Practice Problems

1. We have 2*14 + 14*2 = 28+28 = **$56**. Notice that by commutativity, the total cost of the bowls is the same as the total cost of the jars.
2. For Harry, order of operations gives us 8-(2+5)=8-7=1. For Terry, we have 8-2+5=6+6=11. We then subtract 1-11=**-10**.
3. Let l and w be the length and the width. We are given that l+w=16 and l=3w. We can substitute the second equation into the first and solve to get w=4 and l=12. The rectangle area formula then tells us that the desired area is lw=4*12=**48**.
4. Because of commutativity and associativity, we can change the order that we add the terms in and group them as follows: (1-2)+(3-4)+...+(2017-2018)+2019 = (-1)+(-1)+...+(-1)+2019.. Since this is just adding 1006 -1's and a 2019, we end up with a result of -1006+2019 = **1013**.
5. Let l and w be the length and the width. We are given that l+w=16 and l=3w. We can substitute the second equation into the first and solve to get w=4 and l=12. The rectangle area formula then tells us that the desired area is lw=4*12=**48.**
6. We can rearrange the terms in order to create pairs of numbers that add to -1. We get
   $$(1-2)+(3-4)+(5-6)+...(2017-2018)+2019,$$
   This sum equals -1009 + 2019 = 1010, Therefore making the sum of this equation **1010**.
7. Plugging in b=-1 we get -1 = -1/a. Multiply both sides by a to get -a = -1 → a = **1**.
8. We know ⅓ = 4/12. This makes the right side of the equation 5/12 - 4/12 = 1/12. We also know that ¾ = 9/12. Adding ¾ to both sides of the equation, we get x = 10/12 = ⅚.
9. Let $x = \sqrt{(3-2\sqrt{3})^2} + \sqrt{(3+2\sqrt{3})^2}$, then $x^2 = (3-2\sqrt{3})^2 + 2\sqrt{(-3)^2} + (3+2\sqrt{3})^2$.
   This simplifies to x^2 = 48, so x is **4sqrt(3)**. Alternatively, we may simply simplify the radicals, being careful that 2sqrt(3) is bigger than 3, so we have 2sqrt(3)-3 + 3+2sqrt(3), which is 4sqrt(3).
10. We can plug in 7 for b. This makes the equation 8a + 6 = 70 → 8a = 64 → a = 8. Since we now know that a = 8, we can plug in a=8 and b=3 into our equation to get our answer. This is 24 + 3 + 8 - 1 = **34**.

Challenge Problems

1. We simply add all the equations together to get 2(a+b+c)/2 = 3+4+5, which implies that a+b+c=**12**. Note: This problem, with equations in terms of every possible pair of variables, is an example of symmetry, which will be covered more in future material. When you notice symmetry in a problem, there are a bunch of useful tricks such as combining and substituting that can be applied to make it a lot easier. In fact, symmetry is so powerful that there are times you should look for ways to manipulate the problem to create symmetry. Similar Mathcounts-style problems can be found here.
2. Observing that 256 = 2^8 , and that 8=2*4, multiplication of exponents allows us to rewrite it as $2^8 = 2^{2*4} = (2^2)^4 = 4^4$, so **x=4**.
3. Subtracting the last two equations from each other gives us a=1, which we can plug into the first equation to get b+c=4, so d=3 from the second equation. We plug this into the fourth equation to get e+f = 8, so g=5. Since the absolute difference between a+g and a-g is 2g, we have 2*5=**10**.
4. We start by combining all of the terms into one larger term. We get

$$x + \frac{x^3}{y^2} + \frac{y^3}{x^2} + y = x + \frac{x^3}{y^2} + y + \frac{y^3}{x^2} = \frac{x^3}{x^2} + \frac{y^3}{x^2} + \frac{y^3}{y^2} + \frac{x^3}{y^2}$$

$$\frac{x^3+y^3}{x^2} + \frac{x^3+y^3}{y^2} = \frac{(x^2+y^2)(x^3+y^3)}{x^2y^2} = \frac{(x^2+y^2)(x+y)(x^2-xy+y^2)}{x^2y^2}$$

   Using the two givens in the problem, this is simplified to $x^2y^2 = 4.$

   and: $x^2 + y^2 = 20.$

   We can substitute these values back into

$$\frac{(x^2+y^2)(x+y)(x^2-xy+y^2)}{x^2y^2}$$

   We get 20*4*22/4 = **440**.
5. We know that 9 is (10 - 1), 99 is (10^2 - 1), 999 is (10^3 - 1) etc. Adding all the values in the equation gives us (10 + 10^2 + 10^3 ... 10^321)-321, because there are 321 terms in the expression. All of the powers of 10 add up to 111111...1110, with there being 321 1s before the final zero. We can subtract the 321 from this number. Subtracting 321 would only affect the last 4 digits, so we can just do 1110 - 321 = 789. This makes our final number 11111...10789. This is 318 ones and 7 + 8 + 9 = 24 more. This makes our final answer 318 + 24 = **342.**