt{9} / 9$

23. What is the hundreds digit of 2011^{11}?
A. 1
B. 4
C. 5
D. 6
E. 9

24. Line $y = mx + 2$ passes through no lattice points for $0 < x \leq 100$ if $½ < m < a$. Find the maximum a.
A. 51/101
B. 50/99
C. 51/100
D. 52/101
E. 13/25

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25. Triangle T_1 has sides 2011, 2012, 2013. For $n \ge 1$, $T_{\square+1}$ is formed from the incircle tangency triangle of $T_{\square}$. What is the perimeter of the last triangle ($T_{\square}$)?
A. 1509/8
B. 1509/32
C. 1509/64
D. 1509/128
E. 1509/256

Answer Key:

1. C
2. E
3. A
4. C
5. E
6. A
7. B
8. B
9. D
10. B
11. D
12. A
13. D
14. C
15. E
16. A
17. C
18. E
19. A
20. C
21. B
22. A
23. D
24. B
25. D

AI HOMEWORK

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AMC 2011A

1. A cell phone plan costs \$20 each month, plus 5¢ per text message sent, plus 10¢ for each minute used over 30 hours. In January Michelle sent 100 text messages and talked for 30.5 hours. How much did she have to pay?
A. \$24.00
B. \$24.50
C. \$25.50
D. \$28.00
E. \$30.00

2. A small bottle of shampoo can hold 35 milliliters of shampoo, whereas a large bottle can hold 500 milliliters. Jasmine wants to buy the minimum number of small bottles necessary to completely fill a large bottle. How many bottles must she buy?
A. 11
B. 12
C. 13
D. 14
E. 15

3. Suppose [a b] denotes the average of a and b, and {a b c} denotes the average of a, b, and c. What is {{1 1 0} [0 1] 0}?
A. 2/9
B. 5/18
C. 1/3
D. 7/18
E. 2/3

4. Let X and Y be the following sums of arithmetic sequences:
$X = 10 + 12 + 14 + \dots + 100$
$Y = 12 + 14 + 16 + \dots + 102$
What is the value of $Y – X$?
A. 92
B. 98
C. 100
D. 102
E. 112

5. At an elementary school, third graders, fourth graders, and fifth graders run an average of 12, 15, and 10 minutes per day, respectively. There are twice as many third graders as fourth graders, and twice as many fourth graders as fifth graders. What is the average number of minutes run per day by these students?
A. 12
B. 37/3
C. 88/7
D. 13
E. 14

6. Set A has 20 elements, and set B has 15 elements. What is the smallest possible number of elements in $A \cup B$ (the union of A and B)?

A. 5
B. 15
C. 20
D. 35
E. 300

7. Which of the following equations does not have a solution?
A. $(x + 7)^2 = 0$
B. $|-3x| + 5 = 0$
C. $-x - 2 = 0$
D. $x - 8 = 0$
E. $|-3x| - 4 = 0$

8. Last summer, 30% of the birds on Town Lake were geese, 25% were swans, 10% were herons, and 35% were ducks. What percent of the birds that were not swans were geese?
A. 20
B. 30
C. 40
D. 50
E. 60

9. A rectangular region is bounded by $y = a$, $y = -b$, $x = -c$, and $x = d$, where a, b, c, and d are positive. Which of the following represents the area of this region?
A. $ac + ad + bc + bd$
B. $ac - ad + bc - bd$
C. $ac + ad - bc - bd$
D. $-ac - ad + bc + bd$
E. $ac - ad - bc + bd$

10. A majority of the 30 students in Ms. Demeanor's class bought pencils at the school bookstore. Each bought the same number of pencils (>1). The cost per pencil in cents was greater than the number of pencils bought, and the total cost of all pencils was \$17.71. What was the cost per pencil in cents?
A. 7
B. 11
C. 17
D. 23
E. 77

AI HOMEWORK

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11. Square EFGH has one vertex on each side of square ABCD. Point E is on AB with AE = 7·EB. What is the ratio of the area of EFGH to the area of ABCD?
A. 49/64
B. 25/32
C. 7/8
D. $5\sqrt{2}/8$
E. 14/4

12. A basketball team made three-point, two-point, and one-point shots. They scored as many points from twos as from threes. Their number of free throws was one more than their twos. The team's total was 61 points. How many free throws did they make?
A. 13
B. 14
C. 15
D. 16
E. 17

13. How many even integers are there between 200 and 700 whose digits are all different and come from the set {1, 2, 5, 7, 8, 9}?
A. 12
B. 20
C. 72
D. 120
E. 200

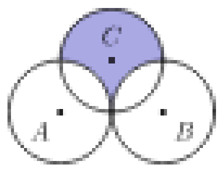
14. A pair of standard 6-sided dice is rolled once. The sum of the numbers rolled determines the diameter of a circle. What is the probability that the circle's area is less than its circumference?
A. 1/36
B. 1/12
C. 1/6
D. 1/4
E. 5/18

15. Roy's hybrid car ran only on its battery for 40 miles, then on gas for the rest of the trip, using 0.02 gallons per mile. He averaged 55 miles per gallon on the trip. How long was the trip?
A. 140
B. 240
C. 440
D. 640
E. 840

16. Which of the following equals $\sqrt{(9 - 6\sqrt{2})} + \sqrt{(9 + 6\sqrt{2})}$?
A. $3\sqrt{2}$
B. $2\sqrt{6}$
C. $7\sqrt{2}/2$
D. $3\sqrt{3}$
E. 6

17. In the sequence A, B, C, D, E, F, G, H, C = 5 and the sum of any three consecutive terms is 30. What is A + H?
A. 17
B. 18
C. 25
D. 26
E. 43

18. Circles A, B, and C each have radius 1. Circles A and B are tangent, and circle C is tangent to the midpoint of AB. What is the area inside circle C but outside A and B?
A. $(3 - \pi)/2$
B. $\pi/2$
C. 2
D. $(3\pi)/4$
E. $(1 + \pi)/2$



19. In 1991, a town's population was a perfect square. After 10 years and an increase of 150 people, it was 9 more than a perfect square. After another 150 in 2011, it became a perfect square again. What was the percent growth over 20 years?
A. 42%
B. 47%
C. 52%
D. 57%
E. 62%

20. Two points are chosen at random on a circle of radius r. From each, a chord of length r is drawn clockwise. What is the probability that the chords intersect?
A. 1/6
B. 1/5
C. 1/4
D. 1/3
E. 1/2

21. Two counterfeit coins are mixed with 8 genuine coins. Two pairs of coins are drawn successively without replacement. The total weight of the first pair equals that of the second pair. What is the probability that all 4 selected coins are genuine?
A. 7/11
B. 9/13
C. 11/15
D. 15/19
E. 15/16

22. Each vertex of convex pentagon ABCDE is assigned a color. There are 6 colors available, and the ends of each diagonal must have different colors. How many different colorings are possible?
A. 2520
B. 2880
C. 3120
D. 3250
E. 3750

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23. Seven students count from 1 to 1000 as follows: Alice says all numbers except skips the middle in each set of three (1,3,4,6,7,9…). Barbara says numbers Alice doesn't, skipping the middle of each triple. Each subsequent student does the same. What number does George, the seventh student, say? A. 37 B. 242 C. 365 D. 728 E. 998	24. Two distinct regular tetrahedra have all their vertices among the vertices of the same unit cube. What is the volume of their intersection? A. 1/12 B. 2/12 C. 3/12 D. 1/6 E. 2/6
25. Let R be a unit square and $n \geq 4$ an integer. A point X is n-ray partitional if there are n rays from X dividing R into n equal-area triangles. How many points are 100-ray partitional but not 60-ray partitional? A. 1500 B. 1560 C. 2320 D. 2480 E. 2500	Answer Key: 1. D 2. E 3. D 4. A 5. C 6. C 7. B 8. C 9. A 10. B 11. B 12. A 13. A 14. B 15. C 16. B 17. C 18. C 19. E 20. D 21. D 22. C 23. C 24. D 25. C

AI HOMEWORK

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AMC 2010B

1. What is $100(100-3)-(100\times100-3)$? A. −20,000 B. −10,000 C. −297 D. −6 E. 0	2. Makarla attended two meetings during her 9-hour work day. The first meeting took 45 minutes and the second took twice as long. What percent of her work day was spent attending meetings? A. 15 B. 20 C. 25 D. 30 E. 35
3. A drawer contains red, green, blue, and white socks with at least one of each color. What is the minimum number of socks that must be pulled to guarantee a matching pair? A. 3 B. 4 C. 5 D. 8 E. 9	4. For a real number x, define ♡(x) as the average of x and x^2. What is ♡(1)+♡(2)+♡(3)? A. 3 B. 6 C. 10 D. 12 E. 20
5. A month with 31 days has the same number of Mondays and Wednesdays. How many of the seven days could be the first day of this month? A. 2 B. 3 C. 4 D. 5 E. 6	6. A circle is centered at O. AB is a diameter and C is a point on the circle with $\angle COB = 50°$. What is the degree measure of $\angle CAB$? A. 20 B. 25 C. 45 D. 50 E. 65
7. A triangle has side lengths 10, 10, and 12. A rectangle has width 4 and area equal to that of the triangle. What is the perimeter of this rectangle? A. 16 B. 24 C. 28 D. 32 E. 36	8. A ticket to a school play costs x dollars (x is a whole number). A group of 9th graders buys tickets totaling $48, and a group of 10th graders buys tickets totaling $64. How many values for x are possible? A. 1 B. 2 C. 3 D. 4 E. 5
9. Lucky Larry substituted numbers for a, b, c, d, and e in the expression $a - (b - (c - (d + e)))$ and ignored parentheses but coincidentally got the correct result. He used a=1, b=2, c=3, d=4. What number did he use for e? A. −5 B. −3 C. 0 D. 3 E. 5	10. Shelby drives 30 mph when not raining and 20 mph when raining. She drove 16 miles in 40 minutes total. How many minutes did she drive in the rain? A. 18 B. 21 C. 24 D. 27 E. 30

AI HOMEWORK

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11. A shopper plans to purchase an item priced over \$100 and can use one of three coupons: Coupon A: 15% off Coupon B: \$30 off Coupon C: 25% off the amount above \$100 Let x and y be the smallest and largest prices for which Coupon A saves at least as much as Coupon B or C. What is $y - x$? A. 50 B. 60 C. 75 D. 80 E. 100	12. At the start of the year, 50% of students said "Yes" to "Do you love math?", and 50% said "No." At year's end, 70% said "Yes" and 30% said "No." x% of students changed their answer. What is the difference between the maximum and minimum possible values of x? A. 0 B. 20 C. 40 D. 60 E. 80
13. What is the sum of all solutions of $x = \|2x - \|60 - 2x\|\|$? A. 32 B. 60 C. 92 D. 120 E. 124	14. The average of 1, 2, 3, …, 98, 99, and x is 100x. What is x? A. 49/101 B. 50/101 C. 1/2 D. 51/101 E. 50/99
15. On a 50-question test: +4 points per correct, −1 per incorrect, 0 for blank. Jesse scored 99 points. What is the maximum number of correct answers? A. 25 B. 27 C. 29 D. 31 E. 33	16. A square of side 1 and a circle of radius $3\sqrt{3}$ share the same center. What is the area inside the circle but outside the square? A. $\pi/3 - 1$ B. $(2\pi/9) - \sqrt{3}/3$ C. $\pi/18$ D. 1/4 E. $2\pi/9$
17. Each high school in Euclid sent a 3-student team to a math contest. Each student had a unique score. Andrea's score was the overall median and highest on her team. Beth and Carla placed 37th and 64th. How many schools are in the city? A. 22 B. 23 C. 24 D. 25 E. 26	18. Positive integers a, b, and c are chosen randomly from {1,2,…,2010}. What is the probability that abc + ab + a is divisible by 3? A. 1/3 B. 29/81 C. 31/81 D. 11/27 E. 13/27
19. A circle has area 156π. Triangle ABC is equilateral, BC is a chord, $OA = 4\sqrt{3}$, and O is outside the triangle. What is the side length of the triangle? A. $2\sqrt{3}$ B. 6 C. $4\sqrt{3}$ D. 12 E. 18	20. Two circles lie outside a regular hexagon ABCDEF. One is tangent to AB, the other to DE. Both are tangent to BC and FA. What is the ratio of their areas (larger/smaller)? A. 18 B. 27 C. 36 D. 81 E. 108

AI HOMEWORK

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21. A palindrome between 1000 and 10,000 is chosen at random. What is the probability it is divisible by 7?
A. 1/10
B. 1/9
C. 1/7
D. 1/6
E. 1/5

22. Seven distinct candies are distributed into three bags. The red and blue bags must each get at least one candy; the white bag may be empty. How many arrangements are possible?
A. 1930
B. 1931
C. 1932
D. 1933
E. 1934

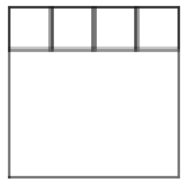
23. A 3×3 array contains digits 1–9 arranged so each row and column increases. How many such arrays exist?
A. 18
B. 24
C. 36
D. 42
E. 60

24. A basketball game was tied after Q1. Raiders' scores follow a geometric sequence; Wildcats' scores an arithmetic one. Raiders won by 1 point. Neither team scored over 100. What was the total number of points scored in the first half?
A. 30
B. 31
C. 32
D. 33
E. 34

25. Let P(x) be a polynomial with integer coefficients such that P(1)=P(3)=P(5)=P(7)=a and P(2)=P(4)=P(6)=P(8)=−a. What is the smallest possible value of a?
A. 105
B. 315
C. 945
D. 7!
E. 8!

Answer Key:
1. C
2. C
3. C
4. C
5. B
6. B
7. D
8. E
9. D
10. C
11. A
12. D
13. C
14. B
15. C
16. B
17. B
18. E
19. B
20. D
21. E
22. C
23. D
24. E
25. B

AMC 2010A

1. Mary's top bookshelf holds five books with the following widths (in centimeters): 6, 0.5, 1, 2.5, and 10. What is the average book width, in centimeters? A. 1 B. 2 C. 3 D. 4 E. 5	2. Four identical squares and one rectangle are placed together to form one large square as shown. The length of the rectangle is how many times as large as its width?  A. 5/4 B. 4/3 C. 3/2 D. 2 E. 3
3. Tyrone had 97 marbles and Eric had 11 marbles. Tyrone gave some of his marbles to Eric so that Tyrone ended with twice as many marbles as Eric. How many marbles did Tyrone give to Eric? A. 3 B. 13 C. 18 D. 25 E. 29	4. A book takes 412 minutes to read aloud. Each CD can hold up to 56 minutes of reading. Using the smallest number of CDs and filling each equally, how many minutes of reading will each CD contain? A. 50.2 B. 51.5 C. 52.4 D. 53.8 E. 55.2
5. The area of a circle whose circumference is 24π is $k\pi$. What is the value of k? A. 6 B. 12 C. 24 D. 36 E. 144	6. For positive numbers x and y, the operation $\spadesuit(x, y)$ is defined as $\spadesuit(x, y) = x - 1/y$. What is $\spadesuit(2, \spadesuit(2, 2))$? A. 3/2 B. 1 C. 4/3 D. 5/3 E. 2
7. Crystal runs 1 mile north, then 1 mile northeast, then 1 mile southeast. The last portion of her run takes her straight back to her starting point. How far, in miles, is this last portion? A. 1 B. $\sqrt{2}$ C. $\sqrt{3}$ D. 2 E. $2\sqrt{2}$	8. Tony works 2 hours a day and is paid $0.50 per hour for each full year of his age. Over 50 workdays he earned $630. How old was Tony at the end of this period? A. 9 B. 11 C. 12 D. 13 E. 14
9. x and $x + 32$ are palindromes, and x is three digits while $x + 32$ is four digits. What is the sum of the digits of x? A. 20 B. 21 C. 22 D. 23 E. 24	10. Marvin's birthday was Tuesday, May 27, 2008 (a leap year). In what year will his birthday next fall on a Saturday? A. 2011 B. 2012 C. 2013 D. 2015 E. 2017
11. The length of the interval of solutions to the inequality $a \le 2x + 3 \le b$ is 10. What is $b - a$? A. 6 B. 10 C. 15 D. 20 E. 30	12. A real water tower is 40 meters tall and holds 100,000 liters. Logan's scale model tower holds 0.1 liters. How tall (in meters) should Logan's tower be? A. 0.04 B. 0.4π C. 0.4 D. 4π E. 4

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13. Angelina drove at 80 km/h, stopped for 20 minutes, then drove at 100 km/h. The total trip was 250 km and lasted 3 hours (including the stop). Which equation can be used to solve for t, the driving time before stopping (in hours)? A. $80t + 100(8/3 - t) = 250$ B. $80t = 250$ C. $100t = 250$ D. $90t = 250$ E. $80(8/3 - t) + 100t = 250$	14. In triangle ABC, AB = 2·AC. Points D and E are on AB and BC, respectively, such that $\angle BAE = \angle ACD$. If $\triangle CFE$ is equilateral, what is $\angle ACB$? A. 60° B. 75° C. 90° D. 105° E. 120°
15. Four amphibians — Brian, Chris, LeRoy, and Mike — make these statements: Brian: "Mike and I are different species." Chris: "LeRoy is a frog." LeRoy: "Chris is a frog." Mike: "Of the four of us, at least two are toads." Toads always tell the truth; frogs always lie. How many of them are frogs? A. 0 B. 1 C. 2 D. 3 E. 4	16. In triangle ABC, BD is an angle bisector, AD = 3, and DC = 8. What is the smallest possible perimeter? A. 30 B. 33 C. 35 D. 36 E. 37
17. A cube has side length 3 inches. A 2×2 square hole is cut through the center of each face. What is the volume (in cubic inches) of the remaining solid? A. 7 B. 8 C. 10 D. 12 E. 15	18. Bernardo picks 3 distinct digits from {1–9} and forms a descending 3-digit number. Silvia picks 3 distinct digits from {1–8} and forms a descending 3-digit number. What is the probability Bernardo's number is larger than Silvia's? A. 47/72 B. 37/56 C. 2/3 D. 49/72 E. 39/56
19. In equiangular hexagon ABCDEF, AB = CD = EF = 1 and BC = DE = FA = r. If the area of $\triangle ACE$ is 70% of the hexagon's area, what is the sum of all possible values of r? A. $4\sqrt{3}/3$ B. 10/3 C. 4 D. 17/4 E. 6	20. A fly in a cube (1m per side) visits each corner exactly once and returns to the start. It travels only straight lines between corners. What is the maximum possible path length (in meters)? A. $4 + 4\sqrt{2}$ B. $2 + 4\sqrt{2} + 2\sqrt{3}$ C. $2 + 3\sqrt{2} + 3\sqrt{3}$ D. $4\sqrt{2} + 4\sqrt{3}$ E. $3\sqrt{2} + 5\sqrt{3}$
21. The polynomial $x^3 - ax^2 + bx - 2010$ has three positive integer roots. What is the smallest possible value of a? A. 78 B. 88 C. 98 D. 108 E. 118	22. Eight points are on a circle, and every pair of points is connected by a chord. No three chords meet at one interior point. How many triangles are formed with all three vertices inside the circle? A. 28 B. 56 C. 70 D. 84 E. 140

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23. There are 2010 boxes in a row. Box k ($1 \le k \le 2010$) has 1 red marble and k white marbles. Isabella draws one marble from each box in order, stopping when she draws a red one. Let P(n) be the probability she stops after exactly n draws. What is the smallest n for which $P(n) < 1/2010$? A. 45 B. 63 C. 64 D. 201 E. 1005	24. The number formed by the last two nonzero digits of 90! is n. What is n? A. 12 B. 32 C. 48 D. 52 E. 68
25. Jim repeatedly subtracts the largest perfect square less than or equal to the current number until reaching 0. His sequence has 8 numbers when starting at N. What is the units digit of N? A. 1 B. 3 C. 5 D. 7 E. 9	Answer Key: 1. D 2. B 3. D 4. B 5. E 6. C 7. C 8. D 9. E 10. E 11. D 12. C 13. A 14. C 15. D 16. B 17. A 18. B 19. E 20. D 21. A 22. A 23. A 24. A 25. B